Laser Feedback Interferometry with Semiconductor Lasers

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Abstract

Self-mixing effect is a remarkably universal phenomena that occurs for any kind of laser and it requires a simple electro-optical circuit. Additionally, the new semiconductors’ technologies enable reducing costs and dimensions while efficiency is, by contrast, augmented. Consequently, a huge amount of research in optics in the last few years has focused in the Laser Feedback Interferometry techniques for Semiconductor Lasers and it is still growing at an accelerated rate.

These techniques only require one optical path so simplicity and reduced dimensions are the biggest advantages compared to other typical interferometric techniques. In some cases, even the photodetector is not used, leading the set-up to be formed exclusively by the laser diode.

The main aim of this thesis was to study the fundamental physics of placing and observing external photodetector and to implement a practical LFI application capable of efficiently detecting bubbles flowing within a microfluidic channel and estimating their volume in real-time. Hence, this thesis will initially focus on what has been already done in this field in order to acknowledge the basics and to reach the main aim afterwards.

During this thesis, we have demonstrated the strong relevance of one of the model’s assumptions that may provide answers to some frequent questions literature has been presenting during the recent years; whereas, on the other hand, we have been able to use self-mixing to estimate the volume of bubbles with less than a 10 % of error within a certain fluid pumping range.
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1 Introduction

The first laser was built by Theodore H. Maiman in 1960, 53 years later than Albert Einstein did the re-derivation of Max Planck’s Law of radiation based upon probability coefficients, the Einstein coefficients, and after the theoretical work carried by Schawlow and Townes [1] in 1958. The invention was quickly followed by demonstrations about their behavior. It was noticed that external feedback coming into the laser cavity induced a modulation of the intensity in its output. Just three years later, King and Steward reported it and were able to use what was considered feedback from a third external mirror, or after known as self-mixing, in the field of laser metrology [2–4] based on the previous demonstrations and measurements in gas lasers. Lots of groups have been carrying research about laser feedback interferometry or LFI for metrology under several different names.

The self-mixing effect is achieved when part of the emitted light of the laser reflects back into its cavity. The reflected electromagnetic wave interacts with the ones inside the cavity of the laser modulating its output power. King and Steward noticed that the effect was observable when the reflected power from an external mirror situated up to 10 m was at least 0.1 % of the emitted one. In 1968, Rudd [5] presented the first Doppler velocimeter with the cavity of the laser as the optical mixer.

Initially, researchers modeled the self-mixing effect as a loss of power. However, self-mixing effect occurs when a previously emitted laser electromagnetic wave is reflected back to the laser cavity after interacting in the external cavity and mixes with the intracavity electromagnetic waves or modes. Hence, they realized that the Fabry-Pérot interferometer with three mirrors, three mirror model, was the most suitable way to understand the self-mixing effect and, in 1980, Lang and Kobayashi [6] developed the model for semiconductor lasers under optical feedback.

Interaction between electromagnetic waves is a really well-known phenomena and it is the principle of interferometry. The Michelson interferometer (See Fig.1.1) is the most classic and standard interferometric setup where the laser output, LD, and the photodetector, PD, are separated.

However, self-mixing enables you having them together as the interferometry phenomena takes place in the laser cavity; just placing the PD behind the emitting cavity of the laser. As a result, the setup is simpler and also permits the total replacement of the PD if the signal monitored is the voltage across the laser terminals.
Chapter 1: Introduction

Figure 1.1: Michelson interferometer (Left) and self-mixing interferometer (Right). (Source: en.wikipedia.org/wiki/Self-mixing_interferometry)

The main goal of this thesis is to understand the self-mixing effect and use it to develop new LFI techniques. Thus, in this thesis there will be seen the basic LFI applications in order to understand the theory introduced at the beginning and, as long as they are introduced, more complexity will be reached, ending in a new application for self-mixing consisting in detecting and analyzing bubbles flowing within a microfluidic channel. Additionally, a fundamental study of about the physics of self-mixing will be also carried.

In order to do so, firstly it will be necessary to study the self-mixing effect and which are the required components to have a LFI set-up. Then, once the LFI is achieved, signal will have to be properly processed. During all this process, it will be required a sense of creativity, showing that the underlying theory is acknowledged and made profit of it. Finally, conclusions of the results obtained and the whole research process will be reached using critical judgment.
2 Theoretical Background

In this chapter, it will be presented the basic theory underneath the self-mixing effect in lasers in order to understand its working principle and its potential applications.

2.1 Laser

Laser is the acronym for "Light Amplification by Stimulated Emission of Radiation" and its working principle is based on the amplification of the emitted light using the stimulated emission of electromagnetic radiation effect.

Einstein derived the theory for the stimulated emission [7, 8] in 1916 from the Max Planck’s law of radiation within the frame of the called old quantum theory, previous results to the modern quantum mechanics. In this derivation, Einstein split Planck’s law in absorption, spontaneous emission and stimulated emission using the Einstein coefficients.

![Figure 2.1: Absorption (Left), spontaneous emission (Center) and stimulated emission (Right) processes.](Source: http://www.optique-ingenieur.org/en/courses/OPI_ang_M01_C01/co/Contenu_05.html)

They can be modeled considering a two electronic energy state atom, a lower state level with energy $E_1$ and an excited state with $E_2$ as shown in Fig.2.1. In the case that the atom is found in its excited state and spontaneously decays to the lower one by the spontaneous emission process, the emitted photon will have an energy equal to the difference between $E_2$ and $E_1$. Quantum mechanics relates the energy of photons with its frequency by using
the Planck’s constant \( h \) or quantum of action.

\[
E_2 - E_1 = h\nu_0
\]  

(2.1)

On the contrary, if the atom is perturbed by an external electric field of the same frequency \( \nu_0 \) whilst in the excited state, an additional photon of the exactly same frequency and phase may be emitted. As a result, the external field is enhanced and the atom decays to the lower level. Whereas in the previously process the emission was spontaneous without the need for an additional electric field, this process needs such stimulus. Therefore, it receives the name of stimulated emission.

The further step is to consider a group of \( N \) atoms with such levels. Some of them will be in their excited state, \( N_2 \) and the rest, \( N_1 \), in their lower one. Einstein expressed the rate at which stimulated emission occurs as:

\[
\frac{\partial}{\partial t} N_2 = -\frac{\partial}{\partial t} N_1 = -B_{21}\rho(\nu)N_2
\]  

(2.2)

Where \( B_{21} \) is the Einstein B coefficient for the stimulated emission and \( \rho(\nu) \) is the radiation density of the incident field at the frequency \( \nu \).

Analogously, an atom from the lower state may absorb a photon and reach its excited state:

\[
\frac{\partial}{\partial t} N_2 = -\frac{\partial}{\partial t} N_1 = B_{12}\rho(\nu)N_1
\]  

(2.3)

It could be easily appreciated that both rates are practically identical. Einstein showed that both absorption and stimulated emission coefficients must be the same, \( B_{12} = B_{21} \). Thus, absorption and stimulated emission are reverse processes happening at different rates, determined by the radiation density and both number of atoms in the different states.

Hence, the rate of atoms in each state must include both phenomenas:

\[
\frac{\partial}{\partial t} N_1 = -\frac{\partial}{\partial t} N_2 = B_{21}\rho(\nu)(N_2 - N_1)
\]  

(2.4)

Leading us to the population inversion condition \( N_2 > N_1 \) in order to have a net stimulated emission, which means that there must be a higher number of excited atoms than relaxed ones. On the contrary, there would be a net absorption reducing the power of the electromagnetic field within the medium.

The biggest benefit of stimulating the emission of the photons is that all those emitted photons have the same frequency, phase, polarization and direction of propagation as the incident photons. As a consequence, the light is highly coherent, much more coherent than other light emitters as lamps. Thus, light emitted by a laser can interfere, enabling the use of interferometric techniques.

Although nowadays there are lots of different types of lasers, they basically consist on:

- Laser pumping energy: necessary energy supplied to the medium in order to stimulate the emission of light.
Chapter 2: Theoretical Background

2.1 Laser

Figure 2.2: Laser general structure. (Source: http://www.thefabricator.com/article/lasercutting/are-you-getting-enough-fiber-in-your-metal-fabricating-dietr)

- Laser gain medium: medium where the population inversion takes place and so, the stimulated emission.

- Laser cavity: formed by the rear mirror and the front one. The first one must be a perfect reflector whereas the front one must be partially reflector as a part of the incident light must leave the cavity whereas the other must reflect back to the laser cavity in order to increase the gain. This last one might be known as Optical coupler.

2.1.1 Semiconductor Lasers

A laser diode or LD is, as its name indicates, a semiconductor diode able to emit light, similar to a light-emitting diode or LED. In this semiconductor laser, the gain medium is formed by a p-n junction with an intrinsic region between them. In this way, carriers – either electrons or holes – are pumped from the P and N regions to the intrinsic area, where they are confined. A photon is emitted each time an electron and a hole annihilate each other or recombine, thus if they are confined in a region, the recombination is enhanced due to the maximization of the chances for recombination.

When talking about efficiency, it is really important which type of semiconductor is used to. In order to recombination take place, one electron from the minimal energy state of the conduction band has to recombine with a hole from the maximal energy state of the valence band. In this case, if both states have the same crystal momentum or \( k \)-vector, they recombine directly and emit a photon of the energy of the band gap. On the contrary, if they belong to different \( k \)-vectors, the recombination will have to emit a phonon to the crystal lattice, in order to transfer momentum and match the crystal momentum of both carriers, and then recombine emitting a photon less energetic - the energy of the band-gap minus the difference of momentum. (See Fig.2.3)

Apart from that both photons will have different frequencies due to their energy difference, the photon emitted directly is more likely to happen than the other one. Consequently, as the aim of a LD is to maximize the recombination, direct band-gap semiconductors must be used rather than indirect ones.

Additionally, the optical cavity of the laser is manufactured by cleaving both crystal’s surface to form parallel edges, forming a Fabry-Pérot resonator. Light is remitted back to the cavity and then amplified by the stimulated emission process. However, a LD has a threshold where it is said that it starts lasing or not. In order to lase, the stimulated emission
must dominate - have a higher rate than the spontaneous emission - and amplification must be greater than the loss from absorption or incomplete reflection.

Nonetheless, the geometry of the laser diode determine some of its important properties as well as its emission direction (see Fig. 2.4). One of the most important one is the width of the layer that acts as a waveguide. If it is thin compared to the emission wavelength, the waveguide can only handle an unique single optical mode whereas, if it is wider compared to the wavelength, it can support multiple transverse optical modes. Hence, there are single-mode LDs and multi-mode ones.

