TENSILE LOADED MEMBRANE CONSTRUCTIONS WITHOUT CUTTING PATTERNS -
COMPUTER BASED MODELING AND ANALYSIS OF HIGH POINTS

STRUCTURAL MEMBRANES 2015

J. SANDMANN*, H. POEPPINGHAUS†

* Hochschule Konstanz- Technik, Wirtschaft und Gestaltung
Brauneggerstr. 55, 78462 Konstanz, Germany
e-mail: jusandma@htwg-konstanz.de, web page: http://www.htwg-konstanz.de

† IF Group, Ingenieure für Flächentragwerke GmbH
Am Dachsberg 3, 78479 Reichenau- Waldsiedlung, Germany
Email: poeppinghaus@if-group.de - Web page: http://www.if-group.de

Key words: membranes without cutting patterns puncture resistance test, stretch fabric.

Summary In this paper, the high points of a stretch-tent construction are analyzed by using two different software programs. The modeling and results are verified by a puncture resistance testing. The possibilities and limits of the software programs are shown as well as the further research opportunities.

1 ABSTRACT

The process of modeling and the structural analysis of light tensile structures is a special discipline in the field of civil engineering. It developed significantly over the last few years and increasingly gained acceptance among architects and engineers. Form-finding and calculation of complex surfaces is no longer limited to just a few specialists but widely available now.

A truly original example of a tensile loaded membrane structure without cutting patterns can be found in the “Black Tents”, the traditional shelter of the nomads in northern Africa and Asia. Adapted to the harsh conditions of the desert and the requirements of the nomads, the Black Tents are reduced to a minimum.

A modern example for the revival and further development of this ancient principle is the analyzed stretch-tent construction. The compression elements consist of central and perimetric poles. The latter are evenly distributed at the corners and boundaries of the cloth. The final shape is gained by putting up the poles and tightening the tension ropes. The membrane material is a composite that consists of two knitted fabrics that are stuck together by means of a flexible and watertight rubber-membrane. Like most of the commonly used textile materials, the mechanical properties of this stretch fabric are anisotropic. In addition to that, it shows a nonlinear deformation behavior and is characterized by its significant deformations.

Today’s software tools offer possibilities to create computational models that include the described material behavior and properties. Starting with the area around one inner high point
the aim is to establish a reasonably accurate model that reflects the strong deformations. Only when this step has been successfully undertaken it is possible to realize a sound analysis of the whole structure.

High stress values and locations of discontinuity due to the concentrated load application will occur around the supporting poles. To be able to verify the analytical results, extensive physical experiments were executed including a specially developed biaxial puncture resistance test.

The structural analysis was made by means of two different software programs. The software EASY by technet represents membranes by a cable net. It was necessary to adapt the form finding process and the particular material parameters to the characteristic properties of the stretch fabric. In addition to that, the accuracy of using a cable net as an approximation to the composite stretch material had to be verified.

The software straus7 by G+D Computing which works with the Finite Element Method (FEM) was also used to analyze the area around the high points. It offers possibilities to realize modeling approaches that include nonlinear behavior and strong deformations of lightweight structures. Modeling the described composite material however included simplifications that had to be evaluated.

As a last step, the results of the calculations using the two software programs were discussed and compared with the physical model. The possibilities and limits of the analysis and the resulting opportunities for further research are presented.

2 DEVELOPMENT AND CHARACTERISTICS OF MEMBRANE STRUCTURES

The development of membrane structures reaches back to the most original housing of humans: the tents. The nomads in northern Africa and Asia developed light tents that can be put up and taken down quickly. They were refined through the flexible use during the tribe’s journeys and adapt to the extreme conditions of the desert environment. Due to this fact, the material consists of the minimum required so that the tent is supported by just a few main poles. The tent cloth is laid over the poles and hold by the lateral tension ropes. Thus, the tents are very light and flexible.

For many years in Europe, tents were not considered as noteworthy in the field of architecture. A significant contribution to the development of the membrane structures and its introduction in architecture was achieved by Frei Otto and his team. These were the basics for today’s use of membrane structures as lightweight structures that can span over large distances without support. Their various applications reach from sports facilities to vertical facades.

The field of membrane constructions was established in the field of civil engineering from the 1950’s onwards but is still a very specific domain. Compared to conventional buildings, membrane materials possess special characteristics. A membrane has a small thickness compared to the dimensions of length and width and can only transfer tension loads. Therefore, the cross sections can be fully exploited which leads to extremely light structures. Pre-tensioning the membrane aims to ensure a continuous stabilization of the shape and has a
stiffening function that prevents folding and fluttering. As the geometry of the membrane structures and the load transfer are directly connected, anticlastic shapes are the natural consequence.

