Learning to Skip State Updates in Recurrent Neural Networks

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Abstract

Recurrent Neural Networks (RNNs) continue to show outstanding performance in sequence modeling tasks. However, training RNNs on long sequences often face challenges like slow inference, vanishing gradients and difficulty in capturing long term dependencies. In backpropagation through time settings, these issues are tightly coupled with the large, sequential computational graph resulting from unfolding the RNN in time. We introduce the Skip RNN model which extends existing RNN models by learning to skip state updates and shortens the effective size of the computational graph. This network can be encouraged to perform fewer state updates through a novel loss term. We evaluate the proposed model on various tasks and show how it can reduce the number of required RNN updates while preserving, and sometimes even improving, the performance of the baseline models.

Keywords: machine learning, deep learning, recurrent neural networks, sequence modeling, conditional computation.
Resum

Les Xarxes Neuronals Recurrents (de l’anglès, RNNs) mostren un alt rendiment en tasques de modelat de seqüències. Tot i així, entrenar RNNs en seqüències llargues sol provocar dificultats com una inferència lenta, gradients que s’esvaeixen i dificultats per capturar depèndencies temporals a llarg terme. En escenaris amb backpropagation through time, aquests problemes estan estretament relacionats amb la longitud i la seqüencialitat del graf computacional resultant de desdoblar la RNN en el temps. Presentem Skip RNN, model que extén arquitectures recurrents existents, permetent-les aprendre quan ometre actualitzacions del seu estat i escurçant així la longitud efectiva del graf computacional. Aquesta xarxa pot ser estimulada per efectuar menys actualitzacions d’estat a través d’un nou terme a la funció de cost. Evaluem el model proposat en una sèrie de tasques i demostrem com pot reduir el nombre d’actualitzacions de la RNN mentre preserva, o fins i tot millora, el rendiment dels models de referència.

Paraules clau: aprenentatge automàtic, aprenentatge profund, xarxes neuronals recurrents, modelat de seqüències, computació condicional.
Resumen

Las Redes Neuronales Recurrentes (del inglés, RNNs) muestran un alto rendimiento en tareas de modelado de secuencias. Aún así, entrenar RNNs en secuencias largas suele provocar dificultades como una inferencia lenta, gradientes que se desvanecen y dificultades para capturar dependencias temporales a largo plazo. En escenarios con backpropagation through time, estos problemas están estrechamente relacionados con la longitud y la secuencialidad del grafo computacional resultante de desdoblar la RNN en el tiempo. Presentamos Skip RNN, un modelo que extiende arquitecturas recurrentes existentes, permitiéndoles aprender cuándo omitir actualizaciones de su estado y acortando así la longitud efectiva del grafo computacional. Esta red puede ser estimulada para efectuar menos actualizaciones de estado a través de un nuevo elemento en la función de coste. Evaluamos el modelo propuesto en una serie de tareas y demostramos cómo puede reducir el número de actualizaciones de la RNN mientras preserva, o incluso mejora, el rendimiento de los modelos de referencia.

Palabras clave: aprendizaje automático, aprendizaje profundo, redes neuronales recurrentes, modelado de secuencias, computación condicional.
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In the first place, I would like to thank my advisors: Shih-Fu, Brendan, Jordi and Xavi. This thesis would not have been possible without them. Thanks Shih-Fu for inviting me to develop this project in your lab at Columbia University, a experience I really enjoyed; Brendan, for your help and advice since my very first steps in deep learning research; Jordi, for giving me the opportunity to join BSC and the great years to come. Xavi, your devotion deserves a special mention. I am very grateful for it and the countless opportunities you always find for your students, making Barcelona a better place for deep learning research every single day.

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<td>NN</td>
<td>Neural Network</td>
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<td>RNN</td>
<td>Recurrent Neural Network</td>
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<td>LSTM</td>
<td>Long Short-Term Memory</td>
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<td>GRU</td>
<td>Gated Recurrent Unit</td>
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<td>GPU</td>
<td>Graphics Processing Unit</td>
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Chapter 1

Introduction

1.1 Motivation

Methods based on deep neural networks have established the state-of-the-art in computer vision tasks [32, 53, 23], machine translation [56], speech generation [43] and even defeated the world champion in the game of Go [50]. Although the development of these algorithms spans over many decades [35], their potential has been unlocked by the increased computational power of specific accelerators, e.g. Graphical Processing Units (GPUs), and the creation of large-scale datasets [15, 28]. Those readers who are not familiar with deep learning techniques may prefer to read Chapter 2 which gives an overview of the most basic concepts related to neural networks, before diving deeper in the contents of this thesis.

Recurrent Neural Networks (RNNs) have become the standard approach for practitioners when addressing machine learning tasks involving sequential data. Besides the increase in computational power and publicly available data, improved architectures and training algorithms have been key for the success of RNNs. Gated units, such as the Long Short-Term Memory [25] (LSTM) and the Gated Recurrent Unit [10] (GRU), were designed to deal with the vanishing gradients problem commonly found in RNNs [8] that precluded their application during years. These architectures have popularized thanks to their impressive results in a variety of tasks such as machine translation [5], language modeling [61] or speech recognition [20].

RNNs process data sequentially in order to capture temporal dependencies. This inherently sequential behavior results in challenging training and deployment, especially when dealing with long sequences. These challenges include throughput degradation, slower convergence during training and memory leakage, even for gated architectures [41]. Sequence shortening techniques, which can be seen as a sort of conditional computation [7, 6, 14] in time, can alleviate these issues. The most common approaches, such as cropping discrete signals or reducing the sampling rate in continuous signals, are heuristics and can be suboptimal. In contrast, we propose a model that is able to learn which samples (i.e. elements in the input sequence) need to be used in order to solve the target task. Consider a video understanding task as an example: scenes with large motion may benefit from high frame rates, whereas only a few frames are needed to capture the semantics of a mostly static scene.

The main goal of this project is the development of an RNN architecture that is able to decide
which samples in a sequence need to be used in order to solve the task at hand. Such subsampling mechanism needs to be integrated in the RNN itself, as opposed to a pre-processing step, and needs to be data-driven instead of being in heuristics. Moreover, the sample selection process has to be dynamic, i.e. each input sequence may be subsampled in a different way depending on their actual content. Our contributions include:

- A novel modification for existing RNN architectures that allows them to skip state updates, decreasing the number of inherently sequential operations to be performed, without requiring any additional supervision signal. This model, called Skip RNN, adaptively determines whether the state needs to be updated or it can be copied to the next time step, thus skipping computation. It can be seamlessly introduced into existing architectures to reduce the number of required RNN updates while preserving, and sometimes even improving, the performance of the baseline models.

- We show how the network can be encouraged to perform fewer state updates by adding a new penalization term during training, allowing to easily train models targeting different computational budgets.

- The proposed modification is implemented on top of well known RNN architectures, namely LSTM and GRU, and the resulting models show promising results in a series of sequence modeling tasks. In particular, the proposed Skip RNN architecture is evaluated on (1) the adding task, (2) sine wave frequency discrimination, (3) digit classification, (4) sentiment analysis in movie reviews, and (5) action classification in video.

This thesis is structured as follows. A brief introduction to deep learning is given in Chapter 2, which may come in handy for readers who are not familiar with the field. Chapter 3 provides an overview of the related work and techniques that bear some similarity with the proposed solution. The Skip RNN model is described in Chapter 4 and evaluated in a series of tasks in Chapter 5. Chapter 6 gives an estimation of the costs associated to the development of the project. Discussion and future research directions are provided in Chapter 7. Finally, appendices include additional qualitative results for the experiments and the pre-print uploaded to arXiv.

1.2 Hardware and Software Resources

This project was developed using the resources provided by the Barcelona Supercomputing Center. In particular, the MinoTaur cluster was been used. The cluster has 39 nodes equipped with two NVIDIA K80 dual-GPUs each, resulting in up to four concurrent processes running in parallel in each node. Having access to several nodes at the same time proved itself crucial for tuning hyperparameters and performing different runs for each experiment. Evaluating the proposed models on such a broad set of tasks would have been unfeasible without access to these computing resources.

Experiments were implemented with TensorFlow using CUDA and cuDNN for fast GPU primitives. This framework was chosen due to previous experience and the fact that it was already available in the MinoTaur cluster. Code is publicly available at https://github.com/imatge-upc/skiprnn-2017-tfm/
1.3 Work Plan

Being this a research-oriented project, it was difficult to define a detailed work plan ahead of time: the next steps to take were generally conditioned on the latest obtained results. Besides, since the field of deep learning is extremely active, new results are published and uploaded to arXiv every day and may affect the development of the project. Without going any further, the work that is most similar to ours \[59\] was published during the development of this project.

The project was organized as follows. After reading and understanding the works related with the goal of the project, the first step consisted in finding a task representing the problem being addressed. This ended up being the adding task in Section 5.1 which was used to evaluate different proposals until the final model presented in Chapter 4 was reached. This final model was the result of several iterations over a design-implement-evaluate-refine process. Once the adding task was solved without using all the input samples, it was time to evaluate the model on harder tasks. Finally, the last month was spent writing the scientific paper and this report.
Chapter 2

Introduction to Deep Learning

This chapter aims to introduce the reader to the most basic concepts in deep learning and, in particular, to the most common used recurrent models which are key to this work. These techniques have become the most common solution for most machine learning tasks in recent years and it is impossible to understand the latest advances in machine learning without deep learning. A detailed explanation of these methods would make this document excessively long and is out of the scope of this thesis. Among the numerous resources that have been published recently, we refer the reader to the book by Goodfellow et al. [18] for an extensive and comprehensive summary of the most important concepts in deep learning.