Figure 2.3: Direct semiconductor band gap recombination process (Left) and indirect one (Right). (Source: https://www.researchgate.net/figure/281067517_fig11_Direct-and-indirect-band-gap)

Figure 2.4: Four different types of diode lasers and their emission. (Top-left) Edge-Emitting Laser, EEL. (Top-right) Quantum Cascade Laser, QCL. (Bottom-left) Vertical-Cavity Surface-Emitting Laser, VCSEL. (Bottom-right) Vertical-External-Cavity Surface-Emitting Laser, VECSEL. (Source: http://www.jos.ac.cn/bdtxbcn/ch/html/15101314.htm)
Chapter 2: Theoretical Background

2.1 Laser

This thesis will focus on the VCSELs (see Fig.2.4) and the DFBs regarding that both of them will be used in the following work, as explained in Chapter 3.

2.1.2 VCSELs

One type of semiconductor laser is the Vertical-Cavity Surface-Emitting lasers, known as VCSELs. The top surface of the diode emits a perpendicular laser beam, contrary to in-plane lasers which emit from surfaces formed by cleaving an individual chip out of a wafer. VCSELs, contrary to other diode lasers, have the optical cavity along the direction of current flow instead of perpendicular to it.

The active region of VCSELs, formed by one or more quantum wells were electrons and holes are confined to annihilate and emit light, is confined between two dielectric mirrors known as Distributed Bragg Reflector, DBR. These reflectors consist in a group of alternated layers of a quarter of the wavelength thickness of low and high refractive index. The alternated and sudden changes in the refractive index yield the mirror to an intensity reflectivity higher than 99 %, property strictly needed for lasing due to the thick axial region of the gain medium.

Common VCSELs form a diode, meaning that they have the upper and lower mirror doped as p-type and n-type materials. The range of wavelengths between 650 nm and 1300 nm VCSELs are typically based on gallium arsenide, GaAs, wafers with GaAs and aluminium gallium arsenide, Al\textsubscript{x}Ga\textsubscript{1-x}As, DBRs due to their manufacturing advantages. Longer wavelengths, from 1300 to 2000 nm, have been performed with active regions based on indium phosphide, InP. The wavelength of the VCSEL can be tuned by modifying the thickness of the layers, as they are wavelength selective due to the strict condition stated above.

The main advantage of VCSELs with respect to Edge Emitting Laser or EEL is that due to their top emitting surface, they can be tested whilst the manufacturing process and being built in two dimensional arrays, reducing the fabrication cost. In addition, VCSELs have a larger output aperture that produces a a lower divergence angle of the output beam, yielding a high coupling efficiency with optical fibers. Furthermore, the high reflectivity drops the threshold current, and so the power consumption, enabling a high intrinsic modulation bandwidth. However, the emission power is lower than the EEL one, although there exist high-power VCSELs.

2.1.3 DFBs

Another type of laser diode, and the second one used, is the Distributed Feedback Laser or DFB. The main characteristic, the one that provides the meaning to its name, is that one layer of the active region is periodically structured to act as a diffraction grating as shown in Fig.2.5. The grating behaves as an optical filter, stabilizing the lasing wavelength because only a narrow broadband around the designed wavelength can feedback the laser.

The grating is a one-dimensional interference, Bragg scattering, and as it is the responsible for the feedback, no reflection from the facets is longer needed. Hence, at least one facet is anti-reflection coated. Due to the non-selective mirrors, DFBs are tunable with
temperature. The semiconductor material changes its refractive index and structure with temperature, changing the grating and so, the selected wavelength. Commonly, they are able to tune about 6 nm for a 50 K change, making them stable and precise.

Consequently, DFBs are widely used in optical communication applications, such as Dense Wavelength Division Multiplexing or DWDM, within the 1550 nm.

Their output power is typically high, as one of the facets, the antireflection-coated one, does not reflect back the beam whereas the other facet prevents losing power from behind. However, their threshold current is about 11 mA and their appropriate bias current in a linear regime could be considered 50 mA.

2.2 Excess phase equation

Generally, all LFI systems work under a quasi-static regime where the photon density $S(t)$ and the carrier density $N(t)$ do not change as fast as the natural phase $\phi(t)$ of the system. The Lang–Kobayashi rate equations derived in Appendix 1 can be reduced to a temporary steady-state whenever the above conditions are satisfied and by substituting $S(t) = S(t - \tau_{ext}) = S_0$, $\phi(t) = (\omega - \omega_{th})t = (\omega - \omega_s)t$ and $N(t) = N_0$, allowing to find the steady-state solutions for the rate equations.

$$\frac{d}{dt} S(t) = 0 = (\Gamma G - \frac{1}{\tau_p}) S_0 + 2\tilde{\kappa} S_0 \cos(\omega \tau_{ext})$$
$$\implies S_0 \left[ (\Gamma G - \frac{1}{\tau_p}) - \tilde{\kappa} \cos(\omega \tau_{ext}) \right] = 0$$
$$\implies (\Gamma G - \frac{1}{\tau_p}) + 2\tilde{\kappa} \cos(\omega \tau_{ext}) = 0$$
$$\implies (\Gamma G - \frac{1}{\tau_p}) = -2\tilde{\kappa} \cos(\omega \tau_{ext})$$

Noting that $(\Gamma G - \frac{1}{\tau_p}) = (N - N_{th}) \Gamma (\frac{\Delta G}{\Delta N}) \approx (N_s - N_{th}) \Gamma \nu_g \alpha$, the first steady-state solution is given by

$$\Gamma G - \frac{1}{\tau_p} = 0$$

Figure 2.5: Example of grating in a DFB diode laser.
(Source: https://www.wsi.tum.de/Research/AmanngroupE26/AreasofResearch/Epitaxy/tabid/104/Default.aspx)
Chapter 2: Theoretical Background

2.2 Excess phase equation

The solution is yield:

\[
(N_s - N_{th}) \Gamma \left( \frac{\partial G}{\partial N} \right) + 2 \kappa \cos(\omega \tau_{ext}) = 0
\]

\[
\rightarrow N_s = N_{th} - \frac{2 \kappa}{\Gamma \nu_g \alpha} \cos(\omega \tau_{ext}) = N_{th} - \frac{2 k}{\Gamma \nu_g \tau_{in}} \cos(\omega \tau_{ext})
\] (2.6)

The photon density steady-solution is directly obtained through the carrier density rate equation as follows:

\[
\frac{d}{dt} N(t) = 0 = \eta I_s \frac{N_s}{\tau_n} - GS_s
\]

\[
\rightarrow S_s = \frac{1}{G} \left[ \frac{\eta I_s}{qV} - \frac{N_s}{\tau_n} \right]
\] (2.7)

Finally, the phase steady-solution must be derived and

\[
\frac{d}{dt} \phi(t) = (\omega - \omega_s) t = \frac{1}{2} (\alpha G - \frac{1}{\tau_p}) - \kappa \sin(\omega \tau_{ext})
\]

\[
\rightarrow \omega - \omega_s = -\alpha \kappa \cos(\omega \tau_{ext}) - \kappa \sin(\omega \tau_{ext}) = 0
\] (2.8)

A useful trigonometric relation for simplifying Eq.2.8 is \( A \cos(x) + B \sin(x) = \sqrt{A^2 + B^2} \cos(x - \arctan(B/A)) \) as well as other basic trigonometric relations:

\[
\omega - \omega_s = -\sqrt{(\alpha \kappa)^2 + \kappa^2} \cos(\omega \tau_{ext}) - \arctan\left( \frac{\kappa}{\alpha \kappa} \right)
\]

\[
= -\kappa \sqrt{1 + \alpha^2} \cos(\omega \tau_{ext}) - \arctan\left( \frac{\pi}{2} + \arctan(\alpha) \right)
\] (2.9)

The last step to yield the excess phase equation is to define \( \omega_s \tau_{ext} \approx \varphi_s \) as the phase stimulus transmitting through the external cavity as if there was no feedback; \( \omega \tau_{ext} \approx \varphi_{FB} \), the phase response to the feedback; and finally the dimensionless feedback level \( C = (k/\tau_{in}) \sqrt{1 + \alpha^2} = \kappa \tau_{ext} \sqrt{1 + \alpha^2} \); and to multiply by \( \tau_{ext} \) the above Eq.2.9.

\[
\varphi_{FB} - \varphi_s + C \sin(\varphi_{FB} + \arctan(\alpha)) = 0
\] (2.10)

Note that the excess phase equation is a transcendental equation, so its solution must be found using numerical methods as the Newton’s algorithm. It describes five different regimes depending on the value of the optical feedback level:

1. \( C \leq 1 \): Weak feedback regime. It is the only regime where it only exists one solution so there is just a unique emission frequency.
2. \( C > 1 \): Moderate feedback regime.

3. \( C \gg 1 \): Strong feedback regime.

4. \( \tilde{\kappa} \simeq \) relaxation oscillation frequency of the laser. Chaotic regime or coherence collapse. The emission line is broadened and the dynamics are chaotic but with some isolated stable regions. This implies that the laser remains partially dependent on the phase of the feedback.

5. \( \tilde{\kappa} \gg \) relaxation oscillation frequency of the laser. Return to stability regime. The laser works independently from the feedback phase because the whole system is behaving as a long external cavity laser optically pumped.

### 2.3 Measurable signal: Output power variation

Although the excess phase equation (Eq.2.10) relates the optical feedback with a physical parameter as the phase, it is not measurable. However, as noticed by several researchers and reported by King and Steward [2–4], the output optical power or the terminal’s voltage of a laser working under optical feedback are modified. Hence, the output power variation is an observable and measurable signal.

The output power is related to the density of photons, which, in a steady-state, is related through the carrier density and a linear gain by using both Eq.2.5 and Eq.2.6.

\[
P_s \propto S_s = \frac{1}{G} \left( \frac{\eta I_s}{qV} - \frac{N_s}{\tau_n} \right) = \frac{\Gamma \tau_p}{1 - 2\tilde{\kappa} \tau_p \cos(\varphi_{FB})} \left( \frac{\eta I_s}{qV} - \frac{N_{th}}{\tau_n} + \frac{2\tilde{\kappa}}{\Gamma \nu \alpha \tau_n} \cos(\varphi_{FB}) \right)
\]  (2.11)

The above expression can be simplified assuming that \( 2\tilde{\kappa} \tau_p \ll 1 \) so it can be applied that \( (1 - x)^{-1} \approx (1 + x) \) for \( x \ll 1 \) and, consequently, the term \( 2\tilde{\kappa} \tau_p \cos(\varphi_{FB}) \) can be neglected compared to the other terms:

\[
P_s \propto \Gamma \tau_p \left( \frac{\eta I_s}{qV} - \frac{N_{th}}{\tau_n} \right) \left( 1 + 2\tilde{\kappa} \tau_p \cos(\varphi_{FB}) \right)
\]  (2.12)

Hence, it can be easily appreciated that the variation in the output power around the steady-state is related to the cosine of the phase response to the optical feedback. Hence, the output power variation is assumed to be:

\[
\Delta P \approx \beta \cos(\varphi_{FB})
\]  (2.13)

Where \( \beta \) refers to the amplitude of the measured self-mixing signal.

