NEW QUASI-MONOLITHIC METHOD TO SOLVE DYNAMIC FLUID-STRUCTURE INTERACTION PROBLEMS ON MEMBRANES

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Summary. We present a new algorithm based on a quasi-monolithic approach to solve strongly coupled fluid-structure interaction problems. This approach is an implicit coupling adapted to a partitioned solver while conserving the property of convergence and stability of the monolithic approach1. The coupling is done between a finite element program ARA developed by K-Epsilon and the Reynolds-averaged Navier-Stokes code, ISIS-CFD, part of the commercial software FINE/MarineTM. The fluid mesh is deformed using a fast, robust and parallelized method which propagates the deformation state. The mesh deformation is taken into account through the ALE method. Validation of the coupling was performed against the experimental results of a flapping membrane. Application of the coupling is made to compare the unsteady flying stability of two downwind sails using an automatic trimming algorithm. Future application to a respiring tube is discussed.

1 INTRODUCTION

Membranes are found frequently in applications where they interact with a fluid such as blimps, sails, tension based or inflated structures, parachutes, and biological flows such as hemodynamics1,2. In many of the above cases a premium is put on the structure being both light and strong. As light structures, the added mass of the fluid may be many times greater than the mass of the structure itself. As an example, the added mass of the air entrained by a gennaker, a type of downwind sail can be three orders of magnitude greater than the mass of the sail3. This added mass effect is destabilizing and dominates the numerical coupling between the fluid and structure problems. Gennakers pose a difficult test case as the flow is separated and they are inherently unsteady in their behaviour. Adding to this difficulty is the fact that the structure has almost no bending stiffness. This makes it a very difficult coupled problem as the sails undergo large deformations and are liable to collapse in on themselves which also challenges the fluid mesh deformation technique.

2 UNSTEADY FSI METHODOLOGY FOR MEMBRANE APPLICATIONS

In order to model the interaction of downwind sails it is necessary to have a strong coupling between the fluid and structure solver. K-Epsilon in cooperation with the DSPM group of Ecole Centrale de Nantes have made such a coupling. The coupling was made...
between the structure solver ARA, developed by K-Epsilon and ISIS-CFD, developed by ECN and sold commercially as part of the software FINE/Marine\textsuperscript{TM}.

4.2 Structure solver ARA

The solver ARA is based on a non-linear finite element formulation derived through the use of the virtual work principle. In order to represent a complete sailboat rig (spars, battens, shrouds and running rigging), cable elements, pulley and 3D Timoshenko beam elements were developed. Sliding and contact elements were developed using a penalization method.

Membranes are modelled using CST (Constant Strain Triangles) membrane elements within the finite strain theory. Large rotations and large strains are then accurately handled. Despite their simplicity (constant stresses, constant strains and uniform stiffness of the material for each element), this choice has proven to give a good trade-off between the accuracy and computational power required. An anisotropic composite material comprised of several layers may be imposed to model the stress-strain relationship of the membrane fabric.

Non-linearities coming from compression in the membranes are taken into account with a wrinkle model permits accurately resolving the local deformations of sails without having a huge number of elements. The model is based on a modification of the stress–strain tensor described in Nakashino and Natori\textsuperscript{4}, according to the definition of three states: taut state, where the sail is completely in tension, wrinkled state, where tension is restricted to one direction, and slack state, where the membrane is completely in compression. The modification leads to a consistent tangent stiffness matrix where changes in both the direction of wrinkling direction and the amount of wrinkling are taken into account.

Structural damping of the tissue is required to correctly capture the dynamic behaviour of a membrane. A Kelvin-Voigt type model while simple to implement has a behaviour which does not correspond closely to real tissues with regards to energy damping. In particular these models fail to capture the additional apparent stiffness which occurs when the material is subjected to a non-negligible velocity\textsuperscript{5}. An alternative model which accounts for variable time scales of the visco-elastic response is used instead. Details of the model developed are presented in Durand\textsuperscript{5}.

The temporal discretization is provided by a Newmark-Bossak scheme\textsuperscript{6} and the resolution is ensured by a Newton method through the computation of the tangent matrix associated with an Aitken relaxation.

