Master thesis of
Automatic control and Robotics

Control of Wind Turbines using Takagi-Sugeno Approach

Ruicong Yang

Supervisors:
Dr. Vicenç Puig Cayuela

Universitat Politècnica de Catalunya
Escola Tècnica Superior d’Enginyeria Industrial de Barcelona
Spain
2017
Acknowledgements

I would first like to thank my thesis supervisor Dr. Vicenç Puig Cayuela of Universitat Politècnica de Catalunya, he reply my email or message very fast whenever I had doubts about my research or writing, and he can always give me very professional conclusion and steered me in the right direction whenever he thought I needed it. Without his patient help and input, this work would not have been successfully conducted.
Abstract

This thesis will investigate the use of the Takagi-Sugeno approach to the control design applied to the wind turbines. The wind turbine model will be transformed to the Takagi-Sugeno representation. From that, control strategies will be developed that will allow the wind turbine to operate in case of faulty situations. The proposed solutions will be tested using a well-known wind turbine case study.
Contents

Abstract ............................................. 5
Index .............................................. 5
Index of figures ................................... 9
Index of tables ................................... 11

1 Introduction ..................................... 13
  1.1 Wind energy world capacity .................. 13
  1.2 Motivation ................................... 14
  1.3 Objectives of project ......................... 15
  1.4 Thesis structure ............................... 15

2 Wind Turbine Modeling .......................... 17
  2.1 Wind turbine Basics ............................ 17
  2.2 Wind Turbine Modeling ....................... 18
    2.2.1 Aerodynamic model ....................... 18
    2.2.2 Pitch system model ....................... 19
    2.2.3 Drive train model ......................... 19
    2.2.4 Generator and converter model ........... 20
  2.3 PI control of wind turbine description ...... 20
  2.4 Data definition ............................... 22
    2.4.1 State space representation of the wind turbine 24
  2.5 Takagi-Sugeno Model .......................... 26
    2.5.1 Takagi-Sugeno approach .................. 26
2.5.2 Wind turbine Takagi-Sugeno model ........................................... 28

3 State feedback control ................................................................. 37
   3.1 Control of Wind Turbines .................................................... 37
      3.1.1 Design fuzzy controller ............................................. 37
      3.1.2 Observer design ..................................................... 40
   3.2 Obtaining the state feedback controller ............................... 43
      3.2.1 Control structure of Wind Turbines ............................ 44
   3.3 Obtaining the observer ..................................................... 46
      3.3.1 Observer based control ............................................. 46
      3.3.2 State feedback using observer .................................. 48

4 Comparison with PI controller .................................................. 51
   4.1 T-S controller .................................................................... 51
   4.2 T-S observer based control .............................................. 54

5 Conclusions ............................................................................. 59
   5.1 Work Summary .................................................................... 59
   5.2 Future work ....................................................................... 60

Bibliography .................................................................................. 79
List of Figures

1.2 Wind Power Capacity and Additions, Top 10 Countries, 2016. figure from [4],
Notes that Germany's additions are net of decommissioning and re-powering.
"∼ 0" denotes capacity additions of less than 50MW. 14
2.1 Wind turbine 17
2.2 Wind turbine components. Figure from [13] 18
2.3 Illustration of the reference power curve for the wind turbine depending on the
wind speed 21
2.4 The wind speed 23
2.5 Reference of the torque 23
2.6 Reference of the pitch angle 24
2.7 Membership Functions $M_1(z_1(t))$ and $M_2(z_1(t))$ 30
2.8 Membership Functions $N_1(z_2(t))$ and $N_2(z_2(t))$ 30
2.9 Membership Functions $L_1(z_3(t))$ and $L_2(z_3(t))$ 31
3.1 LMI region $S(\alpha, r, \theta)$ 40
3.2 Poles of the controller 44
3.3 Wind turbine control feedback loops 44
3.4 Controlled torque 45
3.5 Controlled pitch angle 45
3.6 Poles of the observer 46
3.7 Closed-loop estimation by using the observer 47
3.8 Torque estimated by the observer 47
3.9 Zoom in of the torque estimated by the observer 48
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.10</td>
<td>Pitch angle generated by the observer</td>
<td>48</td>
</tr>
<tr>
<td>3.11</td>
<td>State feedback using the observer</td>
<td>49</td>
</tr>
<tr>
<td>3.12</td>
<td>Controlled torque obtained by state feedback using the observer</td>
<td>49</td>
</tr>
<tr>
<td>3.13</td>
<td>Controlled pitch angle obtained by state feedback using the observer</td>
<td>50</td>
</tr>
<tr>
<td>4.1</td>
<td>Output torque generated by state feedback T-S controller and PI controller</td>
<td>51</td>
</tr>
<tr>
<td>4.2</td>
<td>Output torque generated by state feedback T-S controller and PI controller in time 0 to 400s</td>
<td>52</td>
</tr>
<tr>
<td>4.3</td>
<td>Output torque generated by state feedback T-S controller and PI controller in time 2600s to 3000s</td>
<td>53</td>
</tr>
<tr>
<td>4.4</td>
<td>Output pitch angle generated by T-S controller and PI controller</td>
<td>53</td>
</tr>
<tr>
<td>4.5</td>
<td>Output pitch angle generated by T-S controller and PI controller in time 2600s to 3000s</td>
<td>54</td>
</tr>
<tr>
<td>4.6</td>
<td>Output torque generated by T-S observer based state feedback T-S controller and PI controller</td>
<td>55</td>
</tr>
<tr>
<td>4.7</td>
<td>Output torque generated by T-S observer based state feedback T-S controller and PI controller from time 0 to 400s</td>
<td>55</td>
</tr>
<tr>
<td>4.8</td>
<td>Output torque generated by T-S observer based state feedback T-S controller and PI controller from time 2600s to 3000s</td>
<td>56</td>
</tr>
<tr>
<td>4.9</td>
<td>Output pitch angle generated by T-S observer based state feedback T-S controller and PI controller</td>
<td>56</td>
</tr>
<tr>
<td>4.10</td>
<td>Output pitch angle generated by T-S observer based state feedback T-S controller and PI controller in time 2600s to 3000s</td>
<td>57</td>
</tr>
<tr>
<td>5.1</td>
<td>Wind turbine accidents in year, up to 31 of May 2017. Figure from [13]</td>
<td>60</td>
</tr>
</tbody>
</table>
## List of Tables

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Data of the system</td>
<td>22</td>
</tr>
<tr>
<td>2.2</td>
<td>Fuzzy model</td>
<td>30</td>
</tr>
<tr>
<td>3.1</td>
<td>Fuzzy model with fuzzy control rule</td>
<td>39</td>
</tr>
<tr>
<td>3.2</td>
<td>Fuzzy model with fuzzy observer rule</td>
<td>43</td>
</tr>
<tr>
<td>5.1</td>
<td>Structural failure of wind turbine up to 31 May 2017</td>
<td>61</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Wind energy world capacity

Nowadays, wind energy is world wild used, as an alternative to burning fossil fuels, it is plentiful, renewable, widely distributed, clean, produces no greenhouse gas emissions during operation, consumes no water, and uses little land. [1] The net effects on the environment are far less problematic than those of nonrenewable power sources.