### 2.3.1 Absolute distance measurement

As defined in Sec.2.2, the laser stimulus steady-state phase is \( \varphi_s = \omega_s \tau_{\text{ext}} = 2\pi \nu \tau_{\text{ext}} \) and it is directly related to the frequency of emission of the laser. However, this emitting
frequency $\nu$ varies approximately linearly with the driving current of the laser $I$ as follows:

$$\Delta \omega = 2\pi \Delta \nu = 2\pi \Omega \Delta I$$

(2.14)

Where $\Omega$ is an empirical modulation frequency coefficient that, for semiconductor laser diodes, its value is typically around -3 GHz/mA. This enables sweeping the frequency of the laser by modulating its input current.

$$\phi_s(t) = \omega_s(t) \tau_{\text{ext}} = \tau_{\text{ext}}(\omega_s(0) + 2\pi \Omega \Delta I(t))$$

(2.15)

Let us modulate the laser with a sawtooth signal of the form $\Delta I(t) = At$ where $A$ is the amplitude of the signal and $T$ its period. Consequently, the above expression Eq.2.15 will be periodic in time as well.

$$\phi_s(t) = \tau_{\text{ext}}(\omega_s(0) + 2\pi \Omega At) \rightarrow 2\pi \tau_{\text{ext}}\Omega AT = 2\pi$$

(2.16)

The external cavity round-trip time $\tau_{\text{ext}}$ is related with twice the length of the external cavity, $\tau_{\text{ext}} = n_{\text{ext}} 2L_{\text{ext}} / c$, which is the absolute distance of the target; as photons have to reflect back to the laser terminal. This yields to a relation between $L_{\text{ext}}$ and the known stimulation signal:

$$L_{\text{ext}} = \frac{c}{2n_{\text{ext}}\Omega AT}$$

(2.17)

Beheim and Fritsch [9] were able to develop a counting-fringe algorithm in which knowing the driving parameters of the signal, the number of fringes $N_f$ appearing on the self-mixing signal are directly related to the external cavity length $L_{\text{ext}}$:

$$L_{\text{ext}} = \frac{c(N_f \pm 1)}{2n_{\text{ext}}\Delta \omega}$$

(2.18)

Some simulations following the authors [10] are shown in Fig.2.6, where there can be observed the fringes in the power variation of the laser and how the optical feedback parameter determines the shape of the signal.

### 2.3.2 Harmonic motion measurement

The fact that the external cavity length $L_{\text{ext}}$ determines the external round-trip time of light $\tau_{\text{ext}}$ and, consequently, affects to the excess phase equation; makes obvious that displacing the target will affect somehow the measured signal.

Firstly, let the target move harmonically from its rest position with an amplitude $\Delta L \gg \lambda / (2n_{\text{ext}})$ and a frequency $f$ so the external cavity length can be expressed as follows:

$$L_{\text{ext}}(t) = L_{\text{ext},s} + \Delta L \cos(2\pi ft + \phi) \rightarrow \Delta L_{\text{ext}}(t) = \Delta L \cos(2\pi ft + \phi)$$

(2.19)
Figure 2.6: Simulation results for $\lambda = 850$ nm, $L_{\text{ext}} = 24$ cm, $\alpha = 4.6$ and a triangular stimulation signal of 15 ms of period. The power variation (Left) and its derivative (Right) under weak optical feedback regime of $C = 0.5$ (Top), moderate regime of $C = 2$ (Center) and strong regime of $C = 10$ (Bottom).

Moreover, the round-trip time becomes harmonic and so the stimulus phase $\phi_s$ which can be easily derived:

$$
\phi_s(t) = \omega_s \tau_{\text{ext}}(t) = \frac{2\pi c n_{\text{ext}} 2L_{\text{ext}}(t)}{\lambda} = \frac{4\pi n_{\text{ext}}}{\lambda} \left( L_{\text{ext,s}} + \Delta L \cos(2\pi ft + \phi) \right)
$$

Notice that the term $4\pi n_{\text{ext}} \Delta L / \lambda$ is the total change in $\phi_s(t)$ and must have a periodicity of $2\pi$. Hence, each time the target moves a distance $\Delta L = \lambda / (2n_{\text{ext}})$, the stimulus phase has suffered a $2\pi$ shift. In addition, this leads the signal to become periodic by half-wavelength displacements appearing what are called “fringes” [11], shown in Fig.2.7.
2.3.3 Doppler velocity measurement

Self-mixing is not only limited to range finders or distance measurements applications but also it is useful for the velocimetry or radar applications. [12, 13] It is really well-known that a moving object provokes a shift in the frequency of any incident wave propagation, in this case light. This effect is known as Doppler effect.

Light is an electric field propagating through space with waveforms of a certain wavevector $k$. When incident light strikes the surface of a moving object, the scattered light suffers a shift in its frequency as follows:

$$\Delta f = \frac{\Delta \nu}{c} f_{\text{inc}} = \frac{1}{2\pi} (k_{\text{scatt}} - k_{\text{inc}}) \cdot \vec{v}_{\text{obj}}$$  \hspace{1cm} (2.21)

Furthermore, the expression can be simplified by considering direct back-scattering as it will be the only one able to produce the self-mixing effect. Thus, $k_{\text{scatt}} = -k_{\text{inc}}$, and applying that $|\vec{k}| = 2\pi n / \lambda$ leads to:

$$|\Delta f| = \frac{1}{\pi} k_{\text{inc}} \cdot \vec{v}_{\text{obj}} = \frac{2n_{\text{ext}}}{\lambda} v \cos(\theta)$$  \hspace{1cm} (2.22)

Where $n_{\text{ext}}$ is the refractive index of the external cavity medium, $\lambda$ is the wavelength of the laser and $\theta$ is the angle formed between the propagation of the laser beam and the velocity of the target. Obviously, in order to sense, the sensing surface must not be normal to the incident light so $\cos(\theta) \neq 0$. In other words, the laser beam must have a certain tilt angle with respect to the particles’ movement direction.

Due to the self-mixing effect, the Doppler frequency will be modulating the output power of the laser and, consequently, there will be a higher power spectral density in such frequency. Thus, the Doppler frequency is easily found in the PSD of the signal and applying Eq.2.22 the velocity of the target is sensed.

![P.S.D. (dBV^2/Hz)](image)

Figure 2.8: Power Spectral Density for a simulation with $\lambda = 850$ nm, $f_{\text{Doppler}} = 10$ kHz and $\alpha = 4.6$.

Laser Doppler velocimetry is a widely used technique for measuring displacement, velocity and acceleration of a remote target. Targets can be of any kind of nature as the only requirement is that light must interact by scattering with particles. Hence, LDV is a really powerful technique to sense microfluidic channels where scatterers can be found [14]. Due to the absence of contact, measures are highly precise and of high resolution.
Now that the background for the self-mixing effect and some of the LFI techniques have been explained, this chapter will focus on the different material used during all the laboratory experiences carried during this project that have helped to understand and link the above explained theory as well as to reach the final aim of this thesis.

An opto-electronic circuit is formed by two coupled circuits, an optical and an electronic one. For that reason, we must differ between the optical components and the electrical ones although they are coupled in the same set-up.

Moreover, as there have been done several experiences, the list of components used may be large. However, most of them have been used in different set-ups. Hence, different references to the following experimental sections can be found in the description of each material and component, as well as a final summary in Table 3.1.

### 3.1 Optical components

- Laser diodes and mount

All lasers used have been mounted in Arroyo Instruments laser mounts 224 TEC TO-Can LaserMount. They provide a non-wiring environment, with selectable device configurations and two inputs for the laser driver and the temperature controller.

Configurations may vary depending on the laser diode mount and its connections, defined by the manufacturing process and specified in the data-sheet. It is really important to know which is the inter-connections configuration as it will define the laser mount configuration. This is a consequence of the nature of semiconductor lasers. When diodes are plugged in an electrical circuit, intensity flow plays an important role because diodes let intensity flow from anode to cathode but not otherwise. Hence, one must be aware of which pins for both LD and PD are the cathodes and which one are the anodes to make them work properly. Furthermore, in order to avoid high-frequency noise, both LD mount and laser diode mount are grounded together.

Referring to the laser diodes, there have been used two different wavelengths and types too. The first laser used was a DFB laser with a wavelength of 1550 nm from WaveSpectrum. This has been the only 1550 nm LD used. On the other hand, two different semiconductor lasers have been used with the same wavelength, 850 nm; a Litrax VCSEL and a Eagleyard DFB.
Chapter 3: Materials and Methods

3.1 Optical components

• Photodetectors

LFI techniques usually use the back PD placed behind the LD in order to measure the output power of the diode. This is one of the best advantages of self-mixing effect, as they do not need external PDs or additional optical paths. However, there have been placed external PDs in some of the experiments. When using the 1550 nm LD, the two external PDs were a little silica diodes; whereas for the 850 nm, they were a big GaAs ones.

• Lenses

There have been used Thorlabs lenses for all lasers to collimate the laser beam emitted by the LDs. All of them had a focal length of 8 mm and were aspheric, TME, and coated for a specific range of wavelength. Lenses used for the 1550 nm LD were C240-TME-C, with a coating for wavelengths from 1050 to 1620 nm; whereas for the 850 nm LDs, the lenses were C240-TME-B, a coating range from 600 to 1050 nm.

• Cameras

Cameras are really useful to visualize the laser beam as both wavelengths are not from the visible range. Although two different cameras have been used, a Logitech Webcam C930e and a USB2.0 UVC PC Camera, both were only used for the 850 nm LDs set-ups. with the help of the cameras, proper collimating and focusing procedures have been reached.

• Targets

Depending on the application, different targets have been used. The most used ones were the piezoelectric components driven by a certain known signal generated with the function generator. A membrane and a piezoelectric layer have been used, noting that the layer was much more accurate avoiding second-harmonic generation. It was driven by its specific controller from Physik Instrumente E-665 Piezo Amplifier/ Servo Controller.

It has also been used a rough spinning disk of 5 cm radius driven by a motor and its motor driver, as well as a microfluidic channel from Microfluidic chipshop pumped with two SyringePump pumps NE-1002X pumping syringes of 2.5 cm diameter. The channel width was of 1 mm whereas its depth was only 100 µm. The carrier fluid was sunflower oil, which has a density of 918.8 kg/m$^3$ and a refractive index of $n_{\text{oil}} = 1.4735$, whereas water, 1000 kg/m$^3$ and $n_{\text{water}} = 1.333$; was used for generating bubbles.

• Another important optical components

In order to perform more complex optical set-ups and accurately control the properties of the beam, complementary optical components are needed.