4.2 Fluid solver ISIS

The presence of large separated flow regions violates the assumptions of potential flow and hence a viscous CFD approach is required. ISIS-CFD is an unsteady incompressible Reynolds-averaged Navier-Stokes (URANS) solver. The flow equations are resolved using an unstructured finite-volume method. Cells can be of arbitrary shape and number of faces. A 2-step Backward Differentiation Formula (BDF2) temporal scheme is used. During each time step an inner loop is used to perform the Picard linearization necessary to solve the system non-linearities. The velocity field is obtained momentum equation with the pressure field obtained by the mass continuity equation in the form of a pressure equation in a SIMPLE-like approach. Additional transport equations are solved for the turbulence model variables. All of
the cases presented have been performed with the $k-\omega$ SST model of Menter. An Arbitrary Lagrangian Eulerian (ALE) formulation is used to account for modifications of the fluid spatial domain due body deformation.

4.2 Fluid mesh deformation

In order to account for the considerable structural deformation while avoiding the need to remesh the fluid domain, a new mesh deformation tool was developed. The method is based on a combination of explicit advancing front method and smoothing. It is fully parallelized, and robustly able to accommodate large deformations of an unstructured mesh around multiple bodies like a gennaker and main sail interacting together. The explicit advancing front is based on a computation of the rigid rotation and displacement of each interface element. This rigid motion is then propagated from one cell layer to another all of the way to the boundaries of the fluid domain. This method is fast, however it requires a smoothing algorithm to take into account some cells far from the interfaces, where the propagation method is not well adapted. In some cases, a cell can be influenced by two different fronts of propagation with different deformations resulting in an unacceptable cell. To avoid this an explicit smoothing step based on a weighting neighbour deformation is carried out to improve the robustness and quality of the mesh.

4.2 Quasi-monolithic coupling K-FSI

The fluid-structure interaction between sails and wind is a difficult problem because it is strongly coupled. As stated previously, the added mass on a gennaker is typically three orders of magnitude larger than the mass of the structure. When the added mass effect is strong, weakly coupled methodologies classically used in aeroelasticity fail to reach a stable solution due to the fact that a large part of the fluid force depends on the acceleration of the structure. For such a case, even iterative partitioned approaches (also denoted block-iterative approaches) cannot provide a stable coupling within a reasonable CPU time. To achieve a stable and efficient coupling between the two solvers, the structural resolution is therefore integrated within the non-linear loop of the fluid solver, as was previously done for rigid bodies and bodies with imposed deformation respectively. The fluid non-linear loop becomes the FSI loop when the resolution of the structural part is included the fluid loop as shown in Figure 1. The structural solver is also modified to integrate the small time frame fluid response which is given by the added mass operator. When computing the added mass operator, a second approximation can be made without compromising the efficiency of the coupling: it is diagonalized. Physically, this is equivalent to computing the pressure response from a unit normal acceleration on each sail.
Although not truly monolithic, this algorithm is very stable, fast and parallelized. The number of FSI iterations to converge a time step is similar to the number of non-linear iterations for an unsteady fluid configuration without FSI. Indeed, it can be viewed as an approximated (and then iterative) block-LU factorization of the monolithic system.

Let us represent the linearized monolithic system as Eq.(1).

\[
\begin{bmatrix}
F & C_{fs} \\
C_{sf} & S
\end{bmatrix} \begin{bmatrix}
x_f \\
x_s
\end{bmatrix} = \begin{bmatrix}
s_f \\
s_s
\end{bmatrix}
\]  

(1)

Where \( F \) and \( S \) refer to the linearized fluid and structure operator, respectively, \( x_f \) and \( x_s \) represents the fluid and structure variables. The source term of both solvers are denoted by \( s_f \) and \( s_s \), for the fluid and structure domain, respectively. \( C_{fs} \) and \( C_{sf} \) refer to the coupling operator fluid to structure and structure to fluid, respectively. A block-LU factorization of this monolithic system leads to Eq.(2).

\[
\begin{align*}
(S - C_{fs} \cdot F^{-1} \cdot C_{sf}) \cdot x_s &= s_s - C_{fs} \cdot F^{-1} \cdot s_f \\
F \cdot x_f &= s_f - C_{sf} \cdot x_s
\end{align*}
\]  

(2)

By approximating the Jacobian operator of \( C_{fs} \cdot F^{-1} \cdot C_{sf} \) by the opposite of the added mass operator \(-Ma\) (Eq.(3)), it can be shown that the monolithic problem can be substituted by the iterative resolution of Eq.(4).

\[
\begin{align*}
(C_{fs} \cdot F^{-1} \cdot C_{sf}) \cdot x_s^{k+1} &= (C_{fs} \cdot F^{-1} \cdot C_{sf}) \cdot x_s^k - Ma \cdot (x_s^{k+1} - x_s^k) \\
(S + Ma) \cdot x_s^{k+1} &= s_s^k - C_{fs} \cdot x_f^k + Ma \cdot x_s^k \\
F \cdot x_f^{k+1} &= s_f^k - C_{sf} \cdot x_s^{k+1}
\end{align*}
\]  

(3)  

(4)
As a consequence, the block-LU factorization leads to the two steps of the proposed iterative algorithm, namely: a resolution of a modified structure problem and a resolution of the linearized fluid problem (i.e. one iteration of the non-linear loop).