As of 2015, Denmark generates 40% of its electric power from wind, and at least 83 other countries around the world are using wind power to supply their electric power grids [2]. In 2014, global wind power capacity expanded 16% to 369,553 MW [3]. Moreover, almost 55 GW of wind power capacity was added during 2016, increasing the global total about 12% to nearly 487 GW between 2000 and 2015 (See Figure 1.1), wind increased from 2.4% to 15.6% of total EU power capacity. Germany installed total of almost 50 GW. These installations reflected the grid connection of a large amount of offshore capacity that was constructed in 2015. Spain continued to rank second in the EU for total operating capacity (23 GW) but add wind capacity less than 50 MW in 2016. China added 23.4 GW in 2016, for total installed capacity approaching 169 GW, and accounted for one-third of total global capacity by year’s end [4].
Figure 1.1: Wind Power Global Capacity and Annual Additions, 2006-2016. Figure from [4]

Figure 1.2: Wind Power Capacity and Additions, Top 10 Countries, 2016. Figure from [4], Notes that Germany’s additions are net of decommissioning and re-powering. "∼ 0" denotes capacity additions of less than 50 MW.

1.2 Motivation

With the large capacity of wind turbines, control of wind turbine is important. And with rapidly growing popularity of fuzzy control systems in engineering applications, Tagaki-Sugerno
approach has applied to many applications\textsuperscript{5}: missiles\textsuperscript{6}, aircraft\textsuperscript{7}, energy production systems\textsuperscript{8}, robotic systems\textsuperscript{9}, active suspension of vehicles\textsuperscript{10}, engines\textsuperscript{11} and fault tolerant control\textsuperscript{12}. But there are very few people doing research on wind turbines, Sören Georg\textsuperscript{24} \textsuperscript{25} \textsuperscript{26} \textsuperscript{27} and Urs Giger\textsuperscript{29}, Xiao Xu Liu\textsuperscript{28} etc. So this thesis will introduce the basics of Tagaki-Sugerno approach applied on wind turbine, Which is good way for a beginning understanding.

1.3 Objectives of project

As a size and flexible structures operating in uncertain environments, advanced control technology can improve their performance. For example, advanced controllers can help decrease the cost of wind energy by increasing turbine efficiency, and thus energy capture, and by reducing structural loading, which increases the lifetimes of the components and structures\textsuperscript{15}.

This project will focus on the usage of a fuzzy control technique, Tagaki-Sugerno (T-S) approach for the controller and observer design for a dynamic nonlinear wind turbine model. Both T-S controller and the T-S observer will be implemented and compared with the controller presented in\textsuperscript{14}. The controller and observer gain will be obtained by using LMI\textsuperscript{21}.

All the simulations will be implemented using MATLAB and SIMULINK. The optimizer to be used is SeDuMi (http://sedumi.ie.lehigh.edu/).

1.4 Thesis structure

The structure of the main work is the following:

In Chapter 2 a set of wind turbine models are presented. It is divided in three parts, the first part will describe the wind turbine and its components. The second part presents its mathematics model of each components and transfer the systems to a state-space representation. The third part will compute the T-S model of the wind turbine.

Chapter 3 will present the state feedback control of the wind turbine. It is divided in three parts, the first part introduces the control structure. The second part presents the T-S controller for the wind turbine. The third part will present the state feedback control by using T-S observer.

Chapter 4 will make the comparison between the result with a PI controller and the T-S model and controller, and also the T-S observer based control.
Chapter 2

Wind Turbine Modeling

2.1 Wind turbine Basics

A wind turbine captures the wind kinematic energy and transforms it into mechanical energy (rotating shaft) first and then into electrical energy (generator). The main components of the horizontal-axis wind turbines (HAWT) in Figure 2.1 that are visible from the ground are the tower, nacelle, and rotor, as shown in Figure 2.2.

Figure 2.1: Wind turbine
At first, the wind encounters the rotor on this upwind horizontal-axis turbine and rotates it. The low-speed shaft transfers energy to the gearbox, which steps up in speed and spins the high-speed shaft, which increases the speed and rotates the high-speed shaft. The high-speed shaft causes the generator to spin, producing electricity. In the figure, it is shown that the yaw-actuation mechanism, which is used to turn the nacelle so that the rotor faces into the wind [15].

2.2 Wind Turbine Modeling

In this thesis, the wind turbine model will be used is a three-bladed pitch-controlled variable-speed wind turbine with a nominal power of 4.8MW that is the one described in paper [14]

The description of the model is presented in the following.

2.2.1 Aerodynamic model

The aerodynamics of the wind turbine is modeled as a torque acting on the blades, according to:

$$\tau_r(t) = \sum_{1\leq i \leq 3} \frac{\rho \pi R^3 C_q(\lambda(t), \beta_i(t)) v_{w,i}(t)^2}{6}$$  \hspace{1cm} (2.1)
where \( v_w \) is the wind speed, \( \rho = 1.225 \text{kg/m}^3 \) is the air density, \( R = 57.5 \text{m} \) is the rotor radius, \( \beta_i \) is pitch position, and \( \lambda \) is the Tip Speed Ratio, defined as:

\[
\lambda = \frac{\omega_r \cdot R}{v_w}
\]

### 2.2.2 Pitch system model

For each blade, the hydraulic pitch system is modeled as a closed-loop transfer function between the pitch angle \( \beta_i \) and its reference \( \beta_{i, \text{ref}} \), according to:

\[
\frac{\beta_i(s)}{\beta_{i, \text{ref}}(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \cdot s + \omega_n^2}
\]

which can be written as a differential equation:

\[
\ddot{\beta}_i(t) = -2\xi\omega_n \cdot \dot{\beta}(t) - \omega_n^2\beta(t) + \omega_n^2\beta_{i, \text{ref}}
\]

where \( \xi = 0.6 \) is the damping factor, and \( \omega_n = 11.11 \text{rad/s} \) is the natural frequency, and \( i = 1, 2, 3 \) for three blades.

### 2.2.3 Drive train model

The drive train is modeled by a two-mass model:

\[
J_r \dot{\omega}_r(t) = \tau_r(t) - K_{dt}\theta_\Delta(t) - (B_{dt} + B_r)\omega_r(t) + \frac{B_{dt}}{N_g}\omega_g(t)
\]

\[
J_g \dot{\omega}_g(t) = \frac{\eta_{dt}K_{dt}}{N_g}\theta_\Delta(t) + \frac{\eta_{dt}B_{dt}}{N_g}\omega_r(t) - \left( \frac{\eta_{dt}B_{dt}}{N_g^2} + B_g \right)\omega_g(t) - \tau_g(t)
\]

\[
\dot{\theta}_\Delta(t) = \omega_r(t) - \frac{1}{N_g}\omega_g(t)
\]

where \( J_r = 55 \cdot 10^6 \text{kg} \cdot \text{m}^2 \) is the moment of inertia of the low-speed shaft, \( K_{dt} = 2.7 \cdot 10^9 \text{Nm/rad} \) is the torsion stiffness of the drive train, \( B_{dt} = 775.49 \text{Nms/rad} \) is the torsion damping coefficient of the drive train and \( B_r = 7.11 \text{Nms/rad} \), \( B_g = 45.6 \text{Nms/rad} \) is the viscous friction of the high-speed shaft, \( N_g = 95 \) is the gear ratio, \( J_g = 390 \text{kg} \cdot \text{m}^2 \) is the
moment of the inertia of the high-speed shaft, $\eta_{dt} = 0.97$ is the efficiency of the drive train, and $\theta_\Delta(t)$ is the torsion angle of the drive train.