2.1 Feedforward Neural Networks

Artificial Neural Networks (NNs) have become the de facto solution for many Machine Learning tasks. Despite these methods were proposed decades ago [46, 35], their application to a wide range of problems is relatively recent and has been enabled by the creation of public large scale datasets and more powerful computing devices. In particular, the impressive results achieved by a NN-based model [32] in the 2012 edition of the ImageNet Large Scale Visual Recognition Challenge [15] kick started new research directions based on the usage of NNs.

Neural Networks can be seen as function approximators learning a mapping from their input to their output. In a nutshell, they consist of a stack of non-linear operations whose parameters are trained by means of Stochastic Gradient Descent (SGD) and backpropagation in order to minimize a training objective or cost. Despite achieving outstanding results, such method is very data inefficient and requires from large example collections to be effective. The non-linear nature of these of operations, also known as layers, is the key feature to produce powerful function approximators; in contrast, a stack of linear operations is mathematically equivalent to a single linear operation. A layer in a neural network can be modeled as follows:

\[ h = f(W_h x + b_h) \] (2.1)

where \( x \) is the input vector, \( f \) is a non-linear function (e.g. sigmoid) and \( W_h \) and \( b_h \) are trainable weights and biases, respectively. Additional hidden layers may be added between the input and the output to provide the model with an increased capacity.
2.2 Recurrent Neural Networks

Consider a sequence of input vectors, \( \mathbf{x} = (x_1, ..., x_T) \). The feedforward model described in Section 2.1 would output a sequence of activation vectors, \( \mathbf{h} = (h_1, ..., h_T) \), which are agnostic to previous activations. Recurrent Neural Networks (RNNs) extend the feedforward model by adding a recurrent connection in time:

\[
\mathbf{h}_t = f(W_h \mathbf{x}_t + U_h \mathbf{h}_{t-1} + b_h)
\]

(2.2)

where \( U_h \) operates on the hidden state in the previous time step, \( \mathbf{h}_{t-1} \), allowing information to persist. Providing models with memory and enabling them to model the temporal evolution of signals is a key factor in many sequence classification and transduction tasks where RNNs excel, such as machine translation [5], language modeling [61] or speech recognition [20].

Training recurrent models with backpropagation has been proven difficult for long sequences [8, 25]. The reason for such difficulty lies in the multiplicative dependency between activations over time, which easily leads to vanishing or exploding gradients. This lead to the development of gated units, which aim at alleviating these issues and making training for longer sequences feasible. The most common gated RNN architectures are the Long Short-Term Memory [25] and the Gated Recurrent Unit [10]. The following subsections describe the basics of these two architectures, and we refer the reader to [42] for an extended description and intuition behind them.

2.2.1 Long Short-Term Memory

The core idea behind Long Short-Term Memory (LSTM) is the so called cell state, where information is accumulated over time. Unlike the hidden state in vanilla RNN, the cell state in LSTMs is accessed through addition operations which do not suffer from the same problems as multiplicative relationships during gradient backpropagation. Its update equations can be defined as follows:

\[
\begin{align*}
i_t &= \sigma (W_{xi}x_t + W_{hi}h_{t-1} + b_i) \\
f_t &= \sigma (W_{xf}x_t + W_{hf}h_{t-1} + b_f) \\
c_t &= f_t \odot c_{t-1} + i_t \odot \text{tanh} (W_{xc}x_t + W_{hc}h_{t-1} + b_c) \\
o_t &= \sigma (W_{xo}x_t + W_{ho}h_{t-1} + b_o) \\
h_t &= o_t \odot \text{tanh} (c_t)
\end{align*}
\]

(2.3) - (2.7)

where \( i_t, f_t \) and \( o_t \) represent the input, forget and output gate at time \( t \), respectively, \( \sigma \) is the sigmoid function, \( \odot \) represents element-wise multiplication, \( W \) are trainable weight matrices and \( b \) are trainable bias vectors. The cell state at time \( t \) is represented by \( c_t \), whereas the hidden state (i.e. the cell output) at each time step is \( h_t \).

2.2.2 Gated Recurrent Unit

The Gated Recurrent Unit (GRU) can be understood as a simplified version of LSTM. The simplification is two-fold: (1) erasing information in the cell state is needed whenever a new value is written to it, and (2) the hidden state is directly the cell state. This is implemented
by combining the input and forget gates into a single gate and removing the output gate. The number of learnable parameters is reduced and inference is faster due to the fewer number of operations. Its update equations can be defined as follows:

\[
\begin{align*}
  z_t &= \sigma (W_{xz} x_t + W_{zh} h_{t-1}) \\
  r_t &= \sigma (W_{xr} x_t + W_{hr} h_{t-1}) \\
  \tilde{h}_t &= \tanh (W_{x\tilde{h}} r_t \odot x_t + W_{h\tilde{h}} h_{t-1}) \\
  h_t &= (1 - z_t) \odot h_{t-1} + z_t \odot \tilde{h}_t
\end{align*}
\]

where \(z_t\) and \(r_t\) are the update and reset gates, respectively, \(\sigma\) is the sigmoid function, \(\odot\) represents element-wise multiplication and \(W\) are trainable weight matrices. The hidden state, which in this architecture is merged with the cell state, is represented by \(h_t\).

### 2.2.3 Memory decay in RNNs

Despite providing an outstanding improvement over vanilla RNNs, gated units still have difficulties to remember information during long time lags. Neil et al. \cite{Neil} show how the memory decay rate is exponential for an LSTM on the simple task of keeping an initial memory state \(c_0\) for as long as possible. Assuming that no additional inputs are received (i.e. \(i_t = 0, \forall t\)) and that the forget gate is nearly fully-opened (i.e. \(f_t = 1 - \epsilon, \forall t\)), after \(n\) updates the cell state would contain:

\[
c_n = f_n \odot c_{n-1} = (1 - \epsilon) \odot (f_{n-1} \odot c_{n-2}) = \ldots = (1 - \epsilon)^n \odot c_0
\]

resulting in an exponential memory decay for \(\epsilon < 1\). Please note that the forget gate is the output of a sigmoid function, thus \(f_t < 1\) and memory decays with every time step. The same reasoning can be applied to GRU.
Chapter 3

Related Work

This chapter summarizes works that bear some similarity with the presented model. To the best of our knowledge, only LSTM-Jump [59] addresses a similar problem to the one discussed in this thesis. However, the Skip RNN model lies in the intersection of several techniques from the state of the art in machine learning, which are covered in the following sections.

3.1 Conditional computation

Conditional computation allows to increase model capacity without a proportional increase in computational cost by evaluating only certain computation paths for each input [7, 36, 2, 38, 47]. This concept has been extended to the temporal domain, either by learning how many times an input needs to be pondered before moving to the next one [19] or building RNNs whose number of layers depends on the input data [11]. Some works have addressed time-dependent computation in RNNs by updating only a fraction of the hidden state based on the current hidden state and input [27], or following periodic patterns [31, 41]. However, due to the inherently sequential nature of RNNs and the parallel computation capabilities of modern hardware, reducing the size of the matrices involved in the computations performed at each time step does not accelerate inference. The proposed Skip RNN model can be seen as form of conditional computation in time, where the computation associated to the RNN updates may or may not be executed at every time step. This is related to the UPDATE and COPY operations in hierarchical multiscale RNNs [11], but applied to the whole stack of RNN layers at the same time. Such difference is key to allow our approach to skip input samples, effectively reducing sequential computation and shielding the hidden state over longer time lags. Learning whether to update or copy the hidden state through time steps can be seen as a learnable Zoneout mask [33] which is shared between all the units in the hidden state. Similarly, it can be understood as an input-dependent recurrent version of stochastic depth [26].

3.2 Attention models and subsampling mechanisms

Selecting parts of the input signal is similar in spirit to the hard attention mechanisms that have been applied to image regions [40], where only some patches of the input image are attended in
order to generate captions [57] or detect objects [3]. In a similar fashion, Adaptive Computation Time [19] can be extended to residual networks [23] for image recognition, allowing some image patches to be processed only by a subset of layers [17]. Our model can be understood to generate a hard temporal attention mask on the fly given the previously seen samples, deciding which time steps should be attended and operating on a subset of input samples.

Subsampling input sequences has been explored for visual storylines generation [49], although jointly optimizing the RNN weights and the subsampling mechanism is computationally unfeasible and the Expectation Maximization algorithm is used instead. Similar research has been conducted for video analysis tasks, discovering Minimally Needed Evidence for event recognition [9] and training agents that decide which frames need to be observed in order to localize actions in time [58]. Motivated by the advantages of training recurrent models on shorter subsequences, efforts have been conducted towards learning differentiable subsampling mechanisms [45], although the computational complexity of the proposed method precludes its application to long input sequences. In contrast, our proposed method can be trained with backpropagation and does not degrade the complexity of the baseline RNNs.

3.3 LSTM-Jump

With the goal of speeding up RNN inference, LSTM-Jump [59] augments an LSTM cell with a classification layer that will decide how many steps to jump between RNN updates. Despite its promising results on text tasks, the discrete nature of the jumping actions makes the optimization process more complex. While the LSTM parameters, $\theta_m$, can be directly optimized by backpropagation, the parameters of the new jumping layer, $\theta_a$, need to be trained with REINFORCE [55] on a user-defined reward signal. Determining such reward signal is not trivial and does not necessarily generalize across tasks, e.g. regression and classification tasks may require from different reward signals. In its simplest form, the loss function being minimized is of the form

$$L(\theta_m, \theta_a) = L_m(\theta_m) + L_a(\theta_a)$$

where $L_m$ is the loss function that would be used with the baseline LSTM, e.g. cross-entropy for classification or $L_2$ for regression, and $L_a$ is the loss function defined for the jumping parameters, based on the reward signal. In such formulation, the LSTM parameters and the jumping layer parameters indeed minimize independent loss terms. This forces the practitioner to define an $L_a$ term that mimics the behavior of $L_m$, which may not be trivial for some tasks. In contrast, the parameters in the proposed Skip RNN model are optimized directly from the original loss function and there is no need to define additional loss terms that provide gradients for them.