3 VALIDATION

4.2 Experimental setup of oscillating membrane

In order to validate the code for dynamic membrane FSI a test case consisting of a piece of spinnaker sail cloth between two sail carbon fibre battens was made to oscillate with an imposed rotational velocity about one end of the cloth. The resulting deformations of both the sail cloth and battens was measured. The cloth motions were compared both with video and using a light planning technique to capture the shape along a slice of the cloth 640 mm from the luff of the sail cloth. The camera operated at a speed of 125 images per second. The batten deformations were measured using light emitting diodes attached to their ends. This allows indirect assessment of the fluid forces as they can be inferred by the tip deflections with known structural response. The imposed velocity profile was linearly varying between the maximum rotation velocities in each sense of rotation, resulting in a sinusoidal like rotation profile with maximum rotation angles of ±20°. The experimental setup with dimensions of the respective parts are shown in Figure 2. The structural properties of the cloth were determined by traction test in which a piece of the cloth had 1% tensile deformation imposed and the required force was measured. This was performed in the warp, weft and diagonal directions. The batten structural properties were obtained by performing flexion test on them to obtain EIₓ and EIᵧ with the axial and torsional stiffness derived from them. Further details of the experimental apparatus and setup are given in Durand⁵.

Figure 2: Flapping sail experimental setup
The resulting light plane slices for the start of the oscillatory movement are shown compared to the numerical displacements in Figure 3. It can be seen that in general there is very close agreement with the cloth displacement and curvature. There is some difference at \( t = 0.88 \text{ s} \) and \( t = 1.84 \text{ s} \) due to the appearance of particularly large wrinkles, but the overall shape is correctly maintained throughout.

![Figure 3: Light planning slice position at start of oscillations](image)

The position of the diodes on the ends of the battens permits measuring the tip deflections. The right diode displacements are compared over the periodic portion of the experiment against the numerical response in Figure 4. It can be seen that the maximum Y-displacements are well captured while the X-displacements are comparable with a slight overshoot at the extremes of the motion.
A gennaker is normally affixed to the yacht at the top of the mast and another point near the bow. A third point, called the clew has a line called a sheet attached to it to permit changing the shape of the sail by what is known as trimming the sail. K-FSI was developed with resolving the unsteady problem of a gennaker with an automatic sail trimming algorithm in mind.

Sail designers try to optimize the parameters to maximize the propulsive force, while keeping the most stable flying gennaker. The flying shape stability of the sail is an essential to its performance, and has a particularly large importance on single-handed boats. From a practical point of view, stability can be defined by sailmakers as the capability of the sail to maintain its trimmed shape. It has therefore the meaning of flying shape robustness, resistance to collapse, and minimal need for dynamic trimming. The leading edge of a trimmed gennaker is very light and has a periodic behaviour. When the sail is breaking (i.e. curling) on the luff (see Figure 5), a stable gennaker does not need to have the trim adjusted: it unfolds on its own. In the case of an unstable gennaker, a crew member must adjust the trim or bear away to unfold the gennaker. Unfortunately, this behaviour is very sensitive to wind variations and to the boat motions. This phenomenon cannot be quantified by standard stability assessment procedures. The criterion used here comes from the sailor’s perspective. This is the reason why a specific trimming procedure has also been developed in this study to mimic as much as possible the mechanism affecting the stability of the gennaker. The trimming algorithm used is akin to a PID operating on the gennaker sheet length with feedback from the sail leading edge flow velocity. For further details of the trimming algorithm the reader is referred to Durand et al³.

In this study, we investigate two real gennakers built, tested and used during the 2012-2013 Vendée Globe. The two gennakers are really close in terms of their design, but have different performances. Those differences are small, but significant for both sailors and sailmakers. The two sails are labelled here as gennakers A and B. These two gennakers have been digitized and then compared for one wind condition, taking into account the atmospheric boundary...
layer. The differences between the two sails in their geometry are slight as gennaker B is an evolution of gennaker A. Gennaker B has 1% less luff twist, 0.4% less luff roach, 1% less overall sail twist, a maximum camber 0.7% deeper and 1% further forward than gennaker A, but are otherwise they are identical in their moulded shape.