### 2.2.4 Generator and converter model

The generator and converter dynamics can be modeled by a first transfer function

$$\frac{\tau_g(s)}{\tau_{g,ref}(s)} = \frac{\alpha_{gc}}{s + \alpha_{gc}}$$  \hspace{1cm} (2.8)

The power produced by the generator is given by

$$P_g(t) = \eta_g \omega_g(t) \tau_g(t)$$  \hspace{1cm} (2.9)

where $\alpha_{gc} = 50 \text{rad/s}$ is the generator and converter model parameter, $\eta_g = 0.98$ is the efficiency of the generator. Besides The generator torque $\tau_g$ is controlled by the reference $\tau_{g,ref}$. The dynamics can be approximated by a first order model with time constant $t_g$ \[16\].

$$\dot{\tau}_g(t) = -\frac{\tau_g(t)}{t_g} + \frac{\tau_{g,ref}(t)}{t_g}$$  \hspace{1cm} (2.10)

where $t_g = 20 \cdot 10^{-3}$

### 2.3 PI control of wind turbine description

Figure 2.3 shows the different operating ranges of the wind turbine \[14\].
The controller has two modes. Mode 1 corresponds to the wind zone 2 and mode 2 corresponds to the wind zone 3. Consider our wind data in Figure 2.4, at more or less time 2300s, the wind speed goes from zone 2 to zone 3. Hence, we can assume that from time 0 to 2300s, the PI controller is in mode 1, and after that it goes to mode 2 [14].

The control mode switches from mode 1 to 2 if

$$P_g[n] \geq P_r[n] \vee \omega_g[n] \geq \omega_{nom}$$

(2.11)

where $\omega_{nom} = 162 rad/s$ is the nominal generator speed. The control mode switches from mode 2 to 1 if

$$\omega_g[n] < \omega_{nom} - \omega_{\Delta}$$

(2.12)

**Control Mode 1:**

$$\tau_{g,r}[n] = K_{opt} \cdot \left(\frac{\omega_g[n]}{N_g}\right)^2$$

(2.13)

where

$$K_{opt} = \frac{1}{2} \rho A R^3 C_{P_{max}} \lambda_{opt}^3$$

(2.14)

where $A$ is the area swept by the wind turbine blades, so we have $A = \pi R^2 = 1.0387 \times 10^4 m^2$, and $\lambda_{opt}$ is the optimal value of $\lambda$, $C_{P_{max}}$ is the maximum value of the power coefficient.
Control Mode 2: In this mode, the major control actions are handled by the pitch system using a PI controller trying to keep $\omega_g[n]$ at $\omega_{nom}$

$$\beta_r[n] = \beta_r[n-1] + K_p e[n] + (K_i \cdot T_s - K_p)e[n-1]$$

(2.15)

where $e[n] = \omega_g[n] - \omega_{nom}$, and the controller gain of the PI is $K_p = 4$ and $K_i = 1$. In this case, the converter reference is used to suppress fast disturbances by

$$\tau_{g,r}[n] = \frac{P_r[n]}{\eta_{gc} \cdot \omega_g[n]}$$

(2.16)

2.4 Data definition

The data of the system we are going to use are all described in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>1.225</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$R$</td>
<td>57.5</td>
<td>m</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.6</td>
<td>-</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>11.11</td>
<td>rad/s</td>
</tr>
<tr>
<td>$J_r$</td>
<td>$55 \cdot 10^9$</td>
<td>kg $\cdot$ m$^2$</td>
</tr>
<tr>
<td>$K_{dt}$</td>
<td>$2.7 \cdot 10^9$</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>$B_{dt}$</td>
<td>775.49</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>$B_r$</td>
<td>7.11</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>$B_g$</td>
<td>45.6</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>$N_g$</td>
<td>95</td>
<td>-</td>
</tr>
<tr>
<td>$J_g$</td>
<td>390</td>
<td>kg $\cdot$ m$^2$</td>
</tr>
<tr>
<td>$\eta_{dt}$</td>
<td>0.97</td>
<td>-</td>
</tr>
<tr>
<td>$\eta_g$</td>
<td>0.98</td>
<td>-</td>
</tr>
<tr>
<td>$t_g$</td>
<td>$20 \cdot 10^{-3}$</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2.1: Data of the system

And the wind data we are using is shown in the figure below,
Figure 2.4: The wind speed

the reference of the inputs $\begin{bmatrix} \tau_{g, \text{ref}} & \beta_{1, \text{ref}} & \beta_{2, \text{ref}} & \beta_{3, \text{ref}} \end{bmatrix}^T$ are shown as follow, notice that
the value of reference for each pitch angle to the blade.

Figure 2.5: reference of the torque
2.4.1 State space representation of the wind turbine

In order to use the Takagi-Sugeno Approach, first we need to transform our model into state-space representation. Defining the state and input vectors, as in [16]

\[
x(t) = \begin{bmatrix} \omega_r & \omega_y & \theta_\Delta & \tau_g & \beta_1 & \dot{\beta}_1 & \beta_2 & \dot{\beta}_2 & \beta_3 & \dot{\beta}_3 \end{bmatrix}^T
\] (2.17)

\[
u(t) = \begin{bmatrix} \tau_{g,ref} & \beta_{1,ref} & \beta_{2,ref} & \beta_{3,ref} \end{bmatrix}^T
\] (2.18)

the model of the wind turbine can be written into a state space embedding the non-linearities in the parameters

\[
\dot{x} = Ax(t) + Bu(t)
\] (2.19)

\[
y = Cx(t)
\] (2.20)
where

\[
A = \begin{bmatrix}
-B_{dt} + \frac{B_r}{J_r} & B_{dt} & -\frac{K_{dt}}{J_r} & 0 & z_1(t) & 0 & z_2(t) & 0 & z_3(t) & 0 \\
\frac{\eta_{dt} B_{dt}}{N_g J_g} & -\frac{\eta_{dt} B_{dt}}{N_g^2 J_g} & -\frac{B_g}{J_g} & \frac{\eta_{dt} K_{dt}}{J_g} & -\frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 \\
1 & -\frac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{t_g} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\omega_n^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\omega_n^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi \omega_n & 0 & 0 & 0 \\
\end{bmatrix}
\] (2.21)

where

\[
z_1(t) = \frac{\rho \pi R^3 C_q(\lambda(t), \beta_1(t)) v_w(t)^2}{6 J_r \beta_1}
\] (2.22)

\[
z_2(t) = \frac{\rho \pi R^3 C_q(\lambda(t), \beta_2(t)) v_w(t)^2}{6 J_r \beta_2}
\] (2.23)

\[
z_3(t) = \frac{\rho \pi R^3 C_q(\lambda(t), \beta_3(t)) v_w(t)^2}{6 J_r \beta_3}
\] (2.24)
\[ B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & \frac{1}{I_g} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \omega_n^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_n^2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_n^2 \end{bmatrix} \]  

(2.25)

\[ C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \]  

(2.26)

2.5 Takagi-Sugeno Model

2.5.1 Takagi-Sugeno approach

To apply Takagi-Sugeno (T-S) model, here we are using the method which presented in Chapter 2 of the book [17]. The fuzzy model proposed by Takagi and Sugeno [18] is described by fuzzy IF-THEN rules which represent local linear input-output relations of a nonlinear system. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model.

The \( i \)th rules of the T-S fuzzy models are of the following form, where CFS and DFS denote the continuous fuzzy system and the discrete fuzzy system, respectively.