As a membrane is a composite material, it consists of various components that possess different material properties. It is composed of knitted or orthogonally woven fibers that are embedded in a matrix material—mostly a coating—that works as a protection against environmental impacts. The mechanical strength properties depend on the structure of the fabric, the strength and quantity of the fibers in warp and weft direction as well as the coating. Regarding the characteristic material parameters and their behavior during load impact, membrane materials differ a lot from commonly used materials. They show a complex, nonlinear and anisotropic material behavior, which depends of different parameters like the waviness and the interaction of the fibers, the level of the load impact and the past load history\textsuperscript{2, 3}.

3 ANALYZED TENT CONSTRUCTION

One example for the revival and further development of the lightweight structures is the tent construction whose high points were analyzed. Its design is based on the original model of the black tents. An example of a modern stretch-tent is shown in Figure 1.

![Figure 1: Example of a stretch-tent\textsuperscript{4}](image)

The primary and secondary structure of the stretch tent always work together. Therefore it is not possible to put up one part for itself due to the lack of stability. The final shape of the tent and the pre-stress of the fabric result from the tension forces applied by the boundary cables and tension ropes.

The use of an extremely stretchable membrane without any cutting pattern characterizes the modern stretch-tent construction. Another remarkable aspect are for sure the small heads of the poles that are placed directly under the stretched fabric.

4 NUMERICAL ANALYSIS OF THE HIGH POINTS

The theoretical part of this contribution aims to create a model and realize a structural analysis focusing on the tent’s high points which produce high stress values and locations of discontinuity due to the small load application surface. Numerical analyses were executed by
using two different software programs.

The assumptions and simplifications but also the possibilities and limits of these programs are shown. The results are verified by specially developed puncture resistance testing and are critically discussed.

4.1 Analysis of the material

The stretch material is a composite that consists of three parts. The outer layers are made out of polyester that shows a knitted structure. They are connected by a flexible, watertight rubber-membrane (see Figure 2 and Figure 3). The layer that faces the inner part of the tent shows a lower stiffness than its counterpart and possesses a finishing that protects the membrane against the influence of weather.

In order to be able to carry out the structural analysis, the material parameters of the membrane had to be defined. Therefore, a biaxial testing was executed that loaded the stretch material in two orthogonal directions, warp and weft (see Figure 4 and Figure 5). This should depict the biaxial stresses that the membrane undergoes during the load impact. Different approaches to realize the testing already exist but so far, there are no standards yet in Europe. As a guideline, the MSAJ/M-02-1995 of the Japanese Standard was used. The testing was load-controlled and the stress-strain behavior was documented.
The results show the typical material behavior of membranes combined with the extraordinary capability of stretching. Hence, the mechanical properties of the studied material are non-elastic and anisotropic. Therefore, a nonlinear stress-strain behavior can be observed. The deformation of the material is highly dependent on the previous loading and even after a long time of being unstressed, certain deformations remain in the material. Like most of the membrane materials, the slope of the stress-strain curve is low at the beginning and gets steeper when higher load levels are reached. This phenomenon can be explained by the initially curved fibers that first straighten before they stretch. This behavior is called crimp interchange.

4.2 Constitutive Laws

In order to describe the highly nonlinear, non-elastic material behavior, a hypo elastic material model should be used. Nevertheless, the most common approaches apply an approximation by using a linear elastic, orthotropic material law. There are different procedures to derive the material parameters like the „European Design Guide for Tensile Surface Structures“ published by the TensiNet Association\(^7\) or the Japanese standard MSAJ/M-02-19956. Based on the latter approach, the American Society of Civil Engineers\(^8\) also gives recommendations to determine the material parameters. Furthermore, there exist approaches developed from laboratories like the “Swiss Federal Laboratories for Materials Testing and research”\(^9\) or the laboratory of the DEKRA Industrial International GmbH, former Labor Blum\(^10\). For the following computations, two solutions were chosen that are described below.

Firstly, a linear elastic material model was selected that represents the base for various other approaches. It is represented by (1), where $S$ is the stiffness matrix.
The subscript 1 represents the warp, 2 the fill direction. The Modulus of Elasticity is $E_1$, respectively $E_2$, the Poisson’s ratio are $\nu_{12}$ and $\nu_{21}$. The stiffness matrix has to be symmetrical so that restriction (2) has to be respected.