In LSTM-Jump, some hyperparameters defining the behavior of the subsampling scheme need to be defined ahead of time:

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<tr>
<td>N</td>
<td>number of jumps allowed</td>
</tr>
<tr>
<td>K</td>
<td>maximum size of jumping</td>
</tr>
<tr>
<td>R</td>
<td>number of tokens read before a jump</td>
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</table>

Table 3.1: Hyperparameters defining the jumping behavior of LSTM-Jump.
These hyperparameters define a reduced set of subsequences that the model can sample, instead of allowing the network to learn any arbitrary sampling scheme. This could lead to suboptimal sampling schemes and pushes some of the complexity towards the practitioner’s prior knowledge of the task. For instance, should the optimal policy consist in using every other sample, the model will be unable to learn the best sampling scheme if $R = 2$. Unlike LSTM-Jump, Skip RNN is not limited to a set of sample selection patterns which are predefined ahead of time.
Chapter 4

Model

An RNN takes an input sequence $x = (x_1, \ldots, x_T)$ and generates a state sequence $s = (s_1, \ldots, s_T)$ by iteratively applying a parametric state transition model $S$ from $t = 1$ to $T$:

$$s_t = S(s_{t-1}, x_t)$$  \hspace{1cm} (4.1)

We augment the network with a binary state update gate, $u_t \in \{0, 1\}$, selecting whether the state of the RNN will be updated or copied from the previous time step. At every time step $t$, the probability $\tilde{u}_{t+1} \in [0, 1]$ of performing a state update at $t+1$ is emitted. The resulting architecture is depicted in Figure 4.1 and can be characterized as follows:

$$u_t = f_{\text{binarize}}(\tilde{u}_t)$$  \hspace{1cm} (4.2)

$$s_t = u_t \cdot S(s_{t-1}, x_t) + (1 - u_t) \cdot s_{t-1}$$  \hspace{1cm} (4.3)

$$\Delta \tilde{u}_t = \sigma(W_p s_t + b_p)$$  \hspace{1cm} (4.4)

$$\tilde{u}_{t+1} = u_t \cdot \Delta \tilde{u}_t + (1 - u_t) \cdot (\tilde{u}_t + \min(\Delta \tilde{u}_t, 1 - \tilde{u}_t))$$  \hspace{1cm} (4.5)

where $\sigma$ is the sigmoid function and $f_{\text{binarize}} : [0, 1] \rightarrow \{0, 1\}$ binarizes the input value. Should the network be composed of several layers, some columns of $W_p$ can be fixed to 0 so that $\Delta \tilde{u}_t$ depends only on the states of a subset of layers (see Section 5.5 for an example with two layers). We implement $f_{\text{binarize}}$ as a deterministic step function $u_t = \text{round}(\tilde{u}_t)$, although a stochastic sampling from a Bernoulli distribution $u_t \sim \text{Bernoulli}(\tilde{u}_t)$ would be possible as well.

The model formulation implements the observation that the likelihood of requesting a new input increases with the number of consecutively skipped samples. Whenever a state update is omitted, the pre-activation of the state update gate for the following time step, $\tilde{u}_{t+1}$, is incremented by $\Delta \tilde{u}_t$. On the other hand, if a state update is performed, the accumulated value is flushed and $\tilde{u}_{t+1} = \Delta \tilde{u}_t$.

The number of skipped time steps can be computed ahead of time. For the particular formulation used in this work, where $f_{\text{binarize}}$ is implemented by means of a rounding function, the number of skipped samples after performing a state update at time step $t$ is given by:

$$N_{\text{skip}}(t) = \min\{n : n \cdot \Delta \tilde{u}_t \geq 0.5\} - 1$$  \hspace{1cm} (4.6)
Figure 4.1: Model architecture of the proposed Skip RNN. (a) Complete Skip RNN architecture, where the computation graph at time step \( t \) is conditioned on \( u_t \). (b) Architecture when the state is updated, i.e. \( u_t = 1 \). (c) Architecture when the update step is skipped and the previous state is copied, i.e. \( u_t = 0 \). (d) In practice, redundant computation is avoided by propagating \( \Delta \tilde{u}_t \) between time steps when \( u_t = 0 \).

where \( n \in \mathbb{Z}^+ \). This enables more efficient implementations where no computation at all is performed whenever \( u_t = 0 \). These computational savings are possible because \( \Delta \tilde{u}_t = \sigma(W_p s_t + b_p) = \sigma(W_p s_{t-1} + b_p) = \Delta \tilde{u}_{t-1} \) when \( u_t = 0 \) and there is no need to evaluate it again, as depicted in Figure 4.1d.

There are several advantages in reducing the number of RNN updates. From the computational standpoint, fewer updates translates into fewer required sequential operations to process an input signal, leading to faster inference and reduced energy consumption. Unlike some other models that aim to reduce the average number of operations per step [41, 27], ours enables skipping steps completely. Replacing RNN updates with copy operations increases the memory of the network and its ability to model long term dependencies even for gated units, since the exponential memory decay observed in LSTM and GRU [41] is alleviated. During training, gradients are propagated through fewer updating time steps, providing faster convergence in some tasks involving long sequences. Moreover, the proposed model is orthogonal to recent advances in RNNs and could be used in conjunction with such techniques, e.g. normalization [12, 4], regularization [61, 33], variable computation [27, 41] or even external memory [21, 54].
4.1 Error gradients

The whole model is differentiable except for $f_{\text{binarize}}$, which outputs binary values. A common method for optimizing functions involving discrete variables is REINFORCE \[55\], although several estimators have been proposed for the particular case of neurons with binary outputs \[7\]. We select the straight-through estimator \[24\], which consists in approximating the step function by the identity when computing gradients during the backwards pass:

$$\frac{\partial f_{\text{binarize}}(x)}{\partial x} = 1$$ \hspace{1cm} (4.7)

This yields a biased estimator that has proven more efficient than other unbiased but high-variance estimators such as REINFORCE \[7\] and has been successfully applied in different works \[13, 11\]. By using the straight-through estimator as the backwards pass for $f_{\text{binarize}}$, all the model parameters can be trained to minimize the target loss function with standard backpropagation and without defining any additional supervision or reward signal.

4.2 Limiting computation

The Skip RNN is able to learn when to update or copy the state without explicit information about which samples are useful to solve the task at hand. However, a different operating point on the trade-off between performance and number of processed samples may be required depending on the application, e.g. one may be willing to sacrifice a few accuracy points in order to run faster on machines with low computational power, or to reduce energy impact on portable devices. The proposed model can be encouraged to perform fewer state updates through additional loss terms, a common practice in neural networks with dynamically allocated computation \[36, 38, 19, 27\]. In particular, we will consider the introduction of a cost per sample:

$$L_{\text{budget}} = \lambda \cdot \sum_{t=1}^{T} u_t$$ \hspace{1cm} (4.8)

where $L_{\text{budget}}$ is the cost associated to a single sequence, $\lambda$ is the cost per sample and $T$ is the sequence length. This formulation bears a similarity to weight decay regularization, where the network is encouraged to slowly converge towards a solution where the norm of the weights is smaller. Similarly, in this case the network is encouraged to slowly converge towards a solution where fewer state updates are required.

Despite this formulation has been extensively studied in our experiments, different budget loss terms can be used depending on the application. For instance, a specific number of samples may be encouraged by applying an $L_1$ or $L_2$ loss between the target value and the number of updates per sequence, $\sum_{t=1}^{T} u_t$. 
Chapter 5

Experimental Results

In the following sections, we investigate the advantages of adding the Skip RNN architecture to LSTMs and GRUs for a variety of tasks. Besides the evaluation metric for each task, we report the number of RNN state updates (i.e. the number of elements in the input sequence that are used by the model) as a measure of the computational load for each model. Unlike the wall clock time required by each model to process an input sequence, which we found to be highly dependent on the particular software implementation even for the baseline models, the number of RNN updates provides a meaningful measure of the inherently sequential operations in the computation graph. Since skipping an RNN update results in ignoring its corresponding input, we will refer to the number of updates and the number of used samples (i.e. elements in a sequence) interchangeably throughout this manuscript. For additional qualitative results, see Appendix A.

Training is performed with Adam optimizer \[30\], learning rate of \(10^{-4}\), \(\beta_1 = 0.9\), \(\beta_2 = 0.999\) and \(\epsilon = 10^{-8}\) on batches of 256 samples. Gradient clipping \[44\] with a threshold of 1 is applied to all trainable variables. Bias \(b_p\) in Equation 4.4 is initialized to 1, so that all samples are used at the beginning of training. The initial hidden state \(s_0\) is learned during training, whereas \(\tilde{u}_0\) is set to a constant value of 1 in order to force the first update at \(t = 1\).