**Model Rule \( i \):**

**IF** \( z_1(t) \) is \( M_{i1} \), \ldots and \( z_p(t) \) is \( M_{ip} \),
THEN

\[
\begin{align*}
\dot{x}(t) &= A_i x(t) + B_i u(t) \\
y(t) &= C_i x(t)
\end{align*}
\]

\[i = 1, 2, ... , r \quad (2.27)\]

Here, \(M_{ij}\) is the fuzzy set and \(r\) is the number of model rules; \(x(t) \in \mathbb{R}^n\) and \(x(k) \in \mathbb{R}^n\) are the state vectors, \(u(t) \in \mathbb{R}^m\) and \(u(k) \in \mathbb{R}^m\) are the input vectors, \(y(t) \in \mathbb{R}^q\) and \(y(k) \in \mathbb{R}^q\) are the output vectors, \(A_i \in \mathbb{R}^{n \times n}\), \(B_i \in \mathbb{R}^{n \times m}\) and \(C_i \in \mathbb{R}^{q \times n}\), \(z_1(t), ... , z_p(t)\) are known premise variables that may be functions of the state variables, external disturbances, and/or time.

Given a pair of \(x(t), u(t)\), the final outputs of the fuzzy systems are inferred as follows:

\[
\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z(t))(A_i x(t) + B_i u(t))}{\sum_{i=1}^{r} w_i(z(t))} \quad (2.28)
\]

\[
= \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \quad (2.29)
\]

\[
y(t) = \frac{\sum_{i=1}^{r} w_i(z(t))C_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} \quad (2.30)
\]

\[
= \sum_{i=1}^{r} h_i(z(t))C_i x(t) \quad (2.31)
\]

where

\[
z(t) = [z_1(t), z_2(t), ... , z_p(t)] \quad (2.32)
\]

\[
w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)) \quad (2.33)
\]

\[
h_i(t) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))} \quad (2.34)
\]
for all \( t \). The term \( M_{ij}(z_j(t)) \) is the grade of membership of \( z_j(t) \) in \( M_{ij} \). Since

\[
\begin{align*}
\sum_{i=1}^{r} w_i(z(t)) &> 0 \\
 w_i(z(t)) &\geq 0, \quad i = 1, 2, \ldots, r.
\end{align*}
\]  

(2.35)

we have

\[
\begin{align*}
\sum_{i=1}^{r} h_i(z(t)) &> 0 \\
 h_i(z(t)) &\geq 0, \quad i = 1, 2, \ldots, r.
\end{align*}
\]  

(2.36)

for all \( t \).

### 2.5.2 Wind turbine Takagi-Sugeno model

From equation (2.29) to (2.34), we bound \( z_1(t) \in [z_{1,\text{min}}, z_{1,\text{max}}] \), \( z_2(t) \in [z_{2,\text{min}}, z_{2,\text{max}}] \), \( z_3(t) \in [z_{3,\text{min}}, z_{3,\text{max}}] \).

From the maximum and minimum values, \( z_1(t) \), \( z_2(t) \) and \( z_3(t) \) can be represented by

\[
z_1(t) = \frac{\rho \pi R^3 c_q(\lambda(t), \beta_1(t)) v_w(t)^2}{6 J_r \beta_1} = M_1(z_1(t)) \cdot z_{1,\text{max}} + M_2(z_1(t)) \cdot z_{1,\text{min}}
\]  

(2.37)

\[
z_2(t) = \frac{\rho \pi R^3 c_q(\lambda(t), \beta_2(t)) v_w(t)^2}{6 J_r \beta_2} = N_1(z_2(t)) \cdot z_{2,\text{max}} + N_2(z_2(t)) \cdot z_{2,\text{min}}
\]  

(2.38)

\[
z_3(t) = \frac{\rho \pi R^3 c_q(\lambda(t), \beta_3(t)) v_w(t)^2}{6 J_r \beta_3} = L_1(z_3(t)) \cdot z_{3,\text{max}} + L_2(z_3(t)) \cdot z_{3,\text{min}}
\]  

(2.39)

Therefore the membership functions can be calculated as

\[
M_1 = \frac{z_1 - z_{1,\text{min}}}{z_{1,\text{max}} - z_{1,\text{min}}}
\]  

\[
M_2 = \frac{z_{1,\text{max}} - z_1}{z_{1,\text{max}} - z_{1,\text{min}}}
\]  

\[
N_1 = \frac{z_2 - z_{2,\text{min}}}{z_{2,\text{max}} - z_{2,\text{min}}}
\]  

\[
N_2 = \frac{z_{2,\text{max}} - z_2}{z_{2,\text{max}} - z_{2,\text{min}}}
\]  

(2.40)

(2.41)
\[
L_1 = \frac{z_3 - z_{3,\text{min}}}{z_{3,\text{max}} - z_{3,\text{min}}}, \quad L_2 = \frac{z_{3,\text{max}} - z_3}{z_{3,\text{max}} - z_{3,\text{min}}}
\]

(2.42)

We name the membership functions "Positive", "Negative", respectively. Then, the nonlinear system is represented by the following fuzzy model.

**Model Rule 1:**

\[\text{IF } z_1(t) \text{ is } "\text{Negative}" \quad \text{and} \quad z_2(t) \text{ is } "\text{Negative}" \quad \text{and} \quad z_3(t) \text{ is } "\text{Negative}\"
\]
\[\text{THEN } \dot{x}(t) = A_1 x(t) + Bu(t)\]

**Model Rule 2:**

\[\text{IF } z_1(t) \text{ is } "\text{Positive}" \quad \text{and} \quad z_2(t) \text{ is } "\text{Negative}" \quad \text{and} \quad z_3(t) \text{ is } "\text{Negative}\"
\]
\[\text{THEN } \dot{x}(t) = A_2 x(t) + Bu(t)\]

**Model Rule 3:**

\[\text{IF } z_1(t) \text{ is } "\text{Negative}" \quad \text{and} \quad z_2(t) \text{ is } "\text{Positive}" \quad \text{and} \quad z_3(t) \text{ is } "\text{Negative}\"
\]
\[\text{THEN } \dot{x}(t) = A_3 x(t) + Bu(t)\]

**Model Rule 4:**

\[\text{IF } z_1(t) \text{ is } "\text{Positive}" \quad \text{and} \quad z_2(t) \text{ is } "\text{Positive}" \quad \text{and} \quad z_3(t) \text{ is } "\text{Negative}\"
\]
\[\text{THEN } \dot{x}(t) = A_4 x(t) + Bu(t)\]

**Model Rule 5:**

\[\text{IF } z_1(t) \text{ is } "\text{Negative}" \quad \text{and} \quad z_2(t) \text{ is } "\text{Negative}" \quad \text{and} \quad z_3(t) \text{ is } "\text{Positive}\"
\]
\[\text{THEN } \dot{x}(t) = A_5 x(t) + Bu(t)\]

**Model Rule 6:**

\[\text{IF } z_1(t) \text{ is } "\text{Positive}" \quad \text{and} \quad z_2(t) \text{ is } "\text{Negative}" \quad \text{and} \quad z_3(t) \text{ is } "\text{Positive}\"
\]
\[\text{THEN } \dot{x}(t) = A_6 x(t) + Bu(t)\]

**Model Rule 7:**

\[\text{IF } z_1(t) \text{ is } "\text{Negative}" \quad \text{and} \quad z_2(t) \text{ is } "\text{Positive}" \quad \text{and} \quad z_3(t) \text{ is } "\text{Positive}\"
\]
\[\text{THEN } \dot{x}(t) = A_7 x(t) + Bu(t)\]

