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STRUCTURAL BEHAVIOUR OF PNEUMATIC CUSHIONS

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Summary. *Pneumatic air-inflated cushions are considered. The iteration technique for obtaining equilibrium pressure in their chambers is offered. It is based on the Broyden's method and uses the proposed function of cushion's surface and the differential equation of equilibrium. The technique for obtaining the maximum load, which may be applied on a cushion, is proposed. The research of the influence of temperature variations on the behavior of pneumatic cushions is executed.*

1 INTRODUCTION

Pneumatic cushions consist of polymer envelopes, made of ETFE films[1], forming cavities or chambers, filled with pressurized air. In spite of special equipment, intended for the pressure control inside a cushion, the efficiency of its operation and the reliability of exploitation require precise estimation of pressure changes. Moreover, complex external loadings, temperature variations and physical non-linear properties of the material have to be taken into account in order to achieve a workable construction without excessive stresses and deformations.

There are several approaches[2, 3] intended for analytical estimation of stresses in cushion's envelopes and for the calculation of variations of the internal pressure inside its chambers. On the other hand, the complex nature of cushions, namely the relationship between stresses in longitudinal and transverse directions and deviation of a cushion's cross section from an arc of a circle, require elaboration of an updated procedure for their analysis.

The technique for obtaining equilibrium pressure in cushion's chambers is offered. It is based on the gas law and uses the numerical method for solving the system of implicitly given equations. The technique also uses the proposed function of the envelope's surface with the technique for obtaining its parameters. The latter uses the differential equation of equilibrium[4], the method of least squares and the linearization procedure with Taylor series.

Pneumatic cushions, made of ETFE envelopes arranged on rectangular plan, are considered. The cushions are in the Cartesian coordinate system. The X-axis coincides with the long side of the cushion, while the Y-axis coincides with the short one.

2 EQUILIBRIUM OF PNEUMATIC CUSHIONS

2.1 The pressure in a loaded cushion

The air, trapped in a pneumatic cushion, equally presses its envelopes, while external loads directly influence the upper one only (figure 1). In fact, the remaining envelopes also depend on the loads indirectly, due to variations of the pressure inside cushion's chambers.

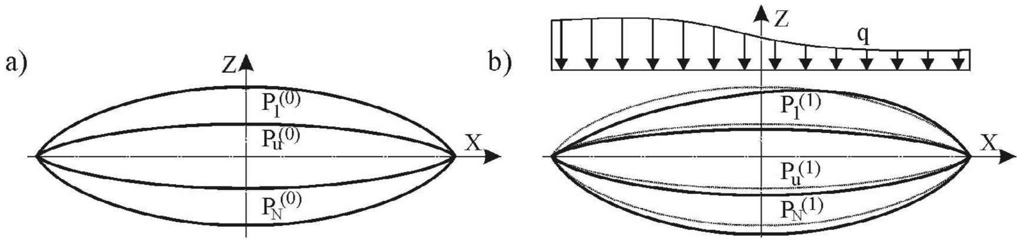

Figure 1. Cross section of a multilayered air inflated cushion: a – internal pressure acts only; b – external load is applied; $P_u^{(0)}$ and $P_u^{(1)}$ are the pressures in chamber u of the cushion in unloaded and loaded conditions; N is the number of chambers

The internal pressure obeys the gas law:

$$P^{(1)} = \left(P^{(0)} + P_a\right) \cdot \frac{t_1 - T_{abs}}{t_0 - T_{abs}} \cdot \frac{V^{(0)}}{V^{(1)}} - P_a , \tag{1}$$

where $P^{(0)}$ is an initial excessive pressure in the chamber, having the volume $V^{(0)}$ at temperature t_0 ,^0C; $P^{(1)}$ is an excessive pressure in the deformed chamber, with the volume $V^{(1)}$ at temperature t_1 ,^0C; $T_{abs} = -273.15\ ^0C$; P_a is an atmospheric pressure, in most cases it is equal to 100 kPa.