5.1 Adding Task

We revisit one of the original LSTM tasks \[25\], where the network is given a sequence of (value, marker) tuples. The desired output is the addition of the only two values that were marked with a 1, whereas those marked with a 0 need to be ignored. In particular, we follow the experimental setup by Neil et al. \[41\], where the first marker is randomly placed among the first 10% of samples (drawn with uniform probability) and the second one is placed among the last half of samples (drawn with uniform probability). This marker distribution yields sequences where at least 40% of the samples are distractors and provide no useful information at all. However, it is worth noting that in this task the risk of missing a marker is very large as compared to the benefits of working on shorter subsequences.

\[1\] In practice, forcing the network to use all samples at the beginning of training improves its robustness against random initializations of its weights and increases the reproducibility of the presented experiments. A similar behavior was observed in other augmented RNN architectures such as Neural Stacks \[22\].
### Table 5.1: Results for the adding task, displayed as mean ± std over four different runs. The task is considered to be solved if the MSE is at least two orders of magnitude below the variance of the output distribution.

<table>
<thead>
<tr>
<th>Model</th>
<th>Task solved</th>
<th>State updates</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSTM</td>
<td>Yes</td>
<td>100.0% ± 0.0%</td>
</tr>
<tr>
<td>Skip LSTM, $\lambda = 0$</td>
<td>Yes</td>
<td>81.1% ± 3.6%</td>
</tr>
<tr>
<td>Skip LSTM, $\lambda = 10^{-5}$</td>
<td>Yes</td>
<td>53.9% ± 2.1%</td>
</tr>
<tr>
<td>GRU</td>
<td>Yes</td>
<td>100.0% ± 0.0%</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 0$</td>
<td>Yes</td>
<td>97.9% ± 3.2%</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 10^{-5}$</td>
<td>Yes</td>
<td>50.7% ± 2.6%</td>
</tr>
</tbody>
</table>

We train RNN models with 110 units each on sequences of length 50, where the values are uniformly drawn from $U(-0.5, 0.5)$. The final RNN state is fed to a fully connected layer that regresses the scalar output. The whole model is trained to minimize the Mean Squared Error (MSE) between the output and the ground truth. We consider that a model is able to solve the task when its MSE on a held-out set of examples is at least two orders of magnitude below the variance of the output distribution. This criterion is a stricter version of the one followed in [25].

While all models learn to solve the task, results in Table 5.1 show that the Skip RNN models are able to do so with roughly half of the updates of their standard counterparts. Interestingly, Skip LSTM tends to skip more updates than the Skip GRU when no cost per sample is set, behavior that may be related to the lack of output gate in the latter. We hypothesize that there are two possible reasons why the output gate makes the LSTM more prone to skipping updates: (a) it introduces an additional source of memory decay, and (b) it allows to mask out some units in the cell state that may specialize in deciding when to update or copy, making the final regression layer agnostic to such process.

We observed that the models using fewer updates never miss any marker, since the penalization in terms of MSE would be very large (see Figure 5.1 for examples). These models learn to skip most of the samples in the 40% of the sequence where there are no markers. Moreover, most updates are skipped once the second marker is found, since all the relevant information in the sequence has been already seen. This last pattern provides evidence that the proposed models effectively learn to decide whether to update or copy the hidden state based on the input sequence, as opposed to learning biases in the dataset only. As a downside, Skip RNN models show some difficulties skipping a large number of updates at once, probably due to the cumulative nature of $\tilde{u}_t$.

#### 5.2 Frequency Discrimination Task

In this experiment, the network is trained to classify between sinusoids whose period is in range $T \sim U(5, 6)$ milliseconds and those whose period is in range $T \sim \{1, 5 \cup (6, 100)\}$ milliseconds [41]. Every sine wave with period $T$ has a random phase shift drawn from $U(0, T)$. At every time step, the input to the network is a single scalar representing the amplitude of the signal. Since sinusoid are continuous signals, this tasks allows to study whether Skip RNNs converge to the same solutions when their parameters are fixed but the sampling period is changed. We study
Figure 5.1: Sample usage examples for the Skip GRU with $\lambda = 10^{-5}$ on the adding task. Red dots indicate used samples, whereas blue ones are skipped.

<table>
<thead>
<tr>
<th>Model</th>
<th>$T_s = 1\text{ms (length 100)}$</th>
<th>$T_s = 0.5\text{ms (length 200)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSTM</td>
<td>Task solved: Yes, State updates: 100.0 ± 0.00</td>
<td>Task solved: Yes, State updates: 200.0 ± 0.00</td>
</tr>
<tr>
<td>Skip LSTM, $\lambda = 0$</td>
<td>Yes, State updates: 55.5 ± 16.9</td>
<td>Yes, State updates: 147.9 ± 27.0</td>
</tr>
<tr>
<td>Skip LSTM, $\lambda = 10^{-5}$</td>
<td>Yes, State updates: 47.4 ± 14.1</td>
<td>Yes, State updates: 50.7 ± 16.8</td>
</tr>
<tr>
<td>Skip LSTM, $\lambda = 10^{-4}$</td>
<td>Yes, State updates: 12.7 ± 0.5</td>
<td>Yes, State updates: 19.9 ± 1.5</td>
</tr>
<tr>
<td>GRU</td>
<td>Yes, State updates: 100.0 ± 0.00</td>
<td>Yes, State updates: 200.0 ± 0.00</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 0$</td>
<td>Yes, State updates: 73.7 ± 17.9</td>
<td>Yes, State updates: 167.0 ± 18.3</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 10^{-5}$</td>
<td>Yes, State updates: 51.9 ± 10.2</td>
<td>Yes, State updates: 54.2 ± 4.4</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 10^{-4}$</td>
<td>Yes, State updates: 23.5 ± 6.2</td>
<td>Yes, State updates: 22.5 ± 2.1</td>
</tr>
</tbody>
</table>

Table 5.2: Results for the frequency discrimination task, displayed as mean ± std over four different runs. The task is considered to be solved if the classification accuracy is over 99%. Models with the same cost per sample ($\lambda > 0$) converge to a similar number of used samples under different sampling conditions.

We train RNNs with 110 units each on input signals of 100 milliseconds. Batches are stratified, containing the same number of samples for each class, yielding a 50% chance accuracy. The last state of the RNN is fed into a 2-way classifier and trained with cross-entropy loss. We consider that a model is able to solve the task when it achieves an accuracy over 99% on a held-out set of examples.

Table 5.2 summarizes results for this task. When no cost per sample is set ($\lambda = 0$), the number of updates differ under different sampling conditions. We attribute this behavior to the potentially large number of local minima in the cost function, since there are numerous subsampling patterns for which the task can be successfully solved and we are not explicitly encouraging the network to converge to a particular solution. On the other hand, when $\lambda > 0$ Skip RNN models with the same cost per sample use roughly the same number of input samples even when the sampling frequency is doubled. This is a desirable property, since solutions are robust to oversampled input signals.
Table 5.3: Accuracy and used samples on the test set of MNIST after 600 epochs of training. Results are displayed as mean ± std over four different runs.

<table>
<thead>
<tr>
<th>Model</th>
<th>Accuracy</th>
<th>State updates</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSTM</td>
<td>0.910 ± 0.045</td>
<td>784.00 ± 0.00</td>
</tr>
<tr>
<td>Skip LSTM, $\lambda = 10^{-4}$</td>
<td>0.973 ± 0.002</td>
<td>379.38 ± 33.09</td>
</tr>
<tr>
<td>GRU</td>
<td>0.968 ± 0.013</td>
<td>784.00 ± 0.00</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 10^{-4}$</td>
<td>0.976 ± 0.003</td>
<td>392.62 ± 26.48</td>
</tr>
</tbody>
</table>

5.3 MNIST Classification from a Sequence of Pixels

The MNIST handwritten digits classification benchmark [35] is traditionally addressed with Convolutional Neural Networks (CNNs) that can efficiently exploit spatial dependencies through weight sharing. By flattening the $28 \times 28$ images into 784-d vectors, however, it can be reformulated as a challenging task for RNNs where long term dependencies need to be leveraged [34]. We follow the standard data split and set aside 5,000 training samples for validation purposes. After processing all pixels with an RNN with 110 units, the last hidden state is fed into a linear classifier predicting the digit class. All models are trained for 600 epochs to minimize cross-entropy loss.

Table 5.3 summarizes classification results on the test set after 600 epochs of training. Skip RNNs are not only able to solve the task using fewer updates than their counterparts, but also show a lower variation among runs and train faster (see Figure 5.2). We hypothesize that skipping updates make the Skip RNNs work on shorter subsequences, simplifying the optimization process and allowing the networks to capture long term dependencies more easily. A similar behavior was observed for Phased LSTM, where increasing the sparsity of cell updates accelerates training for very long sequences [41].

Sequences of pixels can be reshaped back into 2D images, allowing to visualize the samples used by the RNNs as a sort of hard visual attention model [57]. Examples such as the ones depicted in Figure 5.3 (top) show how the model learns to skip pixels that are not discriminative, such as the padding regions in the top and bottom of images. Similarly to the qualitative results for the adding task (Section 5.1), attended samples vary depending on the particular input being given to the network.

With the aim of observing how the attention model changes with the task, we train Skip LSTM ($\lambda = 10^{-4}$) models on two subsets of MNIST. We select pairs of visually similar digits: classes 1 and 7 (MNIST-17) and classes 5 and 6 (MNIST-56). Figure 5.3 (middle and bottom) shows how the network is able to find the most discriminative region for each pair of digits and mostly ignores the rest of the image. These regions differ from the ones attended by the model trained on the full dataset, depicted in Figure 5.3 (top).

5.4 Sentiment Analysis on IMDB

The IMDB dataset [37] contains 25,000 training and 25,000 testing movie reviews annotated into two classes, positive and negative sentiment, with an approximate average length of 240...
Figure 5.2: Accuracy evolution during training on the validation set of MNIST. The Skip GRU exhibits lower variance and faster convergence than the baseline GRU. A similar behavior is observed for LSTM and Skip LSTM, but omitted for clarity. Shading shows maximum and minimum over 4 runs, while dark lines indicate the mean.