**Model Rule 8:**

\[\text{IF } z_1(t) \text{ is } "\text{Positive}" \quad \text{and} \quad z_2(t) \text{ is } "\text{Positive}" \quad \text{and} \quad z_3(t) \text{ is } "\text{Positive}\"
\]
\[\text{THEN } \dot{x}(t) = A_8 x(t) + Bu(t)\]

For illustrative purposes, this can be represented by the following table, where "Positive" can be represented by "+" and "Negative" can be represented by "−".
<table>
<thead>
<tr>
<th>Set</th>
<th>$z_1(t)$</th>
<th>$z_2(t)$</th>
<th>$z_3(t)$</th>
<th>A matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>rule 1</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$A_1$</td>
</tr>
<tr>
<td>rule 2</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
<td>$A_2$</td>
</tr>
<tr>
<td>rule 3</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$A_3$</td>
</tr>
<tr>
<td>rule 4</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$A_4$</td>
</tr>
<tr>
<td>rule 5</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$A_5$</td>
</tr>
<tr>
<td>rule 6</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$A_6$</td>
</tr>
<tr>
<td>rule 7</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
<td>$A_7$</td>
</tr>
<tr>
<td>rule 8</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$A_8$</td>
</tr>
</tbody>
</table>

Table 2.2: Fuzzy model

Figure 2.7 to 2.9 shows the graphical representation of the membership functions.

Figure 2.7: Membership Functions $M_1(z_1(t))$ and $M_2(z_1(t))$

Figure 2.8: Membership Functions $N_1(z_2(t))$ and $N_2(z_2(t))$
Thus, the matrices of the local models are

\[
A_1 = \begin{bmatrix}
\frac{-B_{dt} + B_r}{J_r} & \frac{B_{dt}}{J_g} & -\frac{K_{dt}}{J_g} & 0 & z_{1,\text{min}} & 0 & z_{2,\text{min}} & 0 & z_{3,\text{min}} & 0 \\
\frac{\eta_{dt} B_{dt}}{N_g J_g} & \frac{-\eta_{dt} B_{dt}}{N_g^2 J_g} & \frac{B_g}{J_g} & \frac{\eta_{dt} K_{dt}}{N_g J_g} & -\frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -\frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{t_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\end{bmatrix}
\]
\[ A_2 = \begin{bmatrix} \frac{-B_{dt} + B_r}{J_r} & \frac{B_{dt}}{N_g J_r} & \frac{-K_{dt}}{J_r} & 0 & z_{1,max} & 0 & z_{2,min} & 0 & z_{3,min} & 0 \\ \eta_{dt} \frac{B_{dt}}{N_g J_g} & \eta_{dt} \frac{B_{dt}}{N_g^2 J_g} - \frac{B_g}{J_g} \frac{1}{N_g J_g} & -1 \frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \frac{-1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ A_3 = \begin{bmatrix} \frac{-B_{dt} + B_r}{J_r} & \frac{B_{dt}}{N_g J_r} & \frac{-K_{dt}}{J_r} & 0 & z_{1,min} & 0 & z_{2,max} & 0 & z_{3,min} & 0 \\ \eta_{dt} \frac{B_{dt}}{N_g J_g} & \eta_{dt} \frac{B_{dt}}{N_g^2 J_g} - \frac{B_g}{J_g} \frac{1}{N_g J_g} & -1 \frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \frac{-1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]
\[
\begin{bmatrix}
\frac{B_{dt} + B_r}{J_r} & \frac{B_{dt}}{N_g J_r} & \frac{K_{dt}}{J_r} & 0 & z_{1,max} & 0 & z_{2,max} & 0 & z_{3,min} & 0 \\
\eta_{dt} & \frac{\eta_{dt} B_{dt}}{N_g^2 J_g} & -\frac{B_g}{J_g} & \frac{\eta_{dt} K_{dt}}{N_g J_g} & -\frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 \\
1 & -\frac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{t_g} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\( A_4 = \)

\[
\begin{bmatrix}
\frac{B_{dt} + B_r}{J_r} & \frac{B_{dt}}{N_g J_r} & \frac{K_{dt}}{J_r} & 0 & z_{1,min} & 0 & z_{2,min} & 0 & z_{3,max} & 0 \\
\eta_{dt} & \frac{\eta_{dt} B_{dt}}{N_g^2 J_g} & -\frac{B_g}{J_g} & \frac{\eta_{dt} K_{dt}}{N_g J_g} & -\frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 \\
1 & -\frac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{t_g} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\( A_5 = \)

\[
(2.46)
\]

\[
(2.47)
\]
The matrices $A_6$ and $A_7$ are given by:

$$A_6 = \begin{bmatrix}
\frac{B_{dt} + B_r}{J_r} & \frac{B_{dt}}{N_g J_r} & \frac{K_{dt}}{J_r} & 0 & z_{1,\text{max}} & 0 & z_{2,\text{min}} & 0 & z_{3,\text{max}} & 0 \\
\frac{\eta dt B_{dt}}{N_g J_g} & \frac{\eta dt B_{dt}}{N_g^2 J_g} & \frac{B_g}{J_g} & \frac{\eta dt K_{dt}}{N_g J_g} & -\frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 \\
1 & -\frac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$

$$A_7 = \begin{bmatrix}
\frac{B_{dt} + B_r}{J_r} & \frac{B_{dt}}{N_g J_r} & -\frac{K_{dt}}{J_r} & 0 & z_{1,\text{min}} & 0 & z_{2,\text{max}} & 0 & z_{3,\text{max}} & 0 \\
\frac{\eta dt B_{dt}}{N_g J_g} & \frac{\eta dt B_{dt}}{N_g^2 J_g} & \frac{B_g}{J_g} & \frac{\eta dt K_{dt}}{N_g J_g} & -\frac{1}{J_g} & 0 & 0 & 0 & 0 & 0 \\
1 & -\frac{1}{N_g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$

With:

$$\frac{B_{dt} + B_r}{J_r} = \frac{B_{dt}}{N_g J_r} = \frac{K_{dt}}{J_r} = 0$$

$$\frac{\eta dt B_{dt}}{N_g J_g} = \frac{\eta dt B_{dt}}{N_g^2 J_g} = \frac{B_g}{J_g} = \frac{\eta dt K_{dt}}{N_g J_g} = -\frac{1}{J_g}$$

$$1 = -\frac{1}{N_g}$$

Expressions (2.48) and (2.49) are:

$$B_{dt} + B_r = \eta dt B_{dt} = -\frac{1}{J_g}$$
The defuzzification is carried out as

\[
\dot{x}(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} M_i(Z_1(t)) N_j(Z_2(t)) L_k(Z_3(t)) \cdot A_i x(t) + Bu(t) \quad (2.51)
\]
Chapter 3

State feedback control

3.1 Control of Wind Turbines

3.1.1 Design fuzzy controller

From the wind turbine TS model obtained in previous chapter, we are going to design a state feedback controller. Here we will use a design procedure called "parallel distributed compensation" (PDC) [20]. This model-based design procedure was proposed in [19].