For splitting the laser beam in different optical paths, two non-polarizing cubic beam splitters have been used for both wavelengths. For collimating the laser beam when the
850 nm was used, it has been really useful a Thorlabs Shearing-Plate. Lastly, once has been used an optical attenuator Newport 925B.

Finally, and not the least important, the proper use of the optical mounts facilitate the manipulation of the set-up. For the microfluidic channel has been crucial the design and manufacturing of a custom mount of Poly(methyl methacrylate) or PMMA, commonly known as Achrilic, which is transparent to the used wavelengths. The design was done in DXF and it was manufactured with a laser cutter.

### 3.2 Electronic components

- **Temperature controllers**

  Controlling the temperature of the laser diode is critical in some applications because the diodes’ I–V characteristic changes with temperature. That is why temperature controllers are required in self-mixing. The Arroyo Instruments 5235 TECSource, 3.5A/7V and the 5240 TECSource, 4A/7V have been used in all the set-ups with a range of temperatures from the 20ºC to the 25ºC depending on the laser.

- **Laser drivers**

  In addition, laser diodes must be electrically pumped. In this case, there has been used only once an own custom manufactured laser driver for the 850 nm VCSEL and two laser drivers from Thorlabs, the LDC 202 and the LDC 500.

  The Thorlabs drivers enable controlling either the output intensity or power whilst setting the maximum of its value, however, for self-mixing effect, it is more interesting to control the output intensity rather than the output power, as this last one is the physical measure. Furthermore, two possible configurations for the laser could be assigned to the driver, common anode or cathode.

- **Filters, amplifiers and attenuators**

  Laser diodes have low intensity thresholds, depending on the laser but in the range of mA up to hundreds of mA; and are driven near them in order to be as close as possible to linearity. Hence, signals are usually low and may be noisy. Thus, filters are commonly used for enhancing the Signal to Noise Ratio or SNR while amplifiers and attenuators are used to bring the necessary level of signal to the measure or controller equipment.

  Semiconductor photodiodes’ response is current whereas most of the equipments measure the voltage. Thus, trans-impedance amplifiers or TIA are used in order to transform the current signal into a voltage one thanks to the trans-impedance, and amplify the signal. All the TIAs used were custom and their gains were tunable from no-gain to 100x gains.

  On the other hand, a Stanford Research Systems Low-noise current amplifier Model SR570 was used instead of the TIAs in the first two experiments due to its really good filtering and amplifying response. Occasionally, a Hatfield Instruments attenuator has been used in order to drive the LDs.
Chapter 3: Materials and Methods

3.2 Electronic components

• Function generators

In most of the experiments, there was needed a certain modulation. In that case, the unique function generator that has been used is the Hewlett Packard HP Agilent 33120A. If the laser was modulated, the above mentioned attenuator was needed in order to introduce a certain dB attenuation in the modulation signal.

• Oscilloscopes and Data Acquisition

There have been used two different oscilloscopes and DAQs depending on the requirements of the set-up. A two channels Agilent Technologies DSO-X-2002A scope and a four channels Rohde & Schwarz RTM 1054 one. On the other hand, both DAQs were from National Instruments but they differed on the bus connection, one was the BNC-2110 whereas the other one was the USB-6259 one.

• Actuators

Additionally to the electronic components, some electrically driven mechanical actuators have been used. On one hand, the Zaber actuators are really useful micro-actuators. They are easy to use and introduce in any program from the LabVIEW environment. The model used had a movement range of about 5 cm with a resolution step of 0.04 µm. They have been used for scanning and moving with micro-resolution and no need for any mechanical interaction with the set-up. On the other hand, a motor was used for rotate the spinning disk and the already mentioned Syringe pumps for the microfluidic channel.

• Supplies

A huge amount of components needed a power supply. Since that, a wide variety of power supplies have been used, from analog to digital, and with different operating values. However, it is really important to highlight the low-noise power supplies used.
### 3.2 Electronic components

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Table 3.1: Summary of the materials used.
4 Basic LFI Experiments

Once that all the material and equipment used has been explained, in this section there will be explained the basic LFI experiments performed during the thesis in order to understand self-mixing and being able to apply it in more complex applications.

4.1 Harmonic Motion application

The first step to experience the self-mixing effect is setting-up an oscillator in the external cavity of the laser. According to Sec.2.3.2, the displacement of the target will modify the external cavity length and, as a consequence, the output power of the laser.

On one hand, the optical set-up only needs a laser, a collimating lens and the oscillating target. We used the 1550 nm laser with a 8 mm focal length C240-TME-C lens and the piezoelectric membrane at a distance of 200 mm from the laser diode. The target was stimulated with sinusoidal waves from the function generator while the laser was driven at a constant current of twice the laser threshold, -20 mA; with its temperature maintained at 25ºC with the 5235 TEC Source.

On the other hand, the electrical set-up was one of the simplest. Aside of the above mentioned components, we amplified and filtered the signal coming out from the back PD and it was directly read in a two-channels oscilloscope and a BNC DAQ with a sampling rate of 100 kHz, triggered by the function generator. The filter used was a high pass filter with a cut-off frequency of 10 Hz coupled with a low-noise amplifier with a sensitivity of 20 µA and an offset value of -500 µA.

Despite of the simplicity, and as mentioned in Sec.3.2, the LD and PD anodes and cathodes must be properly connected with the case. In our case, after observing the inter-connections with a microscope, we set the LD anode and the PD cathode in common with the case and ground. On the contrary, there was a leaking current and there was no signal.

The sinusoidal signal which stimulated the membrane had two different effects on the SMI signal. If we increased the amplitude of it, the Volts-Peak-to-Peak or Vpp; the number of fringes increased as well. This is a consequence of the piezoelectric effect, the higher the voltage is, the wider the movement is. Thus, the external cavity range of movement was broader. The second effect was its frequency, which affected the period of the signal. (see Eq.2.19 and Eq.2.20)
With a simple LabVIEW program, the signal was read at the sampling rate of 100 kHz and its Power Spectrum Density or PSD was plotted in order to show the dominant frequency of the signal after averaging it 16 times. The PSD showed a dominant frequency related to the stimulation frequency. However, the membrane had second harmonics in the movement so there were additional powerful frequencies in the PSD.

Several authors have proposed different reconstruction methods for the movement of the target [15]. The developed Matlab code followed the fringe-counting method [16].
The obtained signal is normalized and filtered with a moving mean window and differentiated twice with a simple first-order forward difference method. Thus, fringes are the peaks of the obtained second derivative above an arbitrary threshold. Then, each fringe is related to a $\lambda/2$, as explained above, and a sign is assigned to it. Finally, after all the fringes have been counted, the signal is reconstructed and, afterwards, fitted with a sinus function. This fitting function enables to estimate the frequency of the signal as well as the amplitude.

### 4.2 Range finder

Contrary to the harmonic motion set-up, the laser is driven with a certain stimulation signal from the function generator instead of the piezoelectric target. In order to do so, the set-up had to slightly change. The function generator was connected to the laser driver with the attenuator reducing the signal in a range of 60-80 dB and to the scope as a trigger.

The LD intensity was -54 mA and the stimulated signal was a sawtooth of 1 Vpp at 1.5 kHz. Notice that the value of the intensity had to be much more than twice the threshold in order to appreciate the fringes in the self-mixing signal. Then, we also used the TIA connected to the back PD and filtered with the Low-Noise Amplifier using the same gain.

![Figure 4.3: Scheme of the set-up.](image)

According to Sec.2.3.1, the emitting frequency of the laser will experience a shift whilst its intensity is swept and, as a consequence, the output power will vary due to the changing stimulus phase so the excess phase equation is satisfied (See Eq.2.15 and Eq.2.16).

These fringes are related with the length of the cavity (See Eq.2.18). [17,18] Then, the
Chapter 4: Basic LFI Experiments

4.2 Range finder

(a) Self-mixing signal for a fixed target at 28 cm from the LD.

(b) Scheme of the signal analysis.

Figure 4.4: Signal obtained after stimulating the laser with the parameters above and the scheme of the its processing.

Matlab code has a similar procedure to the harmonic motion processing code because both need to compute the fringes in the signal but the distance is not related with $\lambda/2$.

The noise is removed with a moving mean window in order to, when applying the first-order forward difference method, a threshold value can be applied. The difference with the previous case is that now the constant slope of the signal is removed by software. We could have removed this slope by performing a previous background measurement too. Nonetheless, the chosen option is enough accurate and, in the other case, there would be needed a calibration for changing conditions as temperature and humidity as the background would also vary.

(a) Fitting curve of the frequency shift estimation in red, blue points are the experimental data and dashed blue lines are the upper and lower limits.

(b) Calculated distances compared with the reference $y = x$.

Figure 4.5: Experimental results for several known distances and a sawtooth stimulation signal of 1 Vpp at 1.5 kHz.

However, the optical parameters as the frequency shift must be calculated so the fringes can be related to the external cavity length. (See Eq.2.14). These parameters were
calculated using the self-mixing signal. Hence, we measured the LFI signal for several known distances and counted the number of fringes so a fitting would approximate the frequency shift $\Delta \omega$.

According to Fig.4.5, the frequency shift is calculated with two bond limits. The resulting experimental frequency shift is $\Delta \omega \approx 3.87 \text{ GHz}$ within a trust region of less than 20% of its value. To conclude the accuracy of the absolute distance measurements, the known distances are calculated using the fitted parameters and plotted against the reference as shown above in Fig.4.5.

### 4.3 Radar velocity: Spinning Disk

In addition to the previous applications, LFI can be used as a Doppler velocimeter and flow velocity sensors [13]. This set-up used a fixed spinning disk rotating at a controllable angular velocity. In this case, the external cavity length was not changing but the Doppler effect caused by the movement of the target was the responsible for the measurable self-mixing effect.

Contrary to the above applications, the 1550 nm LD was replaced with the 850 nm VCSEL and we used a custom laser driver, supplied by the low-noise 12 V power supply, to provide a constant current of -5.65 mA, almost twice the -3 mA laser threshold. Consequently, the optical lens was also swapped into the C240-TME-B so the grating met the laser wavelength. Furthermore, we required the shearing plate to properly collimate the laser beam helped with the Logitech Webcam.

The spinning disk, which had an angle of 10° with respect to the laser’s optical axis, was controlled by a motor with different possible angular velocities $\Omega$. The motor driver had a digital signal triggering the scope and the DAQ which was working at a sampling rate of 500 kHz.

Moreover, as the most remarkable aspect in difference with the previous applications, the information is carried in the frequency domain instead of the temporal one. For such reason, the PSD became the meaningful signal. The LabVIEW program was not required to be highly modified as it showed the PSD of the signal and was able to write it in a file. However, we introduced a high-pass filter with a 1 kHz cut-off frequency, in order to remove the noise coming from the motor; and the calculus of the angular velocity of the disk through the Doppler frequency determination.