Figure 5.3: Sample usage examples for the Skip LSTM with $\lambda = 10^{-4}$ on the test set of MNIST (top), MNIST-17 (middle) and MNIST-56 (bottom). Red pixels are used, whereas blue ones are skipped.
words per review. We set aside $15\%$ of training data for validation purposes. Words are embedded into 300-d vector representations before being fed to an RNN with 128 units. The embedding matrix is initialized using pre-trained word2vec\footnote{https://code.google.com/archive/p/word2vec/} embeddings \cite{mikolov2013efficient} when available, or random vectors drawn from $\mathcal{U}(-0.25, 0.25)$ otherwise \cite{srivastava2014dropout}. Dropout with rate 0.2 is applied between the last RNN state and the classification layer in order to reduce overfitting. We evaluate the models on sequences of length 200 and 400 by cropping longer sequences and padding shorter ones \cite{cho2014learning}.

Results on the test are reported in Table \ref{tab:imdb}. In a task where it is hard to predict which input tokens will be discriminative, the Skip RNN models are able to achieve similar accuracy rates to the baseline models while reducing the number of required updates. These results highlight the trade-off between accuracy and the available computational budget, since a larger cost per sample results in lower accuracies. However, allowing the network to select which samples to use instead of cropping sequences at a given length boosts performance, as observed for the Skip LSTM (length 400, $\lambda = 10^{-4}$), which achieves a higher accuracy than the baseline LSTM (length 200) while seeing roughly the same number of words per review. A similar behavior can be seen for the Skip RNN models with $\lambda = 10^{-3}$, where allowing them to select words from longer reviews boosts classification accuracy while using a comparable number of tokens per sequence.

\begin{table}[htb]
\centering
\begin{tabular}{|l|c|c|c|c|c|}
\hline
\textbf{Model} & \textbf{Length 200} & & \textbf{Length 400} & & \\
& \textbf{Accuracy} & \textbf{State updates} & \textbf{Accuracy} & \textbf{State updates} & \\
\hline
LSTM & 0.843 ± 0.003 & 200.00 ± 0.00 & 0.868 ± 0.004 & 400.00 ± 0.00 & \\
Skip LSTM, $\lambda = 0$ & 0.844 ± 0.004 & 196.75 ± 5.63 & 0.866 ± 0.004 & 369.70 ± 19.35 & \\
Skip LSTM, $\lambda = 10^{-5}$ & 0.846 ± 0.004 & 197.15 ± 3.16 & 0.865 ± 0.001 & 380.62 ± 18.20 & \\
Skip LSTM, $\lambda = 10^{-4}$ & 0.837 ± 0.006 & 164.65 ± 8.67 & 0.862 ± 0.003 & 186.30 ± 25.72 & \\
Skip LSTM, $\lambda = 10^{-3}$ & 0.811 ± 0.007 & 73.85 ± 1.90 & 0.836 ± 0.007 & 84.22 ± 1.98 & \\
Skip GRU, $\lambda = 0$ & 0.845 ± 0.006 & 200.00 ± 0.00 & 0.862 ± 0.003 & 400.00 ± 0.00 & \\
Skip GRU, $\lambda = 10^{-5}$ & 0.848 ± 0.002 & 200.00 ± 0.00 & 0.866 ± 0.002 & 399.02 ± 1.69 & \\
Skip GRU, $\lambda = 10^{-4}$ & 0.842 ± 0.005 & 199.25 ± 1.30 & 0.862 ± 0.008 & 398.00 ± 2.06 & \\
Skip GRU, $\lambda = 10^{-3}$ & 0.834 ± 0.006 & 180.97 ± 8.90 & 0.853 ± 0.011 & 314.30 ± 2.82 & \\
& 0.800 ± 0.007 & 106.15 ± 37.92 & 0.814 ± 0.005 & 99.12 ± 2.69 & \\
\hline
\end{tabular}
\caption{Accuracy and used samples on the test set of IMDB for different sequence lengths. Results are displayed as mean ± std over four different runs.}
\end{table}

5.5 Action classification on UCF-101

One of the most accurate and scalable pipelines for video analysis consists in extracting frame level features with a CNN and modeling their temporal evolution with an RNN \cite{cassidy2016temporal} \cite{ross2015inside}. Videos are commonly recorded at high sampling rates, rapidly generating long sequences with strong temporal redundancy that are challenging for RNNs. Moreover, processing frames with a CNN is computationally expensive and may become prohibitive for high framerates. These issues have been alleviated in previous works by using short clips \cite{cassidy2016temporal} or by downsampling the original data in order to cover long temporal spans without increasing the sequence length excessively \cite{ross2015inside}. Instead of addressing the long sequence problem at the input data level, we train RNN models...
Table 5.5: Accuracy and used samples on the validation set of UCF-101 (split 1).

<table>
<thead>
<tr>
<th>Model</th>
<th>Accuracy</th>
<th>State updates</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSTM</td>
<td>0.671</td>
<td>250.0</td>
</tr>
<tr>
<td>Skip LSTM, $\lambda = 0$</td>
<td>0.749</td>
<td>138.9</td>
</tr>
<tr>
<td>Skip LSTM, $\lambda = 10^{-5}$</td>
<td>0.757</td>
<td>24.2</td>
</tr>
<tr>
<td>Skip LSTM, $\lambda = 10^{-4}$</td>
<td>0.790</td>
<td>7.6</td>
</tr>
<tr>
<td>GRU</td>
<td>0.791</td>
<td>250.0</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 0$</td>
<td>0.796</td>
<td>124.2</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 10^{-5}$</td>
<td>0.792</td>
<td>29.7</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 10^{-4}$</td>
<td>0.793</td>
<td>23.7</td>
</tr>
</tbody>
</table>

Table 5.5: Accuracy and used samples on the validation set of UCF-101 (split 1).

Figure 5.4: Accuracy evolution during the first 300 training epochs on the validation set of UCF-101 (split 1). Skip LSTM models converge much faster than the baseline LSTM.

UCF-101 \cite{ucf101} is a dataset containing 13,320 trimmed videos belonging to 101 different action categories. We use 10 seconds of video sampled at 25fps, cropping longer ones and padding shorter examples with empty frames. Activations in the Global Average Pooling layer from a ResNet-50 \cite{resnet} CNN pretrained on the ImageNet dataset \cite{imagenet} are used as frame level features, which are fed into two stacked RNN layers with 512 units each. The weights in the CNN are not tuned during training to reduce overfitting. The hidden state in the last RNN layer is used to compute the update probability for the Skip RNN models.

We evaluate the different models on the first split of UCF-101 and report results in Table 5.5. Skip RNN models do not only improve the classification accuracy with respect to the baseline, but require very few updates to do so. We argue that this is possible due to the low motion between consecutive frames resulting in frame level features with high temporal redundancy \cite{temporal-redundancy}. Moreover, Figure 5.4 shows how models performing fewer updates converge faster thanks to the gradients being preserved during longer spans when training with backpropagation through time.
Chapter 6

Budget

We estimate that the overall GPU usage for the experiments presented in this thesis is roughly 2,880 hours. However, these represent a small fraction of the overall GPU usage, since alternative formulations, hyperparameter settings and debugging experiments were run before achieving the final results included in this document. For the purpose of cost estimation, we assume that the presented experiments comprise around a 20% of the overall computation, thus leading to roughly 14,400 hours of GPU usage. We compute the computation cost based on Amazon Web Services (AWS) rates\footnote{\url{https://aws.amazon.com/ec2/instance-types/p2/?nc1=h_ls}} for \textit{p2.xlarge} instances with one NVIDIA K80 each, which is a dual-GPU and allows to run sets of two experiments in parallel.

The length of the project was 24 weeks. Assuming a commitment of 40 weekly hours and that each advisor spent an average of 1h per week on meetings, the complete costs for the project are the following:

<table>
<thead>
<tr>
<th></th>
<th>Amount</th>
<th>Cost/hour</th>
<th>Time</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWS \textit{p2.xlarge} instance</td>
<td>0.5</td>
<td>$0.90</td>
<td>14,400h</td>
<td>$6,480</td>
</tr>
<tr>
<td>Junior engineer</td>
<td>1</td>
<td>$8.00</td>
<td>960h</td>
<td>$7,680</td>
</tr>
<tr>
<td>Senior engineer</td>
<td>4</td>
<td>$30.00</td>
<td>24h</td>
<td>$2,880</td>
</tr>
<tr>
<td>Other equipment</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$4,000</td>
</tr>
<tr>
<td>\textbf{Total}</td>
<td></td>
<td></td>
<td></td>
<td>$21,040</td>
</tr>
</tbody>
</table>

Table 6.1: Cost of the project. \textit{Other equipment} includes campus services, office supplies and hardware, e.g. the employed laptop and auxiliary monitor.
Chapter 7

Conclusion and Future Work

We presented Skip RNN as an extension to existing recurrent architectures enabling them to skip state updates, thus reducing the number of sequential operations in the computation graph. Unlike previous approaches, all parameters in Skip RNN are trained with backpropagation without requiring to introduce task-dependent modifications. Experiments conducted on LSTM and GRU showed how Skip RNNs match the performance of baseline models while relaxing their computational requirements. Evaluation is performed on a broad set of tasks and datasets involving synthetic data, images, text and video. Skip RNNs provide a faster and more stable training for long sequences and complex models, likely due to gradients being backpropagated through fewer time steps resulting in a simpler optimization task. Moreover, the introduced computational savings are better suited for modern hardware than those methods that reduce the number of computations required at each time step [31, 41, 11].