In the PDC design, each control rule is designed from the corresponding rule of a T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. For the fuzzy model (2.27), we construct the following fuzzy controller via the PDC:

**Control Rule i:**

\[
\text{IF } z_1(t) \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_p(t) \text{ is } M_{ip}, \]

\[
\text{THEN } u(t) = -F_i x(t), \quad i = 1, 2, \ldots, r.
\]

where \( F_i \) is the feedback control gain, it can be described a fuzzy control rule.

The overall fuzzy controller is represented by

\[
u(t) = \frac{\sum_{i=1}^{r} w_i(z(t)) F_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} = -\sum_{i=1}^{r} h_i(z(t)) F_i x(t) \quad (3.1)\]

Now to apply this procedure to our wind turbine case, we have.

**Control Rule 1:**
IF $z_1(t)$ is "Negative", $z_2(t)$ is "Negative" and $z_3(t)$ is "Negative"
THEN $u(t) = -F_1x(t)$

Control Rule 2:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Negative" and $z_3(t)$ is "Negative"
THEN $u(t) = -F_2x(t)$

Control Rule 3:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Positive" and $z_3(t)$ is "Negative"
THEN $u(t) = -F_3x(t)$

Control Rule 4:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Positive" and $z_3(t)$ is "Negative"
THEN $u(t) = -F_4x(t)$

Control Rule 5:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Negative" and $z_3(t)$ is "Positive"
THEN $u(t) = -F_5x(t)$

Control Rule 6:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Negative" and $z_3(t)$ is "Positive"
THEN $u(t) = -F_6x(t)$

Control Rule 7:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Positive" and $z_3(t)$ is "Positive"
THEN $u(t) = -F_7x(t)$

Control Rule 8:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Positive" and $z_3(t)$ is "Positive"
THEN $u(t) = -F_8x(t)$

Thus, we can design the feedback control law $u(t) = -F_i x(t)$ for each model, such that our system $\dot{x} = (A_i + BK_i)x(t)$ is asymptotically stable, where $K_i = -F_i$, therefore in our case, we have $B_1 = B_2 = \cdots = B_i = B$.

We can also present in following table.
The design is based on Lyapunov stability theory and LMI condition for stability of T-S systems in book [21]. We have the LMI region stabilization problem in the case of $S(\alpha, r, \theta)$ has a solution if and only if there exist a symmetric positive definite matrix $P_i$ and a matrix $W_i$ satisfying

$$A_i P_i + BW_i + PA_i^T + W_i^T B^T + 2\alpha P < 0$$

(3.2)

$$
\begin{bmatrix}
-qP_i + A_i P_i + BW_i \\
qP_i + P_i A_i^T + W_i^T B_i^T \\
\end{bmatrix} < 0
$$

(3.3)

$$
\begin{bmatrix}
(A_i P_i + BW_i + PA_i^T + W_i^T B^T) \sin\theta & (A_i P_i + BW_i - (PA_i^T + W_i^T B^T)) \cos\theta \\
(PA_i^T + W_i^T B^T - (A_i P_i + BW_i)) \cos\theta & (A_i P_i + BW_i + PA_i^T + W_i^T B^T) \sin\theta
\end{bmatrix} < 0
$$

(3.4)

In this case, the solution to our problem is given by

$$K_i = W_i P_i^{-1}$$

(3.5)

where $\alpha$ is the minimum speed of the response, $r$ is the maximum speed of the response, and $\theta$ is the overshoot. The LMI region $S$ is shown in the following figure [21].
3.1.2 Observer design

For designing the observer, book [17] has presented the methodologies for designing the T-S fuzzy observer. In linear system theory, one of the most important results on observer design is the so-called separation principle, which means that the controller and observer design can be carried out separately without compromising the stability of the overall closed-loop system. As this point, we can design the observer based on LMIs. As in all observer designs, fuzzy observers [22] [23] are required to satisfy

\[ \lim_{t \to \infty} (x(t) - \hat{x}(t)) = 0 \]  \hspace{1cm} (3.6)

where \( \hat{x}(t) \) denotes the state vector estimated by a fuzzy observer. This condition guarantees that the steady-state error between \( x(t) \) and \( \hat{x}(t) \) converges to 0. As in the case of controller design, the PDC concept is employed to arrive at the following fuzzy observer structures:

**Observer Rule i**

**IF** \( z_1(t) \) is \( M_{i1} \) and ... and \( z_p(t) \) is \( M_{ip} \)
CHAPTER 3. STATE FEEDBACK CONTROL

THEN

\[ \dot{\hat{x}}(t) = A_i \dot{x}(t) + B_i u(t) + L_i (y(t) - \hat{y}(t)) \]
\[ \hat{y}(t) = C_i \dot{x}(t), \quad i = 1, 2, \ldots, r \]

where \( L_i \) is the observer gain. For our wind turbine case, we have fuzzy observer law is given by (notice that in our case \( B_i = B_1 = \ldots = B_8 \) and \( C_i = C_1 = \ldots = C_8 \)).

Observer Rule 1:

IF \( z_1(t) \) is "Negative", \( z_2(t) \) is "Negative" and \( z_3(t) \) is "Negative"

THEN

\[ \dot{\hat{x}}(t) = A_1 \dot{x}(t) + B_1 u(t) + L_1 (y(t) - \hat{y}(t)) \]
\[ \hat{y}(t) = C_1 \dot{x}(t) \]

Observer Rule 2:

IF \( z_1(t) \) is "Positive", \( z_2(t) \) is "Negative" and \( z_3(t) \) is "Negative"

THEN

\[ \dot{\hat{x}}(t) = A_2 \dot{x}(t) + B_2 u(t) + L_2 (y(t) - \hat{y}(t)) \]
\[ \hat{y}(t) = C_2 \dot{x}(t) \]

Observer Rule 3:

IF \( z_1(t) \) is "Negative", \( z_2(t) \) is "Positive" and \( z_3(t) \) is "Negative"

THEN

\[ \dot{\hat{x}}(t) = A_3 \dot{x}(t) + B_3 u(t) + L_3 (y(t) - \hat{y}(t)) \]
\[ \hat{y}(t) = C_3 \dot{x}(t) \]

Observer Rule 4:

IF \( z_1(t) \) is "Positive", \( z_2(t) \) is "Positive" and \( z_3(t) \) is "Negative"

THEN

\[ \dot{\hat{x}}(t) = A_4 \dot{x}(t) + B_4 u(t) + L_4 (y(t) - \hat{y}(t)) \]
\[ \hat{y}(t) = C_4 \dot{x}(t) \]
Observer Rule 5:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Negative" and $z_3(t)$ is "Positive"

THEN

$$\dot{x}(t) = A_5\dot{x}(t) + B_5u(t) + L_5(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C_5\dot{x}(t)$$

Observer Rule 6:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Negative" and $z_3(t)$ is "Positive"

THEN

$$\dot{x}(t) = A_6\dot{x}(t) + B_6u(t) + L_6(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C_6\dot{x}(t)$$

Observer Rule 7:

IF $z_1(t)$ is "Negative", $z_2(t)$ is "Positive" and $z_3(t)$ is "Positive"

THEN

$$\dot{x}(t) = A_7\dot{x}(t) + B_7u(t) + L_7(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C_7\dot{x}(t)$$

Observer Rule 8:

IF $z_1(t)$ is "Positive", $z_2(t)$ is "Positive" and $z_3(t)$ is "Positive"

THEN

$$\dot{x}(t) = A_8\dot{x}(t) + B_8u(t) + L_8(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C_8\dot{x}(t)$$