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$

(2)

Due to the fact that the determinant of the stiffness matrix has to be unequal zero, restriction (3) has to be respected.

$$\nu_{21} < \sqrt{\frac{E_2}{E_1}}$$

(3)

To define the material parameters using the results of the biaxial testing, an optimization process was developed, using the method of the least squares. The material parameters based on the biaxial testing and the described approach are displayed in (4) to (7).

Moduli of elasticity: Poisson’s ratio:

$$E_1 = 48,40 \frac{kN}{m}$$

(4)

$$\nu_{12} = 0.5$$

(5)

$$E_2 = 37.61 \frac{kN}{m}$$

(6)

$$\nu_{21} = 0.36$$

(7)

The second chosen material model was developed by the Laboratory DEKRA, former Laboratory Blum (Linear elastic, combined\textsuperscript{10}). Usually, six different stress ratios are combined with the stress ratio 1:1 in order to define the material parameters. Due to the available biaxial testing, the load ratios 1:1 and 2:1 could be analyzed. The procedure is based on the same approach as the linear elastic material behavior. The results for the moduli of elasticity and the Poisson’s ratio are shown in (8) to (11).

Moduli of elasticity: Poisson’s ratio:

$$E_1 = 72,82 \frac{kN}{m}$$

(8)

$$\nu_{12} = 0.5$$

(9)
These two approaches, respectively the two sets of determined material parameters, demonstrate the great variety of constants that can be derived for one material. This variety is a common problem that also occurs using other approaches, which are confirmed by existing researches\textsuperscript{11}. Another aspect that is not respected using the mentioned approaches is the variation of the Moduli of elasticity. These values depend on the load level and are higher as the imposed strain increases.

Another way of displaying the behavior of membranes is the micromechanical approach that exactly displays the material behavior. Applying this approach demands a deep knowledge in this field and includes a number of variables that need to be defined. In the context of this contribution, the latter approach was not taken into consideration.

4.3 Software EASY

For the approximation via a cable net, the software EASY (release 2015), developed of the technet GmbH, was used. It was established for the proceedings of static analyses of membrane constructions.

The approximation of the membrane is realized by an orthogonally oriented cable net. The software works with the force-density method.

As the procedure of the analysis of the high points does not follow the usual process of computation, these steps had to be adjusted. Firstly, there are neither cutting patterns that have to be defined nor any initial geometry that has to be found. Therefore, the membrane was modeled in the first step of form-finding as a square field with the lowest possible pre-stress. In the area of the pole’s head, the net was refined. The resulting file had to be adjusted manually by moving the appropriate points on the desired level in order to model the surface of the pole’s head. The points are supported by struts and the above determined material parameters had to be defined. This procedure was made for all load steps.

The results of the last load step are shown in Figure 6 and Figure 7. The main stresses follow the orientation of the cable net and have a maximum value of 30kN/m (approach linear elastic material) to 40 kN/m (approach linear elastic, combined). The peaks are observed in the transition sector from the pole’s head to the tensioned membrane.

\[ E_2 = 53.7 \text{kN/m} \quad \text{(10)} \]
\[ \nu_{21} = 0.2 \quad \text{(11)} \]
The approximation of the membrane as a cable net represents the two main directions of the mostly orthogonally woven fabrics but cannot respect any shear forces. It is possible to define two values for the Moduli of elasticity which are the warp- and fill- direction. They can manually be adapted for different load cases. Further material parameters like the Poisson’s ratio as well as aspects like the nonlinear material behavior or the interaction of the yarns are not taken into consideration. The modeling of the high point could only be realized by a rough approximation using struts. Consequently, this software is a powerful tool for modeling whole membrane structures and includes all steps beginning from the form-finding up to the definition of cutting patterns. Though analyzing details as the examined high points that differ from the commonly used ones leads to simplifications whose consequences have to be evaluated.

4.4 Software Straus 7

On the other hand, Straus7 (release 2.4.6) by G+D computing that works with the finite elements method, has been used to focus on the analysis of the high points.

The high points of the tent construction are modeled by representation of only one fourth of the structure (see Figure 8 and Figure 9). This does not change the accuracy of the results but needs less computing time. The membrane has been modeled as a continuum by using 3D membrane plate elements. The net was refined and radially arranged in the area of the column head. Based on the above described constitutive laws, numerical analyses have been made using the two different sets of material parameters.

The modeling using 3D membrane plate elements allows involving several material parameters. The static analysis covers geometric nonlinearities. The modeling of the pole’s head as well as the contact formulations between the column head and the membrane can be calculated in a precise way. Therefore, it can be stated, that the model and the structural analysis using the FE-software lead to more accurate solutions compared to the approximation via cable nets- especially if one goes into the analysis of details.