The laser beam was pointing at a point at 4.5 cm from the radius of the spinning disk. Its velocity, assuming a constant angular velocity is as follows:

$$\vec{v} = \vec{\Omega} \times \vec{r} \rightarrow v_\parallel = \Omega r \cos(\theta)$$  \hspace{1cm} (4.1)

Then, for a fixed point and angle, the velocity depends linearly on the angular velocity of the disk. Additionally, the Doppler frequency is related to the velocity of the target (See Eq.2.22). Hence, by determining the Doppler frequency on the PSD of the signal we were
able to know the spinning velocity of the disk.

\[
\Omega = \frac{\lambda f_D}{2n_{ext} \cos(\theta)}
\]  

(4.2)

Notice that the angular velocity calculated with the expression above is in rev/s. If the revolutions per minute or rpm is the unit of interest, it is easily converted by multiplying the result by 60. Substituting all the known parameters in the previous expression leads us to a simple final expression relating the angular velocity and the Doppler frequency.

\[
\Omega \approx 5.7541 \times 10^{-4} f_D
\]  

(4.3)

Despite of the apparent simplicity of the LabVIEW routine, the Matlab code is more complex, including fittings on the PSD signal. Firstly, after filtering the data with the high-pass filter, the background is removed with a decreasing exponential fitting. Once the background has been removed, it is windowed the Doppler frequency and, as the
background, fitted with a Gaussian peak centered in the Doppler frequency. The fitting is really sensible so the initial parameters must be chosen carefully. Thus, the full width half-maximum of the Gaussian peak is determined with a half of the maximum amplitude threshold and the Doppler frequency is the central frequency of it.

In addition, in some cases there could be a second dominant frequency. In those cases, the optical feedback level can be known by making the quotient between both amplitudes. Thus, another fitting is required. The fitting for the second harmonic follows the exactly same procedure as before. Finally, the resulting fitting consists on two Gaussian peaks and a decreasing exponential background as shown in the figure below, Fig.4.8.

![PSD](image)

Figure 4.8: Fitted PSD signal (red) for a measured velocity of 15.9627 rpm.

The measured Doppler frequency for a velocity of around 20 rpm was at 27.74 kHz and the second harmonic frequency at 55.59 kHz, with a resulting velocity of 15.96 rpm.
5 SMI signal characteristics

During the whole thesis, self-mixing effect was explained for a unique optical arm because LFI can be obtained with a back PD, or even without it, by just measuring the laser voltage junctions; but if an external PD is placed in an extra optical arm, what would happen?

The voltage signal has been demonstrated to be out of phase with the intensity signal coming from the back PD [18]. In this case, it can be deduced because of the semiconductors’ nature. As explained in Sec.2.1.1, a semiconductor laser emits photons after the extinction of an electron with a hole. Thus, semiconductor photodetectors, that are exactly the analogous case, they generate an electron-hole pair when a photon hit onto their surface. Hence, when the LD emits a photon annihilating an electron-hole pair, the PD absorbs it and generates an electron-hole pair; so that voltage decreases whereas intensity increases.

Several authors have reported the external PD signal to be out of phase [19,20] but it is still an unanswered question why. So, once the theoretical background and the basic applications for LFI have been shown, this chapter will be focused on the experiments carried to see the effect of placing an observing external PD. Results showed that the plane wave assumption in the Lang–Kobayashi model is required to be satisfied if a constant phase relation between the output signals is wanted. Consequently, the laser beam must be properly collimated.

5.1 Simple external PD

Firstly, we used the same set-up as for the harmonic motion measurement in Sec.4.1 placing the non-polarizing cubic 1550 nm beam-splitter at 10 cm from the LD. Then, we could situate a silica PD in the new observer optical path on a Zaber micro-actuator at 5 cm from the beam-splitter center. Both PDs’ signals were amplified with two TIAs and connected to the two-channels oscilloscope.

Nonetheless, the output power in each optical arm had to be checked because, as the self-mixing signal must be caused by the harmonic motion of the piezoelectric membrane, the highest amount of optical power must not be in the observer arm. Experimentally, the beam-splitter split the beam in approximately even powers but slightly higher in the piezoelectric arm.

An In-Phase/Out-of-phase relation between two signals can be easily seen in an unitary
X–Y graph because the two diagonals, $y = x$ and $y = -x$ are the respective cases. Thus, both signals were normalized and plotted separately in two graphs after a time average of 8 waveforms, a waveform graph and an X–Y one, where the signal from the back PD was on the x-axis whereas the external PD one was on the y-axis. Moreover, the X–Y result was converted into polar coordinates in order to have an angle relationship between 0 and 180 degrees.

On one hand, LabVIEW’s angle result is the mean value of the angles whereas on the other hand, Matlab fits the X-Y graph and provides the tilt of the minor axis as shown in Fig.5.3. It must be noticed that the resulting tilt indicates if the signals are close to be in-phase or out-of-phase but only we would only be able to say that both signals are clearly in-phase or out-of-phase if the ellipse is close to the diagonal lines. In other words, the minor axis must tend to zero so the ellipse becomes a line.

With a raw scan, we were able to see that the phase was flipping from in-phase to out-
Chapter 5: SMI signal characteristics 5.1 Simple external PD

Figure 5.3: Ellipse fitting results for sinusoidal signals with a phase difference of $\frac{\pi}{2}$ (Left), $\frac{\pi}{4}$ (Center) and $\frac{3\pi}{4}$ (Right).

of-phase as long as we moved the external PD position with the Zaber. As a consequence, we decided to perform a scan in which the step was one micro-step, approximately 47 nm which is a more or less a 3 % of the laser wavelength. The scan was unidimensional, starting in the middle of the 5 cm range of the Zaber and moving forward to the beamsplitter during a $4\lambda/2$ range – a total number of 65 micro-steps.

As we were using the harmonic motion set-up, the 1550 nm LD had a constant current of -20.55 mA and the piezoelectric membrane was stimulated with a sinusoidal function of 5 Vpp of amplitude at 1 kHz whereas the DAQ was reading at 100 kHz. Although the range of the scan was small, due to the movement of the observer PD there was a transitory state until the signal was stable. Hence, we applied a waiting time of 2 minutes for each step to read the data and move to the following one.

Figure 5.4: Experimental data X–Y graphs of both normalized signals in arbitrary units fitted for a $\lambda/2$ scan (a) and the signals obtained for a single scanning point (b).

As Fig.5.4 shows, the phase relationship between both signals changes. Moreover, as there was no clear relation between the position of the external PD and the phase relationship, we decided to make several scans in order to see if the phase was maintained along time for a same point.

Clearly, the phase relationship of the signals was changing in time for a fixed point as it can be appreciated in Fig.5.5. The Lang–Kobayashi model explained in Appendix 1
Chapter 5: SMI signal characteristics

5.2 2D scan external PDs

Following the previous argument, the Lang-Kobayashi model assumes that the electric field that causes the self-mixing effect is contained in a slowly-changing complex envelope and propagating with plane waves. Hence, if a two dimensional scan for a fixed longitudinal point is done, phase should not highly vary.

In order to do that, we placed a 3-dimensional Zaber in the external optical path. Additionally, to have a fixed reference, we placed another Silica PD. To avoid any back reflection from both PDs, the beam splitter had a 10 degrees tilt, so both observing arms were not perpendicular to the external cavity. Nonetheless, both PDs were again 10º tilted so there was no chance to have back reflection to the external cavity.

Moreover, so no second harmonics were produced, the piezoelectric membrane was replaced by the more accurate piezoelectric layers. It was driven with a sinusoidal signal of 2 Vpp at 330 Hz what made an almost sinusoidal movement of amplitude of 10.46 µm, almost 14 λ/2. Between the beam splitter and the target, there was placed the attenuator so we were able to control the amount of feedback.

states that there is no dependence of the phase on the distance, only in time. However, the model assumes plane waves for the electric field. Thus, the only possible explanation we had was that the beam was not perfectly collimated as we did not use a shearing-plate. Consequently, we took the decision to perform a scan of the laser beam with the external PD, as explained beyond in the following section.

Figure 5.5: X–Y graph of both normalized signals in arbitrary units showing the phase relation for L=0.42863 µm at different times.
Chapter 5: SMI signal characteristics

5.2 2D scan external PDs

Most of the electronic components were changed as well. The temperature controller used was the LDC500 at 25°C and all the TIAs were custom with a selected gain of 1V/100mA. Obviously, as we were interested in measuring the signals at the back PD and two external PDs, more channels were required so we used the R&S 4-channels scope. Finally, the DAQ used was the USB one, which offers much more channels, reading at a sampling rate of 500 kHz.

The LabVIEW program reads the three signals and filters them with a band-pass IIR Butter-worth filter with stop frequencies of 100 Hz and 100 kHz. Then, after disregard the transient, the amplitude, the root-mean-square and the phase of the signal are calculated. Notice that the phase of the signal is calculated as the arctangent of the ratio between the imaginary part and the real part of the maximum peak of the FFT of the signal. The scanning area is of N x M points, being the best option to choose square areas of odd points.

Initially, a raw scan was made in order to properly situate the center of the laser beam and to know the scanning area. The first scan took was of an area of 2 mm per 2 mm with a 51 x 51 lattice. The results on Fig.1 shown that the scanning PD was changing differently from the back and fixed PDs, which may had varied due to the long scanning time. However, the fixed and the back PDs were switching between in and out-of-phase.

In Fig.1, the laser beam seemed to be bigger than the scanning area, so we decided to enlarger it. Once the scanning area was selected to be 3000.375 µm x 3000.375 µm, we performed a 151 x 151 points scan, so each pixel corresponded to a displacement of 20.003 µm.

Figure 5.6: Scheme of the set-up.
Chapter 5: SMI signal characteristics

5.2 2D scan external PDs

Results in LabVIEW were graphically meaningful, however, we used Matlab to process the data with a similar procedure. All signals are filtered through a self-designed FIR high-pass filter of stop frequency of 700 Hz and band frequency of 1 kHz using the function `firpm`. Due to the filter action, the transient must be deleted from the signal. Then, as well as in LabVIEW, the RMS and the phase of the signals are calculated (See Fig.2 and Fig.3).

![Diagram of the signal analysis]

Figure 5.7: Scheme of the signal analysis.

However, the phase is calculated with a window containing the frequency of the self-mixing signal. This means that, after counting the number of fringes in a period, it is calculated their frequency because it must be the most powerful in the PSD. If there is no window applied, due to second harmonic coupling, the maximum peak may vary in frequency and then the phase comparison is not consistent.

![Images of phase relation between the scanning PD (Top) and the fixed PD (Bottom). Direct arctangent phase relation (Left) and cross-correlation lag (Right).]

Figure 5.8: Phase relation between the scanning PD (Top) and the fixed PD (Bottom). Direct arctangent phase relation (Left) and cross-correlation lag (Right).