For a better understanding, this can be represented by the following table, where "Positive" can be represented by "+", and "Negative" can be represented by "−".
Set $z_1(t)$ $z_2(t)$ $z_3(t)$ A matrix Observer gain
rule 1 $-$ $-$ $-$ $A_1$ $L_1$
rule 2 $+$ $-$ $-$ $A_2$ $L_2$
rule 3 $-$ $+$ $-$ $A_3$ $L_3$
rule 4 $+$ $+$ $-$ $A_4$ $L_4$
rule 5 $-$ $-$ $+$ $A_5$ $L_5$
rule 6 $+$ $-$ $+$ $A_6$ $L_6$
rule 7 $-$ $+$ $+$ $A_7$ $L_7$
rule 8 $+$ $+$ $+$ $A_8$ $L_8$

Table 3.2: Fuzzy model with fuzzy observer rule

Now in order to obtain the observer gain $L_i$, for a full-order state observers design following the LMIs condition [21]. It has a solution if and only if there exist a symmetric positive definite matrix $P_i$ and a matrix $W_i$ satisfying

$$A_i^T P + C^T W_i + (A_i^T P + C^T W_i)^T + 2\lambda P < 0 \quad (3.7)$$

$$\begin{bmatrix}
-r P_i & q P_i + A_i^T P_i + C^T W_i \\
(q P_i + A_i^T P_i + C^T W_i)^T & -r P_i
\end{bmatrix} < 0 \quad (3.8)$$

$$\begin{bmatrix}
(A_i^T P_i + C^T W_i + (A_i^T P + C^T W_i)^T) \sin\theta & (A_i^T P_i + C^T W_i - (A_i^T P + C^T W_i)^T) \cos\theta \\
-(A_i^T P_i + C^T W_i) + (A_i^T P + C^T W_i)^T) \cos\theta & (A_i^T P_i + C^T W_i + (A_i^T P + C^T W_i)^T) \sin\theta
\end{bmatrix} < 0 \quad (3.9)$$

In this case, the solution to our problem is given by

$$L_i = P_i^{-1} W_i \quad (3.10)$$

Similarly, the poles of the observer should be in the shadow area in Figure 3.1 [24] [25].

### 3.2 Obtaining the state feedback controller

To implement the observer using the methodology introduced in Subsection 3.1.1, the following LMIs parameter are considered: $r = 50$, $q = 0$, $\alpha = 0.5$ and $\theta = \pi/6$, applied to our Wind Turbine case study. The resulting closed loop poles are presented in Figure 3.2.
From that figure, we can see that all the poles are located in the shadow region presented in Figure 3.1.

### 3.2.1 Control structure of Wind Turbines

The considered control structure can be represented by the following diagram:

![Wind Turbine Control Feedback Loops Diagram](image)

Figure 3.3: Wind turbine control feedback loops

The designed control is tested in SIMULINK, leading to the results presented in Figure 3.4.
In this figure, we can see that the output torque.

Similarly, we have the output of the pitch angle,

Figure 3.4: Controlled torque

Figure 3.5: Controlled pitch angle
3.3 Obtaining the observer

To implement the observer using the methodology introduced in Subsection 3.1.2, the following LMI parameters are considered: $r = 500$, $q = 0$, $\alpha = 50$ and $\theta = \pi/3$, applied to our Wind turbine case study. The resulting observer poles are presented in Figure 3.6.

![Figure 3.6: Poles of the observer](image)

From the figure, we can see that all the poles are located in the shadow region that described in figure 3.1.

3.3.1 Observer based control

The observer based estimation scheme considered is presented in Figure 3.7.
This observer scheme is integrated with the state feedback controller designed previously and implemented in SIMULINK, leading to the following result.

In this figure, the estimated torque (in blue) match the reference (in red) very well, we can make a zoom in to see the details.
Also, we can see the estimated pitch angle in Figure 3.10.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{pitch_angle.png}
\caption{Pitch angle generated by the observer}
\end{figure}

\subsection{State feedback using observer}

Based on the model in Figure 3.7, we can use the state feedback controller for it.
Testing this observer with the controller in SIMULINK, we have the following result.

Figure 3.12: Controlled torque obtained by state feedback using the observer

Figure 3.13 shows the pitch angle
Figure 3.13: Controlled pitch angle obtained by state feedback using the observer
Chapter 4

Comparison with PI controller

4.1 T-S controller

Now we can compare the generated torque and pitch angle with the result we obtained from state feedback T-S controller.

In this figure, the torque generated by state feedback T-S controller (blue) is almost match the torque generated by PI controller (red).

We can make a zoom in of mode 1 part. We can see in the figure below.
Theoretically, in this part the two curves should be the same, because in mode 1, system does not have state feedback. We can see that there are small differences between two curves, a possible reason on this maybe is the error of the simulation between differential equation and the state-space model.

Then we can make zoom in on mode 2, we can see the figure below.
In this part, the difference becomes larger. The state feedback of PI (mode 2) starts to work. And the torque under the T-S controller (blue) has a little overshoot.

Additionally, we can see the pitch angle.

In Figure 4.4, we can see the pitch angle of the state feedback T-S controller (blue) is almost
match the pitch angle generated by PI controller (red).

Also we can make a zoom in of this result. We can see that at zone 2, there is no turning on the blade, the pitch angle is 0. So we can see the detail from 2600s.

![Graph showing output pitch angle generated by T-S controller and PI controller in time 2600s to 3000s.](image)

**Figure 4.5**: Output pitch angle generated by T-S controller and PI controller in time 2600s to 3000s

In Figure 4.5 we can see that there are small overshoot.

### 4.2 T-S observer based control

Similarly, we can also compare the result with T-S observer based control.
CHAPTER 4. COMPARISON WITH PI CONTROLLER

Figure 4.6: Output torque generated by T-S observer based state feedback T-S controller and PI controller

The result looks similar with the previous in Figure 4.1, we can also make a zoom in of each mode. Firstly, we can see the mode 1 part in the figure below.

Figure 4.7: Output torque generated by T-S observer based state feedback T-S controller and PI controller from time 0 to 400s
Comparing Figures 4.2 and 4.3 there is no significant improvement, the overshoot is more or less the same, also the settling time, but the curve becomes more smooth.

Also we can take a look for the pitch angle.
For pitch angle there is a significant improvement, we can see the detail from a zoom in.

![Figure 4.10: Output pitch angle generated by T-S observer based state feedback T-S controller and PI controller in time 2600s to 3000s](image)

We can see that the overshoot is smaller than the previous [4.3]
Chapter 5

Conclusions

5.1 Work Summery

In this thesis, a horizontal-axis wind turbine (HAWT) has modeled into a state-space representation and transformed into a Takagi-Sugeno (T-S) model structure. The T-S model exactly represents the nonlinear model as a weighted combination of linear models.

Then a state feedback control schemes for wind turbines were investigated based on a Takagi-Sugeno controller and Takagi-Sugeno observer. The controller and observer were obtained by using LMIs, where the constrains are based on Lyapunov stability theory and LMI region $S(\alpha, r, \theta)$ stabilization [21]. In this part, choosing the suitable parameter $(\alpha, r, \theta)$ is very important. They can directly influence the controller performance, $\alpha$ is the minimum speed of the response, $r$ is the maximum speed of the response, and $\theta$ is the overshoot. These parameter can not set as much as possible, otherwise it will obtain positive poles or the poles are out of the LMI region $S$.