5 VERIFICATION BY TESTING

In order to deduct the maximum breaking strength of the membrane in the area of the high
points, a machine was developed to execute punching shear testing (see Figure 10).

The membrane was cut into pieces of 1, 50 m x 1,50 m. A symmetrical grid of 10 cm x 10 cm in x-(blue), y’- (red, horizontal), y- (red, vertical), and d- (yellow, diagonal) direction was added on the surface (see Figure 11). The testing was undertaken in four load steps, expressed in a vertical displacement of the hydraulic pole. It was conducted in steps of 360 mm, 500 mm, 580 mm and 620 mm. The documentation includes the temperature, the time and the deformations during the four load steps. Hereby, the load was imposed, the deformations were measured and the material was given the possibility to adjust by letting pass some minutes before executing the next load step.

In total, five testing cycles were made. Three of them only consider the maximum breaking strength; two cycles were documented as described above. The material broke at an average force of 28 kN. The fracture appearance was highly similar at every sample- it broke at the transition sector of the column head to area of the tensioned membrane. The failure of the material appeared abruptly.

6 COMPARISON

Comparing the testing and the approximation via a cable net, the shape of the membrane in the lower load steps are similar. Coming to higher load levels, some differences can be observed. Having a look at Figure 12 where the last load step is shown. It can be seen that the curvature of the tested membrane is stronger than the nearly linear shape of the numerical result. This could be explained by tensile ring forces that exist in reality but are not modeled by the software. Also, the model of the pole’s head using beams is an approximation because the individual beams can move away from the centre of the pole’s head.

Concerning the strains, the expression of engineering strains was chosen. Straus 7 can report them directly. The measured strains of the physical model and the calculated strains of EASY were adapted to that definition.

The calculated strains in the area close to the pole’s head (x-direction) differ maximally around 4% from the measured strains. In diagonal direction, the differences are small, adding up to around 1%. This fact leads to the conclusion that the analyzed material barely possesses stiffness in that direction as modeled by the cable net. The two main directions y’ and y show the best results.
During the examination with Straus 7, analyses with full friction and without friction were made. Due to the fact that the results are very close, only the results with friction were considered in the following because there, the load-step curvature comes closer to the one of the physical model.

Having a look at the shape of the deformed membrane (see Figure 13), it can be seen that the modeling of the pole’s head perfectly works. It comes closer to reality but still, the curvature of the stretch material is higher.

Concerning the strains, the results for the linear elastic approach slightly differ more from the physical model than the linear elastic, combined approach. The best results for both approaches are in the main directions $y$ and $y'$ with differences up to around 0.4% ($Y1$, first load step). In x-direction, the results close to the pole show the greatest discrepancy with around 1%. Likewise, the diagonal direction d shows the greatest differences close to the poles during the last load step.

Concerning the supporting forces, the first approach of the linear elastic material behavior differs in the first load step 63% (EASY), respectively 72% (Straus 7) from the measured forces during the testing. Coming to higher load steps, the difference goes down to 28% (EASY), respectively 15.5% (Straus7) in the third load step. Better results could be obtained with the second, linear elastic, combined, approach; the differences are 46.6% (EASY) and 60% (Straus 7) in the first load step but go down to 9% (EASY) and 7.3% (Straus7) in the second load step. In Figure 14 to Figure 17, the resulting forces of the physical model compared with the software results are shown. It can be seen that the linear elastic, combined approach comes closer to the curve of the material testing but still, there is a high discrepancy.

In conclusion, it can be said that the two used software programs are powerful tools for numerical analyses. Compared with EASY, the FE-Software Straus 7 has some additional features to cover the material behavior and can model the pole’s head and the contact formulation more precisely. Nevertheless, one main problem can be found in the present deficiencies of determining the material parameters because of the over-simplification.
**OUTLOOK**

An objective for further works could be the more detailed examination and research of the constitutive laws to be able to formulate material laws that respect the material behavior of membranes in a more accurate way. The constitutive laws that are currently used for the calculation of the material parameters are a strong simplification of the real, highly nonlinear and non-elastic material behavior.
A further step could be the implementation in the already highly developed software programs in order to be able to make the static analysis in a more precise way. Having a look at the analysis of the studied tent construction, the whole structure including the high points could then be analyzed together.

A European design standard for membrane structures is currently under development. It is expected to establish guidelines for testing methods and standards for the analysis of membrane constructions, which leads to a found basis for defining the material parameters.

7 ACKNOWLEDGEMENT

The authors would like to thank Prof. Dr.-Ing. Roman Kemmler (Konstanz University of applied sciences, Faculty of civil engineering) for making this contribution possible with his steady support throughout the master thesis.

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