Windowing, we were able to fix for all the signals the same frequency as the most powerful, which was the self-mixing one. In our case, the number of fringes, as roughly
calculated above, were 17 in 1 millisecond, so the fixed frequency was near the 17 kHz. Finally, the phase was constrained to be in the range of 0º to 180º so they could be directly compared.

Additionally to the comparison between the phases calculated through the FFT, a cross-correlation between each signal and the back LD is made. The cross-correlation between two periodic signals shows which is the necessary lag to have the highest likelihood, so two signals in-phase will have no shift or a lag equal to their period or a multiple of it. Analogously, two out-of-phase signals must be shifted a half of their period.

Clearly, the phase relation between the external PDs and the back PD varies as it can be seen in Fig.5.8, showing that both algorithms present a difference in phase changing through the X-Y plane. With no other explanation that the non-perfect collimation of the beam, and consequently the non-plane waves, we decided to use the 850 nm DFB and collimate the beam using the shearing-plate and the camera.

5.3 Junction voltage across the LD

In addition to the previous signals, we wanted to also measure the voltage of the laser junction and obtain the sinusoidal signal generated by the movement of the piezoelectric driver.

![Power variation (a.u.)](image1.png)  ![PSD](image2.png)  

Figure 5.9: Read signals (Left) and their PSDs (Right) for a single scanning point before being filtered (Top) and after (Bottom). Back PD signal (Blue), Scanning PD signal (Red), Fixed PD (Green) and LD junction voltage (Black).
Using the same set-up, we only had to change the LD and the lenses, as well as the observing PDs. A remarkable feature of the GaAs PDs was that they were bigger, so almost all the beam was collected in the diode surface. Thus, the sampling rate was reduced to 200 kHz. Furthermore, due to the low signal of the voltage, it had to be amplified with three inverting TIAs and so, the read signal was inverted.

After properly collimating the beam with the beam splitter and the help of the camera, the first raw scan made showed that all signals were in phase. Notice that, as the voltage was inverted, it was in fact out-of-phase. The shape of the beam was not circular (See Fig.4), but the previous 3 x 3 mm area scan was enough. Then, the same data processing was made with the LD voltage introduced.

![Figure 5.10: Phase relation between the scanning PD (Top), the fixed PD (Center) and the LD Voltage (Bottom). Direct arctangent phase relation (Left) and cross-correlation lag (Right).](image)

The results are clear, both Fig.5.9 and Fig.5.10 show that the three PDs’ signals are in-phase whereas the LD voltage is out-of-phase with respect all the others. It must be noticed that the noisy signal from the cross-correlation is due to the poor SNR because it is only found in the scanning area where the scanning PD signal is practically non-existent.
and in the LD voltage, really low signal that must have been amplified three times.

As a brief conclusion, the phase relation between the observing PDs and the back PD has been maintained once the laser beam was perfectly collimated, as we supposed. Hence, it is important to have plane waves in the self-mixing set-up, meeting the model’s assumption, if we are interested in place an external arm to our external cavity for signal processing applications such as denoising.
6 Bubble counter

Previous applications have been traditionally the basic research in self-mixing interferometry and are the background for more complex techniques. Thus, this application was based on the set-up simplicity of the shown LFI techniques but with a more advanced working principle in order to reach the main aim of this thesis: being able to design and develop a new LFI application based on the acquired knowledge.

Artificial hearts were clinically introduced in 1982 and have been gaining importance in our society since heart diseases are currently one of the leading causes of death. Consequently, sensing and controlling flow properties are critical. It is well known that bubbles or blood clots within vessels present an extremely risk to health.

Hence, although initially it was only thought to detect bubbles and count them, we present a real-time bubble counter and analyzer that can detect and sense bubbles flowing within a channel using a simple and cheap set-up with a high efficiency and great accuracy limits. In the following sections, there can be found the different steps that have lead us to this useful and interesting application.

6.1 Model and set-up

As for the harmonic motion application and the absolute distance range finder in Sec.4.1 and Sec.4.2 respectively, any change in the external cavity length will vary the stimulus phase $\varphi_s(t)$ and so, the feedback phase and the output power. However, the external cavity length must also change with the refractive index as the propagation velocity of an electromagnetic wave depends on the medium refractive index. Thus, if the medium of the external cavity changes, its effective distance also changes due to the different velocity.

$$\varphi_s(t) = \omega_s \tau_{ext}(t) = \frac{2\pi c n_{ext}(t) 2 L_{ext}}{\lambda} = \frac{4 \pi L_{ext} n_{ext}(t)}{\lambda}$$ (6.1)

Hence, we placed the microfluidic channel as the target with two different fluids flowing through, forming bubbles, regions of different refractive index. The carrier fluid was sunflower oil, with a refractive index of 1.4735, higher than the refractive index of the water, 1.333, which was the one in charge of generating bubbles. Firstly, we observed through the microscope which pumping velocities were the most suitable for the bubbles generation. Then we decided to use pumping velocities, $Q$, of 30 µL/min for the oil and 10 µL/min for the water.
Chapter 6: Bubble counter 6.1 Model and set-up

Once the microfluidic channel was ready, we designed the mounts as explained in Sec. 3.1. The 850 nm DFB laser was mounted with two C240-TME-B so the emitted laser beam was collimated by the first one and focused into the microfluidic channel by the second one. The total length of the set-up was less than 15 cm.

The electronic circuit was simpler, we only required the laser driver controller 500 at -33.33 mA, the 5240 temperature controller at 20ºC, one TIA and its supply and the reading equipment, the R&S scope and the USB DAQ; additionally to the syringe pumps. In order to focus, the USB camera was used saturated with an Olympus LG-PS2 light generator and the focus point was situated at the half of the width of the channel.

First of all, we noticed that each time a bubble crossed the channel through the laser focus, the optical power changed suddenly. However, it must be noticed that the shape of the peak changed depending on which section of the bubble crossed the focal point because the reflected wave will have a different angle so it may not reflect back to the laser cavity. Thus, we made a scan through the width of the channel in order to state which point had the optimal signal.

As it can be appreciated in Fig. 6.2, the region in which the signal was optimal is approximately the middle of the width, what seems obvious so most of the bubbles are sensed. Nonetheless, it could seem an appropriate sensing range the area close to the edge of the channel, but, the peak was lower with respect to the one in the middle and other optical effects may affect. During 6 µm, the peak produced by the refractive index variation was pronounced enough to be easily processed. Nevertheless, we were aware of a second little peak after the huge one. Initially, we thought it was a temporal response of the amplifier because the change in output power was really sudden. However, as it will be demonstrated later, it actually was the transition from water to oil, the bubble’s ending.
6.2 Bubble shape model and measurement

To this extent, a simple bubble counter would be made. Every time the signal reaches a certain threshold above its baseline a bubble has crossed the focal point of the laser. Then, a simple real-time LabVIEW program would count the number of bubbles crossing the channel, warning or informing the user of their presence. Though, a further functionality and performances such as volume estimation can be added by properly interpreting the signal. In order to do so, we first analyzed the bubbles by imaging software taking the width of the channel as the reference (1 pixel $\approx 9.481$ µm and, due to the mean FPS of 13; 1 frame $\approx 76.9$ ms) as shown in Fig. 6.3 below. These are considered the truth values.

Figure 6.2: Power variation for the different positions of the focal point of the laser beam along the channel width.

(a) Major and minor axis measurement with the channel width as reference.

(b) Velocity determination with two consecutive frames.

Figure 6.3: Imaging software bubble analysis. $Q_{Oil} = 30$ µL/min and $Q_{Water} = 10$ µL/min.
There were two necessary measurements, the bubbles’ dimensions and velocity. Although the first one’s purpose is obvious, velocity is required to translate the temporal information from the signal of a bubble crossing the focal point to its length. All the analysis was made with bubbles created at the initial pumping rates, 30 µL and 10 µL per minute for oil and water respectively; hereinafter the calibration parameters.

Moreover, different approximations of the shape were taken in order to see which ones were closer to the dispensed volume of the syringes and which ones had more physical sense. The model assumed would be the one that fits better the experimental data. For instance, considering rectangular bubbles of the measured length and the cross section of the channel would only provide information of its maximum value whereas considering spherical bubbles would only make sense when their length was shorter than the 100 µm of channel depth. Thus, the remaining options are either ellipsoids and elliptical cylinders of the same depth as the channel.

<table>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tr>
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<td>58.69</td>
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</tr>
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<td>5.61</td>
<td>8.227</td>
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</tr>
<tr>
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<td>45.52</td>
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<td>0.0345</td>
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</tr>
<tr>
<td>Ellipsoid</td>
<td>µL</td>
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<td>0.0245</td>
<td>0.0229</td>
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</tr>
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<td></td>
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<td>6.203</td>
<td>6.162</td>
<td>5.7652</td>
<td>5.878</td>
</tr>
</tbody>
</table>

Table 6.1: Bubble measurements using Vision Assistant imaging software. Volume of an ellipsoidal cylinder: $V = \pi abd$. Volume of an ellipsoid: $V = \frac{4}{3}\pi ab d^2$; where $d$ is the depth of the channel.

There are two surprising results from Table 6.1. On one hand, the elliptic cylinders model reaches with less than a 15% error the total volume of water dispensed per minute. Contrary to what we initially thought, the ellipsoids model greatly differs from the dispensed 10 µL/min of water, leading us to approximate the deviation by a factor $(1 - Q_{\text{Water}}/Q_{\text{Oil}})$, showing that the first model fits better the experimental data. Hence, the model taken to estimate the shape of the bubbles is the elliptic cylinders one.

On the other hand, if we assume a constant velocity through the channel of the form:

$$v = \frac{\text{Total Pumping Rate} (\mu L/min)}{\text{Cross – section} (\mu m^2)} \left(\frac{10^9 \mu m^3}{1 \mu L} \right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \frac{10^9 (30 + 10) \mu m}{60 \cdot 10^5 \text{ s}} = 6.667 \text{ mm/s}$$

(6.2)

A value which is deviated from the mean of the measured velocities less than a 1%.
6.2.1 Bubble length measurement

Once we had some references, we were able to discern if the second peak of the signal was due to an hysteresis of the electronic amplification process or the physical change of the refractive index; as suggested previously. In order to do so, two different signal processing methods were developed. Both of them share the same principle, measuring the huge peak with a threshold; however, the difference is if the length of the bubble is the length of the peak or the length between the two peaks.

Firstly, the data is smoothed with a moving average window filter of 5 ms (See Fig.6.2). Then, a threshold value is applied to convert the analogical signal into a digital one; so, when derived, peak limits are the only non-zero values. The threshold value is an important parameter for accuracy and consistency of the algorithm because, the lower, the more accurate measure of the temporal length of the bubble. However, the lower, the closer to be inconsistent due to the noise of the baseline. Hence, it is a trade between how close the bubble is intended to be sensed and which level of reliability is desired.