The presented results motivate several new research directions towards designing efficient RNN architectures. Introducing stochasticity in neural network training has proven beneficial [52, 33], so that replacing the deterministic rounding operation with a stochastic sampling is a natural extension to the current formulation. This may improve generalization while allowing the model to escape poor local minima more easily. The loss term penalizing the number of updates is an important element in the performance of Skip RNN and different formulations can be tailored for specific tasks. For instance, the cost could increase at each time step to encourage the network to emit a decision earlier [11], or a particular number of updates can be enforced if such information is available. Finally, understanding and analyzing the patterns followed by the model when deciding whether to update or copy the RNN state may provide insight for developing better and more efficient architectures.

This project was aimed at designing a recurrent architecture capable of skipping state updates and showing its advantages in a series of tasks. Once the formulation is mature enough and has been proven effective, future work should focus on developing efficient implementations pushing the limits in terms of wall clock time. This may include the development of low level CUDA kernels that speedup the execution of Skip RNN models, as well as porting the code to a framework based on dynamic computational graphs (e.g. PyTorch [1], DyNet [2], Chainer [3]). The latter are more suitable for the problem at hand than frameworks based on static computation graphs, since the

1 http://www.pytorch.org/
2 https://github.com/clab/dynet/
3 https://chainer.org/
actual computation graph is conditioned on the model outputs at each time step and cannot be completely defined before being run.
Bibliography


[37] Andrew L Maas, Raymond E Daly, Peter T Pham, Dan Huang, Andrew Y Ng, and Christopher Potts. Learning word vectors for sentiment analysis. In *ACL*, 2011.


Appendix A

Qualitative results

This appendix contains additional qualitative results for the Skip RNN models.
A.1 Adding task

Figure A.1: Sample usage examples for the Skip GRU with $\lambda = 10^{-5}$ on the adding task. Red dots indicate used samples, whereas blue ones are skipped.
A.2 Frequency discrimination task

Figure A.2: Sample usage examples for the Skip LSTM with $\lambda = 10^{-4}$ on the frequency discrimination task with $T_s = 0.5$ms. Red dots indicate used samples, whereas blue ones are skipped. The network learns that using the first samples is enough to classify the frequency of the sine waves, in contrast to a uniform downsampling that may result in aliasing.
Appendix B

ArXiv pre-print

This appendix includes the pre-print that has been uploaded to arXiv and that will be submitted to a Machine Learning conference this Fall.
Skip RNN: Learning to Skip State Updates in Recurrent Neural Networks

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Abstract

Recurrent Neural Networks (RNNs) continue to show outstanding performance in sequence modeling tasks. However, training RNNs on long sequences often face challenges like slow inference, vanishing gradients and difficulty in capturing long term dependencies. In backpropagation through time settings, these issues are tightly coupled with the large, sequential computational graph resulting from unfolding the RNN in time. We introduce the Skip RNN model which extends existing RNN models by learning to skip state updates and shortens the effective size of the computational graph. This model can also be encouraged to perform fewer state updates through a budget constraint. We evaluate the proposed model on various tasks and show how it can reduce the number of required RNN updates while preserving, and sometimes even improving, the performance of the baseline RNN models.

1 Introduction

Recurrent Neural Networks (RNNs) have become the standard approach for practitioners when addressing machine learning tasks involving sequential data. Such success has been enabled by the appearance of larger datasets, more powerful computing resources and improved architectures and training algorithms. Gated units, such as the Long Short-Term Memory [24] (LSTM) and the Gated Recurrent Unit [11] (GRU), were designed to deal with the vanishing gradients problem commonly found in RNNs [8]. These architectures have become popularized thanks to their impressive results in a variety of tasks such as machine translation [5], language modeling [53] or speech recognition [19].

Some of the main limitations of RNNs are their challenging training and deployment when dealing with long sequences, due to their inherently sequential behaviour. These challenges include throughput degradation, slower convergence during training and memory leakage, even for gated architectures [38]. Sequence shortening techniques, which can be seen as a sort of conditional computation [7, 6, 15] in time, can alleviate these issues. The most common approaches, such as cropping discrete signals or reducing the sampling rate in continuous signals, are heuristics and can be suboptimal. In contrast, we propose a model that is able to learn which samples (i.e. elements in the input sequence) need to be used in order to solve the target task. Consider a video understanding task as an example: scenes with large motion may benefit from high frame rates, whereas only a few frames are needed to capture the semantics of a mostly static scene.

The main contribution of this work is a novel modification for existing RNN architectures that allows them to skip state updates, decreasing the number of sequential operations to be performed, without requiring any additional supervision signal. This model, called Skip RNN, adaptively determines whether the state needs to be updated or copied to the next time step, thereby allow a “skip” in the computation graph. We show how the network can be encouraged to perform fewer state updates by

∗Work done while Víctor Campos was a visiting scholar at Columbia University.
adding a penalization term during training, allowing us to train models of different target computation budgets. The proposed modification is implemented on top of well known RNN architectures, namely LSTM and GRU, and the resulting models show promising results in a series of sequence modeling tasks. In particular, the proposed Skip RNN architecture is evaluated on five sequence learning problems: an adding task, sine wave frequency discrimination, digit classification, sentiment analysis in movie reviews and action classification in video.

This paper is structured as follows: Section 2 provides an overview of the related work, Section 3 describes the proposed model, experimental evaluation of Skip RNN in a series of sequence modeling tasks is presented in Section 4, and Section 5 summarizes the main results and some potential extensions of this work.

2 Related work

Conditional computation has been shown to allow gradual increases in model capacity without a proportional increases in computational cost by exploiting certain computation paths for each input [7, 33, 2, 35, 41]. This idea has been extended in the temporal domain, either by learning how many times an input needs to be pondered before moving to the next one [18] or building RNNs whose number of layers depends on the input data [12]. Some works have addressed time-dependent computation in RNNs by updating only a fraction of the hidden states based on the current hidden state and input [26], or following periodic patterns [29, 38]. However, due to the inherently sequential nature of RNNs and the parallel computation capabilities of modern hardware, reducing the size of the matrices involved in the computations performed at each time step does not accelerate inference. The proposed Skip RNN model can be seen as form of conditional computation in time, where the computation associated to the RNN updates may or may not be executed at every time step. This is related to the UPDATE and COPY operations in hierarchical multiscale RNNs [12], but applied to the whole stack of RNN layers at the same time. This difference is key to allowing our approach to skip input samples, effectively reducing sequential computation and shielding the hidden state over longer time lags. Learning whether to update or copy the hidden state through time steps can be seen as a learnable Zoneout mask [30] which is shared between all the units in the hidden state. Similarly, it can be interpreted as an input-dependent recurrent version of stochastic depth [25].

Selecting parts of the input signal is similar in spirit to the hard attention mechanisms that have been applied to image regions [37], where only some patches of the input image are attended in order to generate captions [49] or detect objects [3]. Our model can be understood to generate a hard temporal attention mask on the fly given the previously seen samples, deciding which time steps should be attended and operating on a subset of input samples. Subsampling input sequences has been explored for visual storylines generation [43], although jointly optimizing the RNN weights and the subsampling mechanism is computationally unfeasible and the Expectation Maximization algorithm is used instead. Similar research has been conducted for video analysis tasks, discovering minimally needed evidence for event recognition [9] and training agents that decide which frames need to be observed in order to localize actions in time [50, 46]. Motivated by the advantages of training recurrent models on shorter subsequences, efforts have been conducted towards learning differentiable subsampling mechanisms [40], although the computational complexity of the proposed method precludes its application to long input sequences. In contrast, our proposed method can be trained with backpropagation and does not degrade the complexity of the baseline RNNs.

Accelerating inference in RNNs is difficult due to their inherently sequential nature, leading to the design of Quasi-Recurrent Neural Networks [10], which relax the temporal dependency between consecutive steps. With the goal of speeding up RNN inference, LSTM-Jump [51] augments an LSTM cell with a classification layer that will decide how many steps to jump between RNN updates. Despite its promising results on text tasks, the model needs to be trained with REINFORCE [48], which requires the definition of a reward signal. Determining such reward signal is not trivial and does not necessarily generalize across tasks, e.g. regression and classification tasks may require from different reward signals. Moreover, the number of tokens read between jumps, the maximum jump distance and the number of jumps allowed need to be chosen ahead of time. These hyperparameters define a reduced set of subsequences that the model can sample, instead of allowing the network to learn any arbitrary sampling scheme. Unlike LSTM-Jump, our proposed approach is differentiable, thus not requiring any modifications to the loss function and simplifying the optimization process, and is not limited to a predefined set of sample selection patterns.
3 Model Description

An RNN takes an input sequence $x = (x_1, \ldots, x_T)$ and generates a state sequence $s = (s_1, \ldots, s_T)$ by iteratively applying a parametric state transition model $S$ from $t = 1$ to $T$:

$$s_t = S(s_{t-1}, x_t)$$  \hspace{1cm} (1)

We augment the network with a binary state update gate, $u_t \in \{0, 1\}$, selecting whether the state of the RNN will be updated or copied from the previous time step. At every time step $t$, the probability $\tilde{u}_{t+1} \in [0, 1]$ of performing a state update at $t + 1$ is emitted. The resulting architecture is depicted in Figure 1 and can be characterized as follows:

$$u_t = f_{\text{binarize}}(\tilde{u}_t)$$  \hspace{1cm} (2)
$$s_t = u_t \cdot S(s_{t-1}, x_t) + (1 - u_t) \cdot s_{t-1}$$  \hspace{1cm} (3)
$$\Delta \tilde{u}_t = \sigma(W_p s_t + b_p)$$  \hspace{1cm} (4)
$$\tilde{u}_{t+1} = u_t \cdot \Delta \tilde{u}_t + (1 - u_t) \cdot (\tilde{u}_t + \min(\Delta \tilde{u}_t, 1 - \tilde{u}_t))$$  \hspace{1cm} (5)

where $\sigma$ is the sigmoid function and $f_{\text{binarize}} : [0, 1] \rightarrow \{0, 1\}$ binarizes the input value. Should the network be composed of several layers, some columns of $W_p$ can be fixed to 0 so that $\Delta \tilde{u}_t$ depends only on the states of a subset of layers (see Section 4.5 for an example with two layers). We implement $f_{\text{binarize}}$ as a deterministic step function $u_t = \text{round}(\tilde{u}_t)$, although a stochastic sampling from a Bernoulli distribution $u_t \sim \text{Bernoulli}(\tilde{u}_t)$ would be possible as well.