The new volume $V^{(1)}$ depends on the shape of cushion's envelopes, which are deformed under the influence of external loads and due to variations of pressure and temperature. For any given pressure P^g, changing the volume $V^{(1)}$, there is a discrepancy Θ in comparison to the equilibrium pressure $P^{(1)}$:

$$\Theta = P^g - P^{(1)} . \tag{2}$$

The proposed iteration technique (figure 2), based on the Broyden's method[5], allows to obtain the pressure in cushion's chambers and to minimize the error (2). The technique includes numerical calculation of partial derivatives A of the error Θ with respect to the given pressure P^g by adding a tiny variation to the pressure in the corresponding chamber and by calculating the variations of the discrepancy.

For multilayered cushions, [A] is a square matrix of $A_{u,w}$ components and $\overrightarrow{P^g}$, $\overrightarrow{P_{init}}$, $\overrightarrow{P^{(1)}}$, $\overrightarrow{V^{(1)}}$, $\overrightarrow{\Psi_k}$, $\overrightarrow{\Theta}$, $\overrightarrow{\Delta p}$ are vectors of u elements, where u and w are numbers of chambers:

$u \in [1,\ N]$, $w \in [1,\ N]$. Initial pressures are equal to pressures in unloaded cushion: $P_{init,u} = P_u^{(0)}$. In case of a single-layered cushion, which consists of only one chamber ($N = 1$), the vectors and matrixes in figure 2 become scalars and for $k > 0$ the value of A is the following: $A = Y / \Delta p$.

In order to secure the reliability of the iteration process, the correction of pressures should be confined with the parameter λ, (e.g. $\lambda = 0.2$). The limit value ε of the relative discrepancy is used to stop the process and to get the result.

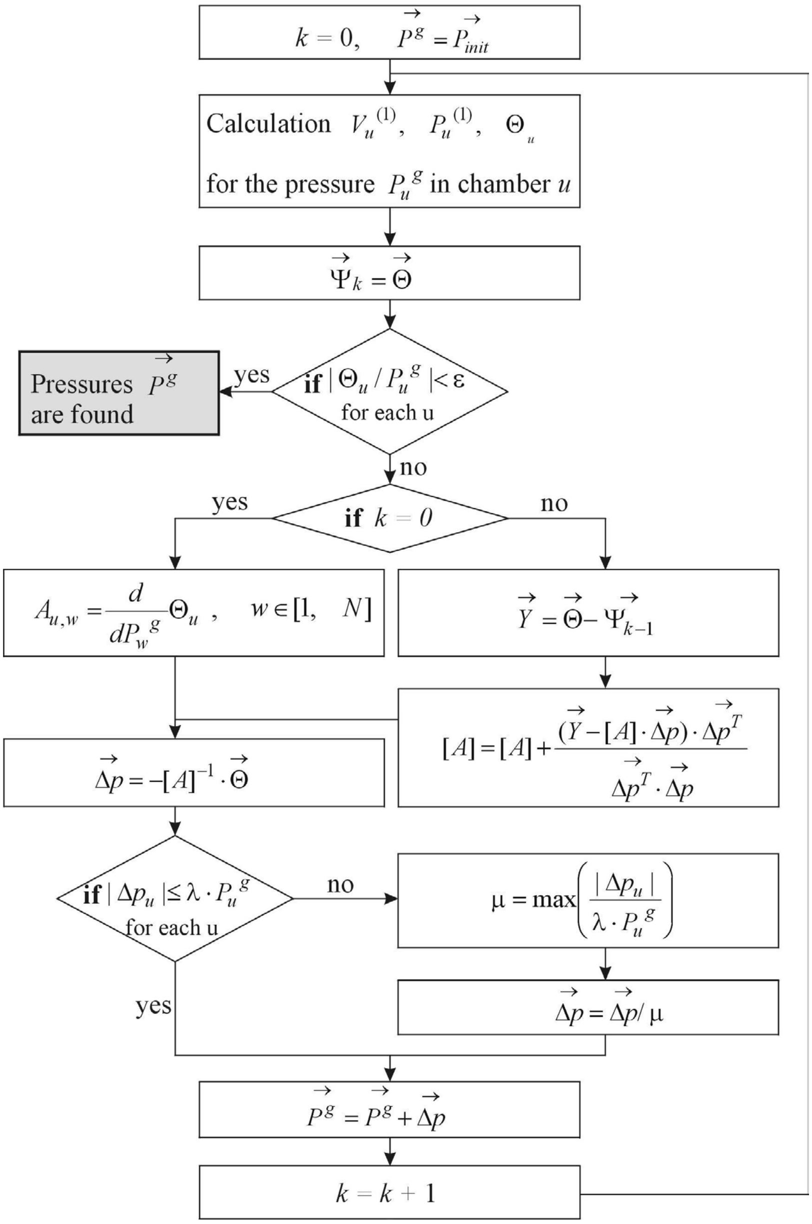

Figure 2. The iteration technique for obtaining the equilibrium pressure in a cushion