Then, if the length of the bubble is considered to be the width of the first peak, it is only needed to transform this temporal duration into length by simply multiplying by the fluid velocity. On the contrary, for measuring the length between the two peaks, another step must be done. The easiest way to implement it is by realizing that the second peak always has an inverted part or valley lower than the baseline. Consequently, the algorithm looks for the minimum value of the consecutive signal. Then, the nearest maximum is found using a window of 50 ms so end of the bubble is meant to be within this two peaks; as shown in Fig.6.5 below.

Results were clear, the width of the first peak method sensed a much more shorter length than the second algorithm. The mean length measured with the first one was 302.032 µm whereas the second one calculated a mean length of 843.3 µm. The last one fitted the analyzed lengths within less than a 10% of error. Consequently, we disregarded the first algorithm and, from then on, the measure of interest was the distance between both peaks.

In order to provide a deeper analysis of the algorithm’s accuracy, we changed the water pumping velocity whilst keeping fixed the oil one at 30 µL/min, so different pumping
relations were obtained and, consequently, different bubble’s aspect ratios. For each pumping relation, 5 bubbles were measured with the camera and compared to 10 seconds of self-mixing signal measurements. Thus, results were treated as independent Normal Random Variables with a certain mean and variance value as shown in Fig.6.6 below.

(a) Minor (Left) and Major (Right) axis measured with the camera software (Blue) and with the self-mixing signal (Red) fitted with Normal Random Variables for a pumping relation of 10/30.

(b) Relative error of the LFI mean major axis with respect to truth and its standard deviation for different pumping relations.
It is clearly seen in Fig. 6.6 that when the pumping relation is beyond a half, the measured lengths are much more accurate than when the water’s pumping velocity tends to equal the oil one; so bubbles are bigger. A relative error of less than a 10% can be considered a good result as bubbles measured are within the micro and millimeters range. Furthermore, the standard deviations in most of the cases are less than the 10% as well.

However, as one can notice, these measurements only provide the major axis of the bubble. In order to sense the volume, assuming the elliptical cylinders of depth equal to the depth of the channel; a value for the minor axis is required. There could be two ways of solve it, a mathematical one by estimating it or a technological one using an array of LDs along the width of the channel which would additionally assure that every bubble is detected no matter its volume. Both of them are feasible and easy to implement, however, we chose the estimation method following some literature [21].

### 6.2.2 Minor axis estimation

Considering that our measured parameters are highly accurate and within reasonable errors, we could estimate the shape of the elliptical face of the bubble. Firstly, we considered the two extremes:

1. Water pumping rate « Oil pumping rate: Bubbles would be closer to spheres or circular cylinders, so the minor axis would be equal to the measured major axis.

2. Water pumping rate \( \approx \) Oil pumping rate: Bubbles would have larger major axis than minor axis, so they would be completely elliptical, with the channel width as a constraint.

The parameter that characterizes this behavior is the eccentricity of the ellipse, \( \varepsilon = \sqrt{1 - (b^2/a^2)} \). This parameter is ranged between 0 and 1, being a circle when it reaches 0 or an ellipse when 1. For the analyzed values of the major and minor axis, \( \varepsilon \approx 0.6 \). Hence, we decided to make an estimation of the minor axis using a multi-step finite variation of the eccentricity by comparing the water volume dispensed.

The estimation process starts with a rough guess of \( \varepsilon \) using the pumping rates of both fluids; \( \hat{\varepsilon} = \sqrt{Q_{\text{Water}}/Q_{\text{Oil}}} \). For instance, with the calibration values, \( \hat{\varepsilon}_0 = 0.5774 \). Then, the minor axis is directly:

\[
b = \min\left(\sqrt{1 - \hat{\varepsilon}^2} a, \text{Width}_{\text{channel}}/2 \right)
\]

Where \( a \) is the half of the measured length. Once the minor axis is estimated, the volume of the bubble can be calculated. The multi-step method compares the volume dispensed each 5 bubbles with the pumping rate volume dispensed during the that period and modifies the eccentricity of the ellipse according to the relation between by an arbitrary finite difference \( de \) them as follows:

\[
\hat{\varepsilon}_{n+1} = \hat{\varepsilon}_n + \text{sign}\left(V_{5\text{bubbles}} - Q\right)de
\]

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As a consequence, if the calculated volume is higher than the expected dispensed one, \( \hat{\varepsilon} \to 0 \). On the contrary, \( \hat{\varepsilon} \to 1 \). It may seem contrary to the desired result but one must be aware that the minor axis is constrained by the channel width, as expressed in Eq.6.3; and extremely high volumes mean that this limit has been reached. Additionally, in the case of spherical bubbles, the measured length would be shorter than the channel width.

Finally, Eq.6.4 is only applied when the relative difference of the volumes is above the desired accuracy and the eccentricity is within \([d\varepsilon, 1 - d\varepsilon]\).

The estimator efficiency was tested in Matlab, providing different values of accuracy and \( d\varepsilon \). (See Fig.6.8)

Obviously, when evaluating the efficiency of the estimator, speed and accuracy must be taken into account. Hence, based on the results obtained, the most efficient \( d\varepsilon \) was decided to be \( d\varepsilon = 0.01 \) because in less than 15 iterations, the estimated volume is within our accuracy without any oscillatory response. Results in Fig.6.8 also showed that the shape of the bubbles were of high likeness.

Additionally, and once again, the estimator was tested for different pumping relations in order to compare the different dispensed volumes. As it can be appreciated in Fig.6.9,
Chapter 6: Bubble counter

6.2 Bubble shape model and measurement

(a) Relative error for the estimation process of the mean volume dispensed by 5 bubbles (Top) and the estimated minor axis (Bottom) for different \( \epsilon \).

(b) Estimated bubble (Red) and analyzed or truth one (Green).

Figure 6.8: Estimation of the shape of the bubble assuming elliptical cylinders.

and as previously commented in Fig. 6.6, the major axis measures are practically identical for low pumping relations whereas for higher ones, there is a higher discrepancy.

Figure 6.9: Top: Mean major axis and its standard deviation measured with the camera (Blue) and the LFI (Red) for different pumping relations. Bottom: Water volume dispensed by the pump (Green), bubbles’ volume measured with the camera (Blue) and bubble’s volume estimated with the LFI (Yellow) and their standard deviations for different pumping relations.

Nonetheless, the same behavior was obtained for the volume estimation, which is logic as the minor axis is an estimated parameter that also depends on the measured major axis. However, it is surprising that the pump dispensed volume greatly differed from the
volumes calculated with the camera for high pumping relations. This loss of accuracy may have been caused by the large diameter of the syringes and the dispensing method. If the 2.5 cm diameter is reduced, the dispensed flow by the pumps will be more constant and, desirably, volume values will approach. On the other hand, constant pressure pumps can be used in order to obtain even a more constant flow.

Summing up, the elliptical cylinder model with the implemented minor axis estimator enabled us to precisely estimate the volume of the bubbles for low pumping relations whereas for high ones, although the estimation was close to the volume value dispensed by the pump, it was not that accurate with respect to the camera values.

6.3 Real–time bubble volume estimator

Finally, the main aim of this project could be reached after all the modeling and analyzing processes. We implemented the Matlab algorithm in a LabVIEW real-time program so, not only bubbles were detected but also sensed; giving to the user information of presence and dimension.

Figure 6.10: Snapshot of the Front Panel of the LabVIEW real-time bubble counter software.

In order to do so, we had to carefully choose the proper way to read data and process it. LabVIEW environment was chosen to be the producer by reading the data from the
DAQ and the display using its Front Panel whereas Matlab was the consumer in charge of processing the data and calculating the measurements. However, the estimator was exclusively implemented in LabVIEW.

The signal can be split in three blocks, the baseline, the first and the second peak. Hence, LabVIEW reads the data in three separated but dependent blocks. The DAQ worked at a sampling rate of 500 kHz, so LabVIEW reads between 1 and 10 thousand samples, which are 2 and 20 ms reading windows. Two interesting consequences are obtained with these values, the first one is that the program is practically real-time based and the second one is that smoothing filters such as Savitzky-Golay[22]. Thus, a 3rd order Savitzky-Golay filter is applied with a windowing of 201 samples (100 per side) in order to smooth the noise as several algorithms for electrocardiography (ECG) have reported [23]. However, it must be noticed that the reading window length is, again, a trade between real-time acquisition and displaying and false readings due to noise. The longer the program reads, the more effective is the denoising filtering; though, it is strictly necessary that the averaging is not too much wide so the second peak is detectable.

The first block is in charge of reading while there is no bubble crossing the focal point.
Once the threshold is reached, the second block starts reading and saving in an array at the same time while the threshold value is exceeded. Then, and latest, the third block reads and saves in the array until the threshold is again reached or a maximum time of 500 ms has elapsed. The sum of these three blocks is the producer.

Sequentially, the array is sent to Matlab, the consumer, in order to process the length of the bubble with the previous minimum finding algorithm and calculate its volume following the model. Nonetheless, the array must be flipped due to different writing and reading protocols. Results are displayed in LabVIEW. Additionally, every 5 bubbles and under the following conditions, the eccentricity is estimated in LabVIEW:

1. The estimated dispensed volume of the 5 bubbles is out of the desired accuracy with respect to the dispensed volume in that time

2. The eccentricity is between \( de \) and \( 1 - de \)

As a summary, we have been able to detect the presence of bubbles and estimate their shape and volume with a simple LFI set-up formed by a single diode laser, its back photodetector and two lenses. This application can lead to two different but related ones, a simple bubble counter as suggested initially where the user is warned when a bubble is detected or a bubble volume sensor for a wide range of applications; advising the user depending on his interests: maximum volume per bubble allowed, aimed dispensed volume and maximum bubble rate permitted.
7 Conclusions

Self-mixing effect was firstly reported as a power loss but, during this thesis, it has been shown to be useful in a huge range of applications, leading LFI applications to compete against other traditional techniques due to its simplicity. Additionally, semiconductor lasers bring reduced dimensions and low-cost components as powerful advantages. These two facts are of huge impact now that photonics and optical circuits are between the newest and most advanced technological challenges for researchers and industries.

Although it has been more than 50 years since self-mixing was firstly reported for metrology applications, it is an extremely recent field of research as it can be noticed in the literature and, as a consequence, further theoretical developments are needed because there are still unanswered questions. One of them has been addressed in Chapter 5, leading us to conclude that the Lang–Kobayashi model assumption of plane waves is strictly necessary for keeping phase relation after adding observing arms to the external cavity. Hence, self-mixing and LFI has not reached its roof, presenting year after year new designs for more advanced applications such as skin cancer diagnosis and imaging.

Additionally, semiconductor lasers adaptability enables enough freedom to create and design completely new LFI set-ups. In that case, we have shown in Chapter 6 that a simple set-up can have a wide versatility within different applications keeping its consistency and working principle. The bubble counter was initially thought as a simple detector for an artificial heart controller but, as long as we have introduced new features, its functionalities started to become important for other applications as biomedicine, microfluidics and microparticles manufacturing.