The model formulation implements the observation that the likelihood of requesting a new input increases with the number of consecutively skipped samples. Whenever a state update is omitted, the pre-activation of the state update gate for the following time step, $\tilde{u}_{t+1}$, is incremented by $\Delta \tilde{u}_t$. On the other hand, if a state update is performed, the accumulated value is flushed and $\tilde{u}_{t+1} = \Delta \tilde{u}_t$.

The number of skipped time steps can be computed ahead of time. For the particular formulation used in this work, where $f_{\text{binarize}}$ is implemented by means of a rounding function, the number of skipped samples after performing a state update at time step $t$ is given by:

$$N_{\text{skip}}(t) = \min\{n : n \cdot \Delta \tilde{u}_t \geq 0.5\} - 1$$  \hspace{1cm} (6)

where $n \in \mathbb{Z}^+$. This enables more efficient implementations where no computation at all is performed whenever $u_t = 0$. These computational savings are possible because $\Delta \tilde{u}_t = \sigma(W_p s_t + b_p) = \Delta \tilde{u}_{t-1}$ when $u_t = 0$ and there is no need to evaluate it again, as depicted in Figure 1d.

There are several advantages in reducing the number of RNN updates. From the computational standpoint, fewer updates translates into fewer required sequential operations to process an input signal, leading to faster inference and reduced energy consumption. Unlike some other models that aim to reduce the average number of operations per step [38, 26], ours enables skipping steps completely. Replacing RNN updates with copy operations increases the memory of the network and its ability to model long term dependencies even for gated units, since the exponential memory decay observed in LSTM and GRU [38] is alleviated. During training, gradients are propagated through fewer updating time steps, providing faster convergence in some tasks involving long sequences. Moreover, the proposed model is orthogonal to recent advances in RNNs and could be used in conjunction with such techniques, e.g., normalization [13, 4], regularization [53, 30], variable computation [26, 38] or even external memory [20, 47].

3.1 Error gradients

The whole model is differentiable except for $f_{\text{binarize}}$, which outputs binary values. A common method for optimizing functions involving discrete variables is REINFORCE [48], although several estimators have been proposed for the particular case of neurons with binary outputs [7]. We select the straight-through estimator [23], which consists in approximating the step function by the identity when computing gradients during the backward pass:

$$\frac{\partial f_{\text{binarize}}(x)}{\partial x} = 1$$  \hspace{1cm} (7)
This yields a biased estimator that has proven more efficient than other unbiased but high-variance estimators such as REINFORCE [7] and has been successfully applied in different works [14, 12]. By using the straight-through estimator as the backward pass for $f_{\text{binarize}}$, all the model parameters can be trained to minimize the target loss function with standard backpropagation and without defining any additional supervision or reward signal.

### 3.2 Limiting computation

The Skip RNN is able to learn when to update or copy the state without explicit information about which samples are useful to solve the task at hand. However, a different operating point on the trade-off between performance and number of processed samples may be required depending on the application, e.g., one may be willing to sacrifice a few accuracy points in order to run faster on machines with low computational power, or to reduce energy impact on portable devices. The proposed model can be encouraged to perform fewer state updates through additional loss terms, a common practice in neural networks with dynamically allocated computation [33, 35, 18, 26]. In particular, we consider a cost per sample:

$$L_{\text{budget}} = \lambda \sum_{t=1}^{T} u_t$$  \hspace{1cm} (8)$$

where $L_{\text{budget}}$ is the cost associated to a single sequence, $\lambda$ is the cost per sample and $T$ is the sequence length. This formulation bears a similarity to weight decay regularization, where the network is encouraged to slowly converge towards a solution where the norm of the weights is smaller. Similarly, in this case the network is encouraged to slowly converge towards a solution where fewer state updates are required.
Despite this formulation has been extensively studied in our experiments, different budget loss terms can be used depending on the application. For instance, a specific number of samples may be encouraged by applying an $L_1$ or $L_2$ loss between the target value and the number of updates per sequence, $\sum_{t=1}^{T} u_t$.

4 Experiments

In the following section, we investigate the advantages of adding this state skipping to LSTMs and GRUs for a variety of tasks. In addition to the evaluation metric for each task, we also report the number of RNN state updates (i.e. the number of elements in the input sequence that are used by the model) as a measure of the computational load for each model. Since skipping an RNN update results in ignoring its corresponding input, we will refer to the number of updates and the number of used samples (i.e. elements in a sequence) interchangeably.

Training is performed with Adam [28], learning rate of $10^{-4}$, $\beta_1 = 0.9$, $\beta_2 = 0.999$ and $\epsilon = 10^{-8}$ on batches of 256. Gradient clipping [39] with a threshold of 1 is applied to all trainable variables. Bias $b_p$ in Equation 4 is initialized to 1, so that all samples are used at the beginning of training\(^2\). The initial hidden state $s_0$ is learned during training, whereas $\tilde{u}_0$ is set to a constant value of 1 in order to force the first update at $t = 1$.

Experiments are implemented with TensorFlow\(^3\) and run on a single NVIDIA K80 GPU.

4.1 Adding Task

We revisit one of the original LSTM tasks [24], where the network is given a sequence of (value, marker) tuples. The desired output is the addition of only the two values that are marked with a 1, whereas those marked with a 0 need to be ignored. We follow the experimental setup by Neil et al. [38], where the first marker is randomly placed among the first 10% of samples (drawn with uniform probability) and the second one is placed among the last half of samples (drawn with uniform probability). This marker distribution yields sequences where at least 40% of the samples are distractors and provide no useful information at all. However, it is worth noting that in this task the risk of missing a marker is very large as compared to the benefits of working on shorter subsequences.

We train RNN models with 110 units each on sequences of length 50, where the values are uniformly drawn from $\mathcal{U}(-0.5, 0.5)$. The final RNN state is fed to a fully connected layer that regresses the scalar output. The model is trained to minimize the Mean Squared Error (MSE) between the output and the ground truth. We consider that a model is able to solve the task when its MSE on a held-out set of examples is at least two orders of magnitude below the variance of the output distribution. This criterion is a stricter version of the one followed in [24].

While all models learn to solve the task, results in Table 1 show that Skip RNN models are able to do so with roughly half of the updates of their corresponding counterparts. Interestingly, Skip LSTM tends to skip more updates than the Skip GRU when no cost per sample is set, behavior that may be related to the lack of output gate in the latter. We hypothesize that there are two possible reasons why the output gate makes the LSTM more prone to skipping updates: (a) it introduces an additional source of memory decay, and (b) it allows to mask out some units in the cell state that may specialize in deciding when to update or copy, making the final regression layer agnostic to such process.

We observed that the models using fewer updates never miss any marker, since the penalization in terms of MSE would be very large (see Figure 2 for examples). These models learn to skip most of the samples in the 40% of the sequence where there are no markers. Moreover, most updates are skipped once the second marker is found, since all the relevant information in the sequence has been already seen. This last pattern provides evidence that the proposed models effectively learn to decide whether to update or copy the hidden state based on the input sequence, as opposed to learning biases in the dataset only. As a downside, Skip RNN models show some difficulties skipping a large number of updates at once, probably due to the cumulative nature of $\tilde{u}_t$.

\(^2\)In practice, forcing the network to use all samples at the beginning of training improves its robustness against random initializations of its weights and increases the reproducibility of the presented experiments. A similar behavior was observed in other augmented RNN architectures such as Neural Stacks [21].

\(^3\)https://www.tensorflow.org
<table>
<thead>
<tr>
<th>Model</th>
<th>Task solved</th>
<th>State updates</th>
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<tr>
<td>LSTM</td>
<td>Yes</td>
<td>100.0% ± 0.0%</td>
</tr>
<tr>
<td>Skip LSTM, $\lambda = 0$</td>
<td>Yes</td>
<td>81.1% ± 3.6%</td>
</tr>
<tr>
<td>Skip LSTM, $\lambda = 10^{-5}$</td>
<td>Yes</td>
<td>53.9% ± 2.1%</td>
</tr>
<tr>
<td>GRU</td>
<td>Yes</td>
<td>100.0% ± 0.0%</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 0$</td>
<td>Yes</td>
<td>97.9% ± 3.2%</td>
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<td>Skip GRU, $\lambda = 10^{-5}$</td>
<td>Yes</td>
<td>50.7% ± 2.6%</td>
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Table 1: Results for the adding task, displayed as mean ± std over four different runs. The task is considered to be solved if the MSE is at least two orders of magnitude below the variance of the output distribution.