The volumes ${V_u}^{(1)}$ are calculated as follows:

$$ {V_u}^{(1)} = \pm V^{(e)}{}_s - (\pm V^{(e)}{}_{s+1}), \tag{3} $$

where $V^{(e)}$ is a volume, between the corresponding envelope and the plane *XOY*; u is a number of the chamber, for which the volume is calculated; s and $s+1$ are numbers of envelopes, which surround the corresponding chamber, $s = u$; the sign "+" is applied for envelopes, situated above the plane *XOY*, and the sign "-" is applied for envelopes, situated below it.

Obtaining of the volume V^e is performed by integration of the function of surface $Z(x, y, \overrightarrow{Hs})$ of the corresponding envelope, where x and y are coordinates in plane *XOY* and $\overrightarrow{Hs}$ is a vector.

Vector $\overrightarrow{Hs} = (H_0, H_{05}, H_{10}, H_{105}, V_x, V_y, V_{xy})^T$ contains parameters of the function, describing the shape of the surface of the envelope. The first four of them relate to uniformly distributed loads, while the remaining three parameters describe inverse-symmetric loads. It is obvious, that the whole set of these seven parameters is only used for the upper envelope, influenced by external loads. For all remaining envelopes, the last three components of $\overrightarrow{Hs}$ may be omitted.

For obtaining of $\overrightarrow{Hs}$ the differential equation of equilibrium[4] is proposed to be used:

$$ F = \frac{A}{B} = 1, \tag{4} $$

where

$$ A = Z_{xx} \cdot \sigma_x \cdot \frac{\sqrt{1 + Z_y^2}}{\sqrt{1 + Z_x^2}} + Z_{yy} \cdot \sigma_y \cdot \frac{\sqrt{1 + Z_x^2}}{\sqrt{1 + Z_y^2}} + 2 \cdot Z_{xy} \cdot \sigma_{xy}, \tag{5} $$

$$ B = -(p_z - p_x \cdot Z_x - p_y \cdot Z_y) \cdot \sqrt{1 + Z_x^2 + Z_y^2}, \tag{6} $$

where Z_x, Z_y, Z_{xx}, Z_{yy}, Z_{xy} are partial derivatives, calculated numerically; σ_x, σ_y, σ_{xy} are stresses at a point of the envelope; p_x, p_y, p_z are total loads, acting on the envelope, including excessive pressure P^g and external loads q_x, q_y, q_z.

The non-linear equation (4) is linearized with the Taylor series and the solution is found with the method of least squares.

2.2 Examples

Rectangular cushions with sizes 3 x 12 m and 4 x 8 m are considered. The thickness of the envelopes' film is 0.25 mm. Dependencies of tensile stress vs. strain[6] for the material are in figure 3. The coefficient of linear expansion of the membrane is assumed[7] the following: $\alpha = 1.31 \cdot 10^{-4}\ 1/{}^0C$.

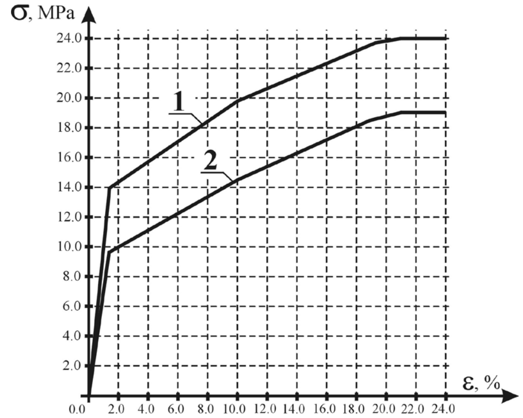

Figure 3. The diagrams of the dependencies of tensile stress vs. strain for ETFE[6]: curve 1 is for temperature 23^{0}C, curve 2 is for temperature 50^{0}C