It has been demonstrated to be greatly effective to detect bubbles and accurate enough to estimate their shape and sense their volume. Furthermore, future work would be to introduce velocimetry through Doppler effect, so a more accurate velocity can be calculated regardless the type of channel; and to measure with an array of lasers diodes along the width of the channel in order to precisely measure the minor axis of bubbles and assure their detection as well.

Finally, I would like to mention that the objectives presented in the introduction, Chapter 1, have been achieved, as this thesis reflects a deep sight to self-mixing effect and its applications, in addition to new LFI features designed once the background was acknowledged. Moreover, I have had the chance to work in a great and professional researching group and to participate in its development by actively participate in its researching discussions.
Appendix 1: Lang–Kobayashi model

Electric fields are both transmitted and reflected when a change in the refractive index in the medium is reached. They are split into an amplitude envelope and a variable phase and Lang and Kobayashi assumed plane wave and slowly varying envelope approximations of the electric field, \( E(t)e^{j\omega t} \) where \( E(t) \) is the slowly varying and complex envelope of the electric field which oscillates rapidly as \( e^{j\omega t} \); after neglecting multiple reflections in the external cavity for a diode laser under feedback for their model:

\[
\frac{d}{dt}(E(t)e^{j\omega t}) = [j\omega_m + \frac{1}{2}(\Gamma G - \frac{1}{\tau_p})]E(t)e^{j\omega t} + \tilde{\kappa}E(t - \tau_{ext})e^{j\omega(t-\tau_{ext})} \tag{1}
\]

Leading to the following expression after applying the time derivation and rearranging terms:

\[
\frac{d}{dt}E(t) = [j(\omega_m - \omega) + \frac{1}{2}(\Gamma G - \frac{1}{\tau_p})]E(t) + \tilde{\kappa}E(t - \tau_{ext})e^{-j\omega\tau_{ext}} \tag{2}
\]

The last term represents the back injection of light and \( \tilde{\kappa} \) is a measure of the coupling strength between the laser and the external cavities. For notation, \( \text{in} \) refers to the laser internal cavity whereas \( \text{ext} \), to the external one.

\[
\tilde{\kappa} = \frac{\kappa}{\tau_{in}} = e(1 - R_2)\sqrt{\frac{R}{R_2}}\frac{1}{\tau_{in}} \tag{3}
\]

Where \( \tau_{in} \) is the laser internal cavity round-trip time and corresponds to \( \tau_{in} = 2n_{in}L_{in}/c = 2L_{in}/\nu_g \). Then, Eq.2 is coupled with the carrier density in a laser through the gain term \( G \) and they both describe the laser dynamics under optical feedback.

\[
\frac{d}{dt}N(t) = \frac{\eta_i I(t)}{qV} - \frac{N(t)}{\tau_n} - GS(t) \tag{4}
\]

In order to reach a real system of equations, the complex rate equation Eq.2 is split into two equations, one for the photon density in the laser cavity, \( S \), and one for the phase of the electric field, \( \phi \).

The photon density in the laser cavity is defined as \( S(t) = |E(t)|^2 = E(t)E^*(t) \), where the electric field \( E(t) \) has been scaled with its complex \( E^*(t)e^{-j\omega t} \), denoted by *.

\[
\frac{d}{dt}S(t) = E(t)\frac{d}{dt}E^*(t) + \frac{d}{dt}E(t)E^*(t) \tag{5}
\]

\[
\frac{d}{dt}S(t) = E(t)[-j(\omega_m - \omega) + \frac{1}{2}(\Gamma G - \frac{1}{\tau_p})]E^*(t) + \tilde{\kappa}E^*(t)e^{j\omega\tau_{ext}}
+ [j(\omega_m - \omega) + \frac{1}{2}(\Gamma G - \frac{1}{\tau_p})]E(t) + \tilde{\kappa}E(t - \tau_{ext})e^{-j\omega\tau_{ext}} E^*(t) \tag{6}
\]

\[
= (\Gamma G - \frac{1}{\tau_p})S(t) + \tilde{\kappa}E^*(t)E(t - \tau_{ext})e^{-j\omega\tau_{ext}} + \tilde{\kappa}E(t)E^*(t - \tau_{ext})e^{j\omega\tau_{ext}}
\]

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Finally, applying that $E(t) = |E(t)|e^{j\varphi(t)}$ where $\varphi(t) = \arctan(\Im(E(t))/\Re(E(t)))$ and $\cos(x) = (e^{ix} + e^{-ix})/2$, the rate equation for the photon density is reached:

$$\frac{d}{dt} S(t) = (\Gamma G - \frac{1}{\tau_p}) S(t) + 2\kappa \sqrt{S(t)S(t - \tau_{ext})} \cos(\omega\tau_{ext} + \varphi(t) - \varphi(t - \tau_{ext}))$$ \hspace{1cm} (7)

On the other hand, in order to reach the rate equation for the phase, the direct relation of $\varphi(t)$ with $E(t)$ must be derived and applied in Eq.2. The following expression $\Im(x\ast y) = \Im(y) \Re(x) - \Im(x) \Re(y)$ is useful to simplify the rate equation.

$$\frac{d}{dt} \varphi(t) = \frac{1}{1 + (\Im(E(t)))^2} \frac{\frac{d}{dt}(\Im(E(t)))\Re(E(t)) - \Im(E(t)) \frac{d}{dt}\Re(E(t))}{\Re(E(t))^2}$$

$$= \frac{1}{\Re(E(t))^2 + \Im(E(t))^2} \left[ \Im \frac{d}{dt} E(t) \Re(E(t)) - \Im(E(t)) \Re(\frac{d}{dt} E(t)) \right]$$ \hspace{1cm} (8)

Applying the Eq.2 in the above equation yields:

$$E^\ast(t) \frac{d}{dt} E(t) = [j(\omega_m - \omega) + \frac{1}{2} (\Gamma G - \frac{1}{\tau_p})] S(t) + \kappa E^\ast(t) E(t - \tau_{ext}) e^{-j\omega\tau_{ext}}$$

$$= [j(\omega_m - \omega) + \frac{1}{2} (\Gamma G - \frac{1}{\tau_p})] S(t) + \kappa \sqrt{S(t)S(t - \tau_{ext})} e^{-j\omega\tau_{ext} + \varphi(t) - \varphi(t - \tau_{ext})}$$ \hspace{1cm} (9)

Where it only remains to apply the relations for $e^{ix} = \cos(x) + j\sin(x)$ and $\sin(-\alpha) = -\sin(\alpha)$ and finally select uniquely the imaginary terms:

$$\frac{d}{dt} \varphi(t) = (\omega_m - \omega) - \kappa \sqrt{S(t - \tau_{ext})/S(t)} \sin(\omega\tau_{ext} + \varphi(t) - \varphi(t - \tau_{ext}))$$ \hspace{1cm} (10)

However, the term $\omega_m - \omega$ can be even more simplified if it is assumed that $\omega \approx \omega_{th}$ and, as a consequence, it is linearized. In order to do so, both laser gain index $n_i$ and laser gain $G$ must be linearized with respect to the carrier density $N$ at the threshold value.

$$n_i = n_{th} + (N - N_{th}) \frac{\partial n_{in}}{\partial N}$$ \hspace{1cm} (11)

$$\Gamma G = \Gamma G_{th} + \Gamma (N - N_{th}) \frac{\partial G}{\partial N} = \frac{1}{\tau_p} \Gamma (N - N_{th}) \frac{\partial G}{\partial N}$$

$$\rightarrow (N - N_{th}) = \frac{\Gamma G - \frac{1}{\tau_p}}{\Gamma \frac{\partial G}{\partial N}}$$ \hspace{1cm} (12)
From the expression $\omega_{th} = m\pi c / (n_{th}L_{in})$:

$$\omega_m - \omega_{th} = \frac{m\pi c}{L_{in}} \left( \frac{1}{n_{in}} - \frac{1}{n_{th}} \right) = \frac{m\pi c}{L_{in}} \left[ \frac{n_{th} - n_{in}}{n_{in}n_{th}} \right]$$

$$= -\frac{m\pi c}{L_{in}n_{in}n_{th}} (N - N_{th}) \frac{\partial n_{in}}{\partial N} = -\frac{\omega_m}{n_{th}} (N - N_{th}) \frac{\partial n_{in}}{\partial N}$$

(13)

Now, substituting the last relationship found in Eq.12 in the above Eq.13 and noting that $G = \nu_{a}a(N - N_{th})$, the final expression is yield by using the linewidth enhancement factor of the laser $\alpha$:

$$\omega_m - \omega_{th} = -\frac{\omega_m}{n_{th}} \frac{(\Gamma G - \frac{1}{\tau_p})}{\Gamma} \frac{\partial n_{in}}{\partial N} = \frac{1}{2} \alpha \left( \Gamma G - \frac{1}{\tau_p} \right)$$

(14)

Finally, the set of rate equations Eq.4, Eq.7 and Eq.10 for the number of electrons are simplified by applying the linearized relationship found above and the fact that $\omega \approx \omega_{th}$.

$$\frac{d}{dt} S(t) = (\Gamma G - \frac{1}{\tau_p}) S(t) + \tilde{\kappa} \sqrt{S(t)S(t - \tau_{ext})} \cos(\omega \tau_{ext} + \phi(t) - \phi(t - t_{ext}))$$

(15)

$$\frac{d}{dt} \phi(t) = \frac{1}{2} \alpha \left( \Gamma G - \frac{1}{\tau_p} \right) - \tilde{\kappa} \frac{\sqrt{S(t - \tau_{ext})}}{S(t)} \sin(\omega \tau_{ext} + \phi(t) - \phi(t - \tau_{ext}))$$

(16)

$$\frac{d}{dt} N(t) = \frac{\eta_{i} I(t)}{qV} - \frac{N(t)}{\tau_{n}} - GS(t)$$

(17)
Appendix 2: Additional figures

Additional figures for the better understanding of the different sections can be found below.

Figure 1: LabVIEW front diagram showing from top to bottom the amplitude, the RMS and the phase of each signal; from left to right, the back, the scanning PD and the fixed PDs. [Chapter 5. Sec.5.2]
Figure 2: RMS of each PD, back (Left), scanning (Center) and fixed (Right). [Chapter 5. Sec.5.2]

Figure 3: Phase relation of each PD, back (Left), scanning (Center) and fixed (Right). [Chapter 5. Sec.5.2]

Figure 4: 850 nm DFB laser beam onto the scanning PD surface. [Chapter 5. Sec.5.3]
Figure 5: Front image of the bubble counter mount. [Chapter 6. Sec.6.1]

Figure 6: Bird’s eye view of the bubble counter mount. [Chapter 6. Sec.6.1]
Bibliography