4.2 Frequency Discrimination Task

In this experiment, the network is trained to classify between sinusoids whose period is in range $T \sim U(5, 6)$ milliseconds and those whose period is in range $T \sim \{(1, 5) \cup (6, 100)\}$ milliseconds [38]. Every sine wave with period $T$ has a random phase shift drawn from $U(0, T)$. At every time step, the input to the network is a single scalar representing the amplitude of the signal. Since sinusoid are continuous signals, this task allows to study whether Skip RNNs converge to the same solutions when their parameters are fixed but the sampling period is changed. We study two different sampling periods, $T_s = \{0.5, 1\}$ milliseconds, for each set of hyperparameters.

We train RNNs with 110 units each on input signals of 100 milliseconds. Batches are stratified, containing the same number of samples for each class, yielding a 50% chance accuracy. The last state of the RNN is fed into a 2-way classifier and trained with cross-entropy loss. We consider that a model is able to solve the task when it achieves an accuracy over 99% on a held-out set of examples.

Table 2 summarizes results for this task. When no cost per sample is set ($\lambda = 0$), the number of updates differ under different sampling conditions. We attribute this behavior to the potentially large number of local minima in the cost function, since there are numerous subsampling patterns for which the task can be successfully solved and we are not explicitly encouraging the network to converge to a particular solution. On the other hand, when $\lambda > 0$ Skip RNN models with the same cost per sample use roughly the same number of input samples even when the sampling frequency is doubled. This is a desirable property, since solutions are robust to oversampled input signals.

4.3 MNIST Classification from a Sequence of Pixels

The MNIST handwritten digits classification benchmark [32] is traditionally addressed with Convolutional Neural Networks (CNNs) that can efficiently exploit spatial dependencies through weight sharing. By flattening the $28 \times 28$ images into 784-d vectors, however, it can be reformulated as a challenging task for RNNs where long term dependencies need to be leveraged [31]. We follow the standard data split and set aside 5,000 training samples for validation purposes. After processing all pixels with an RNN with 110 units, the last hidden state is fed into a linear classifier predicting the digit class. All models are trained for 600 epochs to minimize cross-entropy loss.
Table 2: Results for the frequency discrimination task, displayed as mean ± std over four different runs. The task is considered to be solved if the classification accuracy is over 99%. Models with the same cost per sample (λ > 0) converge to a similar number of used samples under different sampling conditions.

Table 3: Accuracy and used samples on the test set of MNIST after 600 epochs of training. Results are displayed as mean ± std over four different runs.

Table 3 summarizes classification results on the test set after 600 epochs of training. Skip RNNs are not only able to solve the task using fewer updates than their counterparts, but also show a lower variation among runs and train faster (see Figure 3). We hypothesize that skipping updates make the Skip RNNs work on shorter subsequences, simplifying the optimization process and allowing the networks to capture long term dependencies more easily. A similar behavior was observed for Phased LSTM, where increasing the sparsity of cell updates accelerates training for very long sequences [38].

Sequences of pixels can be reshaped back into 2D images, allowing to visualize the samples used by the RNNs as a sort of hard visual attention model [49]. Examples such as the ones depicted in Figure 4 show how the model learns to skip pixels that are not discriminative, such as the padding regions in the top and bottom of images. Similarly to the qualitative results for the adding task (Section 4.1), attended samples vary depending on the particular input being given to the network.

4.4 Sentiment Analysis on IMDB

The IMDB dataset [34] contains 25,000 training and 25,000 testing movie reviews annotated into two classes, positive and negative sentiment, with an approximate average length of 240 words per review. We set aside 15% of training data for validation purposes. Words are embedded into 300-d vector representations before being fed to an RNN with 128 units. The embedding matrix is initialized using pre-trained word2vec embeddings [36] when available, or random vectors drawn from \( \mathcal{U}(-0.25, 0.25) \) otherwise [27]. Dropout with rate 0.2 is applied between the last RNN state and the classification layer in order to reduce overfitting. We evaluate the models on sequences of length 200 and 400 by cropping longer sequences and padding shorter ones [51].

Results on the test are reported in Table 4. In a task where it is hard to predict which input tokens will be discriminative, the Skip RNN models are able to achieve similar accuracy rates to the baseline models while reducing the number of required updates. These results highlight the trade-off between accuracy and the available computational budget, since a larger cost per sample results
Figure 3: Accuracy evolution during training on the validation set of MNIST. The Skip GRU exhibits lower variance and faster convergence than the baseline GRU. A similar behavior is observed for LSTM and Skip LSTM, but omitted for clarity. Shading shows maximum and minimum over 4 runs, while dark lines indicate the mean.

Figure 4: Sample usage examples for the Skip LSTM with $\lambda = 10^{-4}$ on the test set of MNIST. Red pixels are used, whereas blue ones are skipped.

in lower accuracies. However, allowing the network to select which samples to use instead of cropping sequences at a given length boosts performance, as observed for the Skip LSTM (length 400, $\lambda = 10^{-4}$), which achieves a higher accuracy than the baseline LSTM (length 200) while seeing roughly the same number of words per review. A similar behavior can be seen for the Skip RNN models with $\lambda = 10^{-3}$, where allowing them to select words from longer reviews boosts classification accuracy while using a comparable number of tokens per sequence.

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<td>Accuracy</td>
<td>State updates</td>
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<td>200.00 ± 0.000</td>
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<td>0.844 ± 0.004</td>
<td>196.75 ± 5.63</td>
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<td>Skip LSTM, $\lambda = 10^{-5}$</td>
<td>0.846 ± 0.004</td>
<td>197.15 ± 3.16</td>
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<tr>
<td>Skip LSTM, $\lambda = 10^{-4}$</td>
<td>0.837 ± 0.006</td>
<td>164.65 ± 8.67</td>
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<tr>
<td>Skip LSTM, $\lambda = 10^{-3}$</td>
<td>0.811 ± 0.007</td>
<td>73.85 ± 1.90</td>
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<tr>
<td>GRU</td>
<td>0.845 ± 0.006</td>
<td>200.00 ± 0.000</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 0$</td>
<td>0.848 ± 0.002</td>
<td>200.00 ± 0.000</td>
</tr>
<tr>
<td>Skip GRU, $\lambda = 10^{-5}$</td>
<td>0.842 ± 0.005</td>
<td>199.25 ± 1.30</td>
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<td>Skip GRU, $\lambda = 10^{-4}$</td>
<td>0.854 ± 0.006</td>
<td>180.97 ± 8.90</td>
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<td>Skip GRU, $\lambda = 10^{-3}$</td>
<td>0.800 ± 0.007</td>
<td>106.15 ± 37.92</td>
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Table 4: Accuracy and used samples on the test set of IMDB for different sequence lengths. Results are displayed as mean ± std over four different runs.
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<td>0.757</td>
<td>24.2</td>
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<td>0.791</td>
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<td>23.7</td>
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Table 5: Accuracy and used samples on the validation set of UCF-101 (split 1).

4.5 Action classification on UCF-101

One of the most accurate and scalable pipelines for video analysis consists in extracting frame level features with a CNN and modeling their temporal evolution with an RNN [17, 52]. Videos are commonly recorded at high sampling rates, rapidly generating long sequences with strong temporal redundancy that are challenging for RNNs. Moreover, processing frames with a CNN is computationally expensive and may become prohibitive for high framerates. These issues have been alleviated in previous works by using short clips [17] or by downsampling the original data in order to cover long temporal spans without increasing the sequence length excessively [52]. Instead of addressing the long sequence problem at the input data level, we train RNN models using long frame sequences without downsampling and let the network learn which frames need to be used.

UCF-101 [44] is a dataset containing 13,320 trimmed videos belonging to 101 different action categories. We use 10 seconds of video sampled at 25fps, cropping longer ones and padding shorter examples with empty frames. Activations in the Global Average Pooling layer from a ResNet-50 [22] CNN pretrained on the ImageNet dataset [16] are used as frame level features, which are fed into two stacked RNN layers with 512 units each. The weights in the CNN are not tuned during training to reduce overfitting. The hidden state in the last RNN layer is used to compute the update probability for the Skip RNN models.

We evaluate the different models on the first split of UCF-101 and report results in Table 5. Skip RNN models do not only improve the classification accuracy with respect to the baseline, but require very few updates to do so, possibly due to the low motion between consecutive frames resulting in frame level features with high temporal redundancy [42]. Moreover, Figure 5 shows how models performing fewer updates converge faster thanks to the gradients being preserved during longer spans when training with backpropagation through time.
5 Conclusion

We presented Skip RNNs as an extension to existing recurrent architectures enabling them to skip state updates thereby reducing the number of sequential operations in the computation graph. Unlike other approaches, all parameters in Skip RNN are trained with backpropagation without requiring the introduction of task-dependent hyperparameters like a dropout rate. Experiments conducted with LSTMs and GRUs showed that Skip RNNs can match or in some cases even outperform the baseline models while relaxing their computational requirements. Skip RNNs provide faster and more stable training for long sequences and complex models, likely due to gradients being backpropagated through fewer time steps resulting in a simpler optimization task. Moreover, the introduced computational savings are better suited for modern hardware than those methods that reduce the amount of computation required at each time step [29, 38, 12].

The presented results motivate several new research directions toward designing efficient RNN architectures. Introducing stochasticity in neural network training has proven beneficial for generalization [45, 30], and in this work we propose a deterministic rounding operation with stochastic sampling. We showed that the addition of a loss term penalizing the number of updates is important in the performance of Skip RNN and allows flexibility to specialize to tasks of varying budget requirements, e.g. the cost can be increased at each time step to encourage the network to emit a decision earlier [1], or the number of updates can be strictly bounded and enforced. Finally, understanding and analyzing the patterns followed by the model when deciding whether to update or copy the RNN state may provide insight for developing better and more efficient architectures.

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