2.2.1 Single-layered cushion at variable temperature

The cushion consists of two films, separated by an air gap. Its dimensions are 3 x 12 m. It is inflated at the room temperature ($t_0 = 23^0$ C) and exploited at an elevated temperature $t_1 = 50^0$ C. The initial pressure in the chamber is $P^{(0)} = 1.0$ kPa. The envelopes are initially equally stressed up to 3 kN/m. No external load influences the cushion.

The new pressures ($P^{(1)}{}_{dif} = 0.7943$ and $P^{(1)}{}_{eas} = 0.8102$ kPa) in the heated cushion are found in accordance with the proposed iteration technique (figure 2). The first value ($P^{(1)}{}_{dif}$) is calculated by numerical solution of the differential equation of equilibrium (4). The second one ($P^{(1)}{}_{eas}$) is obtained with the special computer program EASY. The discrepancy between them is 2%. Cross sections of the upper envelope of the cushion are in figure 4.

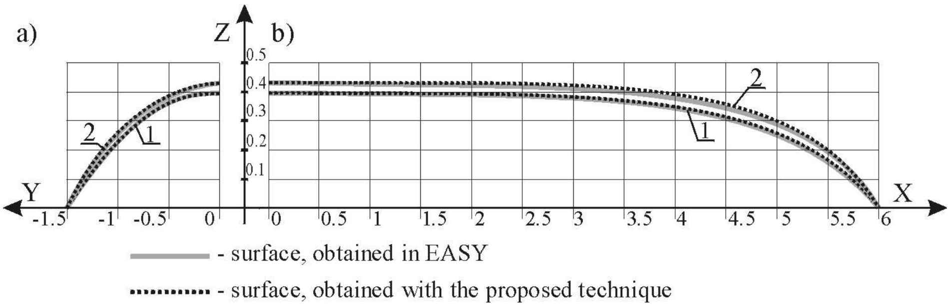

Figure 4. Cross sections of the upper envelope of the cushion, which is inflated and exploited at different temperatures: 1 – curves for the temperature t_0; 2 – curves for the temperature t_1

2.2.2 Two-layered cushion under non-uniform load

The cushion consists of three films. Its dimensions are 4 x 8 m. The cushion is inflated and exploited at equal temperature 23^{0}C. The initial pressures are the following: $P_1^{(0)} = 0.9$ kPa – in the upper chamber and $P_2^{(0)} = 0.6$ kPa – in the lower one. Every envelope is initially equally stressed up to 2kN/m. The upper envelope is influenced by non-uniform load (figure 5). Cross sections of the loaded cushion are in figure 6.

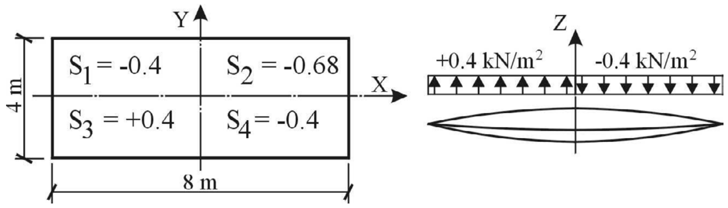

Figure 5. Loads on the upper envelope of the cushion, kN/m^2: positive value means that the load coincides with the vertical axis Z

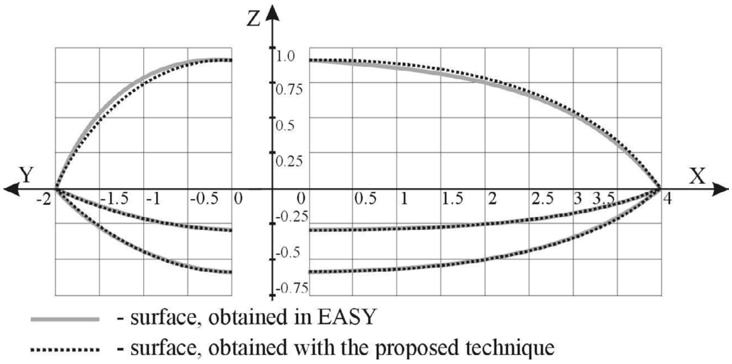

Figure 6. Cross sections of the loaded cushion

New pressures in the loaded envelope's chambers are found in accordance with the proposed iteration technique (figure 2). They are: $P_1^{(1)} = 1.0173$ kPa – in the upper chamber and $P_2^{(1)} = 0.684$ kPa – in the lower one. Discrepancies of these pressures and results obtained with the special computer program EASY are less than 3%.