![Figure 5: Luff curling which precurses collapse requiring trimming](image)

## 4.2 Setup

In order to perform the FSI computation it is necessary to pass through a number of intermediary steps. The sail shape is given in the moulded shape in which it was constructed, however this does not correspond to the flying shape due to the fluid loads. To reduce the total mesh deformation and pass through the initial transient stage of the computation more quickly, a uniform pressure load is applied in ARA to the sail to deform it closer to a flying shape. The fluid mesh is then generated around the pre-deformed sail. An initial fluid computation is then performed to initialize the flow fields, before the FSI computation is launched.

The fluid mesh is an unstructured, fully hexahedral mesh. A graded refinement in the vertical direction is applied to resolve the atmospheric boundary layer. The wind enters the domain diagonally, hence two velocity inlet patches are used in conjunction with a zero pressure gradient condition at the top of the domain, slip condition at the sea level and two fixed pressure outlet patches. A wall function approach with a Y+ of 30 is used to provide the fluid boundary condition on the sail.

The fluid computation is initialized with a velocity distribution which accounts for the combination of the atmospheric boundary layer and the boat speed. A boat speed of 5.92 m/s is used in conjunction with a logarithmic boundary layer (Z0=0.002 m); true wind speed measured at 30 m is 7.72 m/s, true wind angle is 150 degrees. The apparent wind speed at z=15 m is about 2.6 m/s. The computations on both sails were run for 25 seconds in order to obtain a quasi-periodic behaviour.

## 4.2 Results

The trimming behaviour which the algorithm applied during the two computations are shown in Figure 6. Gennaker A is found to require significantly larger sheet changes than gennaker B. In order to quantify this an a-dimensional stability parameter S obtained by
dividing the sail height by the trimming amplitude. The term stability is used here from a practical point of view while sailing, with the meaning of flying shape robustness and its resistance to collapse, with a minimal need for dynamic trimming or over-sheeting.

The force in the three directions is given along with the stability parameter $S$ for both gennakers in Table 1. It apparent that gennaker B is far more stable while also generating slightly more propulsive force. However the side force is detrimental to performance. Hence a velocity prediction program (VPP) would be necessary to quantify this trade-off in greater detail to determine if the overall propulsive performance of gennaker B is better than gennaker A.

![Figure 6: Sheet length over time for the two gennakers: variations show the instability of the gennakers.](image)

<table>
<thead>
<tr>
<th></th>
<th>Gennaker A</th>
<th>Gennaker B</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propulsive force [N]</td>
<td>3625</td>
<td>3737</td>
<td>+3.1%</td>
</tr>
<tr>
<td>Side force [N]</td>
<td>1555</td>
<td>1684</td>
<td>+8.3%</td>
</tr>
<tr>
<td>Vertical force [N]</td>
<td>1223</td>
<td>1335</td>
<td>+9.2%</td>
</tr>
<tr>
<td>Stability parameter $S$</td>
<td>34</td>
<td>64</td>
<td>+85%</td>
</tr>
</tbody>
</table>

The average flying shape of the two sails is shown in Figure 7. It can be seen that the two sails differ visibly in their flying shapes. Gennaker A requires a greater average sheet length and hence takes a deeper shape.
1 WORKS IN PROGRESS

At the time of the paper submission, K-Epsilon is in the midst of further adapting the coupling of K-FSI to better handle internal flows such as the flexible, seawater filled, membrane tube in Figure 8. The tube is undergoing a rapid depressurization of the internal fluid with an external fluid present. The advancing pressure pulse from the lower end of the tube leads to a very strong added mass effect for which the presented method was not sufficiently well adapted. A new segregated monolithic approach is hence in development.
1 CONCLUSIONS

A new FSI coupling algorithm based on quasi-monolithic approach has been presented, validated and applied to strongly coupled membrane FSI. The algorithm is applied using a URANS fluid solution with ALE approach to handle the mesh deformation. A wrinkle model and visco-elastic damping model are used to accurately capture the membrane behaviour.

The tool was experimentally validated by the use of a periodically oscillating sail cloth. Very good agreement was found in the cloth displacements as well as the diode displacements. This indicates that both the membrane model and the FSI coupling are successfully recreating the experiment in terms of the fluid forces, the wrinkles and structural dampening of the cloth.

The tool was then applied to assess the stability and performance of two gennakers using a unique dynamic trimming algorithm. The tool is able to distinguish stability and performance characteristics between the two closely related sails. Gennaker B was found to require significantly less trimming and generates 3.1% more propulsive force, but also generates 8.3% more side force which is detrimental to yacht performance.

1 REFERENCES