3 ESTIMATION OF THE MAXIMUM LOAD, WHICH MAY BE APPLIED ON THE CUSHION

External load, applied on the cushion, changes stresses in its envelopes. If the resulting stresses approach to zero or exceed the strength of the polymer membrane, the load has to be confined.

Conditions of reliable behaviour of the envelope are the following:

$$\Omega^{(1)} = 1 - \sigma_{\min} / \sigma_{\lim,\min} \le 0 , \tag{7}$$

$$\Omega^{(2)} = 1 - \sigma_{\lim,\max} / \sigma_{\max} \le 0 \tag{8}$$

where $\sigma_{\min}$, $\sigma_{\max}$ are the minimum and maximum stresses in the envelope of the loaded cushion; $\sigma_{\lim,\min}$, $\sigma_{\lim,\max}$ are permissible stresses: $\sigma_{\lim,\min}$ ensures the absence of wrinkles on the surface, while $\sigma_{\lim,\max}$ depends on the tensile strength of the polymer membrane.

The iteration technique for obtaining the maximum vertical load *qz*, which may be applied on the cushion, is in figure 7.

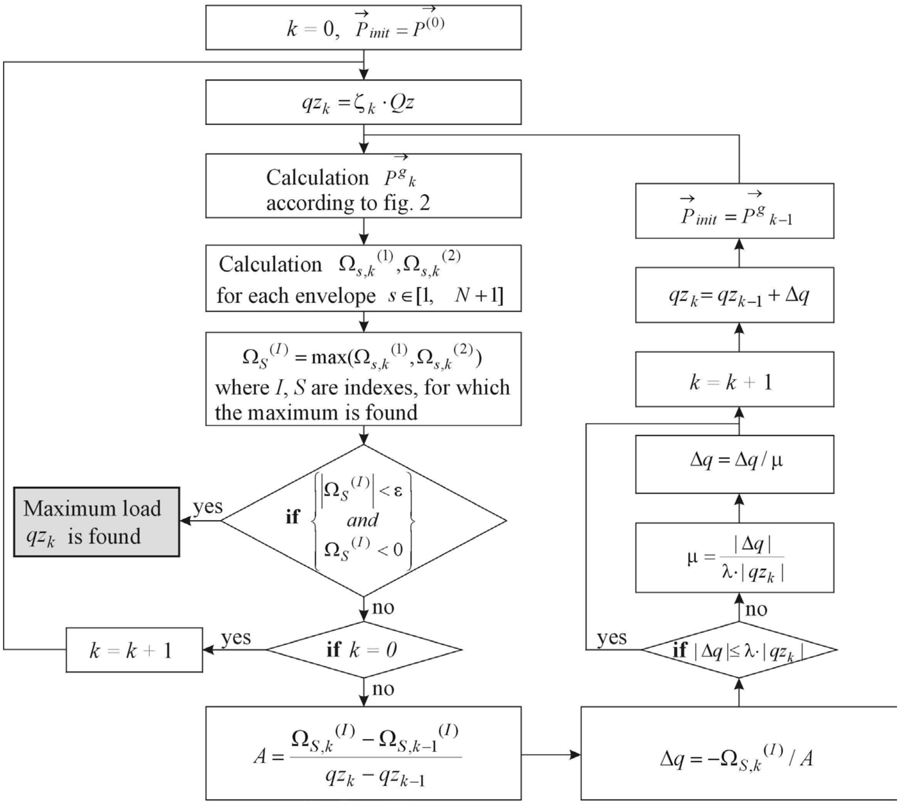

Figure 7. The iteration technique for estimation of the maximum load on the cushion

In the figure, the parameter ε is a limit value of the discrepancy and λ is a limit value for correction of the load, permissible at iteration. The values of the load (qz_0 and qz_1), calculated from preliminary given approximation *Qz*, are needed to obtain the initial value of derivative *A*. Coefficients ζ_0 and ζ_1 may be assumed equal to 0.97 and 1.0 accordingly.

Equilibrium pressures $\vec{P^g}_k$ at the iteration *k* are calculated according to figure 2, using the initial pressures $\vec{P}_{init}$.

4 RESEARCHES

4.1 Research of the maximum load, which may be applied on the cushion

Single-layered cushion with dimensions 3 x 12 m is considered. It is inflated and loaded at the room temperature ($t_0=23^0$ C). The permissible stresses in cushion's envelopes are assumed the following: $\sigma_{\lim,\min}=0.25$ and $\sigma_{\lim,\max}=4.0$ kN/m. Initial stresses σ_{init} in the envelopes of the unloaded cushion are in the range: $[1.0;\quad 3.0]$ kN/m. Initial internal pressures $P^{(0)}$ in the cushion's chamber are assumed according to (9):

$$P^{(0)}=k_{\sigma}\cdot\sigma_{init}\,, \tag{9}$$

where k_σ are the following: 1/3, 1/2, 2/3 m^{-1}.

It is assumed, that loads are uniformly distributed on the upper envelope only. They are directed opposite to the Z-axis (figure 1,b). Initial approximation of the loads is assumed the following: $Qz = 0.75 \cdot P^{(0)}$.

The graphs of maximum loads qz (kN/m^2), obtained in accordance with figure 7, are shown in figure 8,a. Equilibrium pressures P^g in the loaded cushion are illustrated in figure 8,b by means of the ratio k_P:

$$k_P = P^g / P^{(0)}. \tag{10}$$

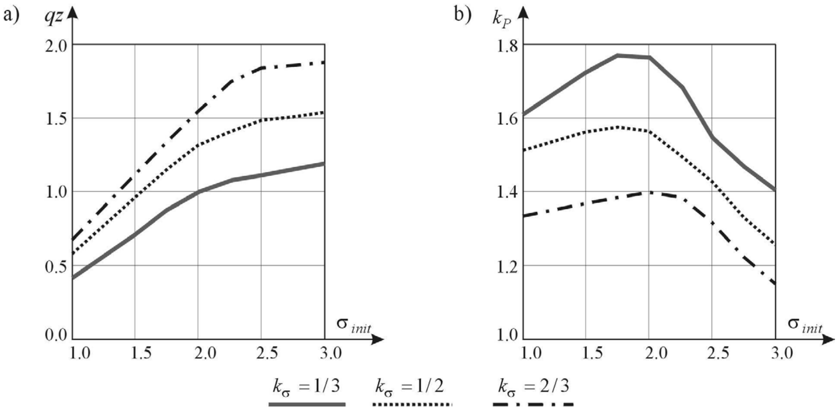

Figure 8. Maximum loads on the cushion:
a – load values qz; b – ratios k_P

Figure 8 shows the following:

- the increase of the initial stress σ_{init} and initial pressure $P^{(0)}$ brings about the growth of absolute values of loads qz, which may be applied on the cushion;
- on the other hand, the smaller the initial pressure $P^{(0)}$, the more substantially the equilibrium pressure P^g grows.

4.2 Research of the effect of temperature variations on the behavior of pneumatic cushions

Temperature variations influence pneumatic cushions by the following manner:

- the diagram of dependence of stress vs. strain for a polymer material depends on temperature (figure 3);
- additional deformations due to the expansion of the material occur;
- the air, trapped in a pneumatic cushion, obeys the gas law (1).

The influence of temperature on the cushion may be estimated with the following ratios:

$$K_H = H_{t1} / H_{t0}, \tag{11}$$

$$K_{\Pr} = P_{t1} / P_{t0}, \tag{12}$$

$$K_\sigma = \sigma^{\max}{}_{t1} / \sigma^{\max}{}_{t0}, \tag{13}$$

where H, P and $\sigma^{\max}$ are the height, internal excessive pressure and maximum stress in the cushion, accordingly; indexes $t0$ and $t1$ correspond to the initial temperature of inflation and the temperature of exploitation of the cushion.

It is assumed, that single-layered cushions with the dimensions 3 x 12 m are inflated at temperature 23^{0}C and exploited in the temperature range: $[-30,\ \ +50]^0$ C. For temperatures, not equal to 23 or 50^{0}C, the dependences of stress vs. strain are calculated with interpolation or extrapolation, using data of figure 3.

The initial internal pressure is $P_{t0} = 1$ kPa, while initial stresses σ_{t0} in cushions' envelopes are 2.0, 2.5, 3.0, 3.5 and 4.0 kN/m. No external load influences the cushions.

Graphs of ratios (11), (12) and (13), describing cushions' heights, new internal pressures and surface stresses are in figure 9. The research is executed with the help of the technique (figure 2), intended for obtaining the equilibrium pressure in cushion's chambers.

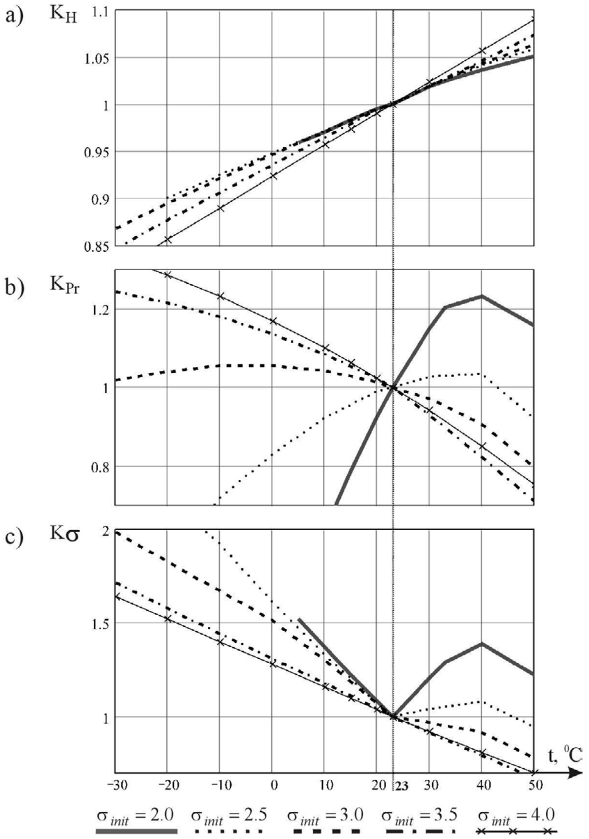

Figure 9. Behaviour of the cushions in case on non-constant temperature

Figure 9 shows that initial stresses in the cushion's envelopes influence the cushion's behaviour substantially:

- for relatively low initial stresses, namely 2.0 and 2.5 kN/m, the behaviour of the cushion at low temperatures becomes instable due to diminishing of the internal pressure and local slack of the envelopes;

- on the other hand, in case of relatively increased initial stresses low temperature leads to the increase of the internal pressure;
- at the temperatures less, than initial ($t_0 = 23^0$ C), the maximum stresses occur in the *X*-direction, while at temperatures above t_0 - in the *Y*-direction (figure 9,c);
- the smaller initial stress, the more significantly the maximum stress increases and the minimum stress (not shown in the figure) decreases;
- in compliance with the growth of initial stresses, the curves tend to approach straight lines.

5 CONCLUSIONS

1. The iteration technique for obtaining the equilibrium pressure in multilayered pneumatic cushions is offered. It is based on the Broyden's method and uses the differential equation of equilibrium and the proposed function of the cushion's surface. The results of the technique are in a good coincidence with the special computer program EASY, which is intended for structural analysis of systems, made of flexible fabric membranes.
2. The technique for estimation of maximum loads, which may be applied on a cushion, is proposed.
3. The research of the influence of temperature variations on the behavior of pneumatic cushions is executed. It is found, that the temperature growth always causes the increase of cushion's height. The equilibrium pressure in a cushion and maximum stresses in its envelopes significantly depend on the initial stresses.
4. The research may be used for designing and further development of single and multilayered air-inflated cushions.

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