By tested on T-S wind turbine model. The wind speed we are using include low speed and high speed, which means that it include Zone 2 and Zone 3 (See Figure 2.3 and 2.4). The performance is very well, with only the T-S controller, the outputs keep reaching the reference and no too much overshoot, then the observer based state feedback control were tested, the performance is similar like the previous, approximately same overshoot, same setting time, but more smooth, where the performance is similar with the PI controller in [14].

For a conclusion, we can say that Takagi-Sugeno approach is a good way for presenting the nonlinear system of wind turbine. The T-S controller can give a very good performance under a suitable LMI condition. The T-S observer estimate the states very perfect. For wind turbine
case study, T-S approach can be a powerful tool for the future research.

5.2 Future work

This T-S model can be improved, for decreasing the error.

The performance of the controller can be improved, and also it can apply by other control methodology on T-S model, for example sliding model control, $H_\infty$ control, MPC, etc.

For the simulation the 4.8MW HAWT by using SIMULINK, this T-S model can be embedded in the benchmark model [14], and replace the controller Mode 2 by T-S controller. Then see if there are better performance.

Additionally this work can be tested on The FAST (Fatigue, Aerodynamics, Structures, and Turbulence) Code, it should be more accurate for wind turbine case study.

Furthermore, this can be a starting point for FDI (Fault detection and isolation) and FTC (Fault Tolerant Control) concepts, because now the accidents on wind turbine are getting increase. The following figure shows the accidents up to May 2017.

![Figure 5.1: Wind turbine accidents in year, up to 31 of May 2017.](image)

Many cases can cause the wind turbine accident, blade failure, fire, structural failure, Ice throw, transport, environmental damage (including bird deaths) and other miscellaneous (Component or mechanical failure, lack of maintenance, electrical failure, Construction and construction support accidents, lightning strikes). In these accidents, poor quality control can cause a portion of structural failure.
### Table 5.1: Structural failure of wind turbine up to 31 May 2017

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Accidents</td>
<td>15</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>7</td>
<td>4</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Year</td>
<td>2007</td>
<td>2008</td>
<td>2009</td>
<td>2010</td>
<td>2011</td>
<td>2012</td>
<td>2013</td>
<td>2014</td>
</tr>
<tr>
<td>Number of Accidents</td>
<td>13</td>
<td>9</td>
<td>16</td>
<td>9</td>
<td>13</td>
<td>10</td>
<td>14</td>
<td>13</td>
</tr>
<tr>
<td>Year</td>
<td>2015</td>
<td>2016</td>
<td>2017</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Accidents</td>
<td>12</td>
<td>11</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For decrease this kind of accident, keeping the wind turbine works in a normal and stable status seems very important, especially FDI and FTC technique.
Appendix

MATLAB code for TS controller design

Notice that the Aerodynamics data is required, which it contains $\lambda$, $\beta$, $C_q$ and $C_p$ (See in Section 2.2).

```matlab
1 % TS model for controller design
2 clear all; clc; close all;
3
4 load AeroDynamics.mat
5 [ANGLE,LAMBDA] = meshgrid(Angle,Lambda);
6 ANGLE = ANGLE(:,:,11:end);
7 LAMBDA = LAMBDA(:,:,11:end);
8 Cq = Cq(:,:,11:end);
9 CqLAMBDA = Cq./ANGLE;
10
11 % wind turbine parameter
12 omega_n=11.11; xi=0.6; rho=1.225; R=57.5; J_r=55e6; B_dt=775.49; B_g=45.6;
13 B_r=7.11; N_g=95; K_dt=2.7e9; eta_dt=0.97; J_g=390; vwmax = 25; tau_g = 20e-3;
14
15 thetamin = rho*pi*R^3*vwmax^2*min(min(CqLAMBDA))/(6*J_r);
16 thetamax = rho*pi*R^3*vwmax^2*max(max(CqLAMBDA))/(6*J_r);
17
18 thetarange = [thetamin thetamin thetamin ;
19 thetamax thetamax thetamax]';
20
21 amatcaixa = pvec('box',thetarange);
22 caixavertex = polydec(amatcaixa);
23
24 nx = 10; ny = 6;
25 Avertex = zeros(nx,nx,size(caixavertex,2));
26 ATVvertex = zeros(nx,nx,size(caixavertex,2));
27 Cvertex = zeros(ny,nx,size(caixavertex,2));
28 CTvertex = zeros(nx,ny,size(caixavertex,2));
29
30 a11 = -(B_dt+B_r)/J_r; a12 = B_dt/(N_g*J_r); a13 = -K_dt/J_r;
31 a21 = eta_dt*B_dt/(N_g*J_g); a22 = -(eta_dt*B_dt/(N_g^2+3.9)+B_g/J_g); a23 = eta_dt*K_dt/(N_g*J_g);
```
a24 = -1/j_g; a32 = -1/N_g; a44 = -1/tau_g; b41 = 1/tau_g; a88 = -2*x*omega_n;
a65 = -omega_n^2; a66 = -2*x*omega_n; b62 = omega_n^2; a87 = -omega_n^2;
b83 = omega_n^2; a109 = -omega_n^2; a1010 = -2*x*omega_n; b104 = omega_n^2;

for k=1:size(caixavertex,2)
  Avertex(:,:,k) = [a11 a12 a13 0 caixavertex(1,k) 0 caixavertex(2,k) 0 caixavertex(3,k) 0 ;
  a21 a22 a23 a24 0 0 0 0 0 0 ;
  0 0 0 a44 0 0 0 0 0 0 ;
  0 0 0 0 1 0 0 0 0 0 ;
  0 0 0 a65 a66 0 0 0 0 0 ;
  0 0 0 0 0 1 0 0 0 0 ;
  0 0 0 0 0 a87 a88 0 0 0 0 0 0 0 1 ;
  0 0 0 0 0 0 a109 a1010]

  ATvertex(:,:,k)= Avertex(:,:,k)';
  Bvertex(:,:,k) = [0 0 0 b41 0 0 0 0 0 0 ;
  0 0 0 0 0 b62 0 0 0 0 ;
  0 0 0 0 0 0 b83 0 0 0 0 ;
  0 0 0 0 0 0 0 b104]';
  BTvertex(:,:,k) = Bvertex(:,:,k)';
  Cvertex(:,:,k) = [1 0 0 0 0 0 0 0 0 0 ;
  0 1 0 0 0 0 0 0 0 0 ;
  0 0 1 0 0 0 0 0 0 0 ;
  0 0 0 0 1 0 0 0 0 0 ;
  0 0 0 0 0 0 1 0 0 0 ;
  0 0 0 0 0 0 0 0 1 0];
  CTvertex(:,:,k) = Cvertex(:,:,k)';
end

vertices = size(Avertex,3); % 8

\% DESIGN OF THE OBSERVER

rL = 50; % r
qL = 0; % q
lambdaL = 0.5; % alpha
thetaL = pi/6; % theta
Kvertex = zeros(4,nx,vertices); % (4,10,8)
PolesK = zeros(nx,vertices); % (10,8)
XL = sdpvar(nx); % P
W = cell(vertices,1); % W
for k=1:vertices
  W{k} = sdpvar(nx);
end
clear F
tic
F = [XL>0];

% LMI condition D-stability
for ii = 1:vertices
    F = [F, Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii)+2*lambdaL*XL <0];
    F = [F, [-rL*XL qL*XL Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}; qL*XL+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii) -rL*XL]<0];
    F = [F, [sin(thetaL)*(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii))... cos(thetaL)*(-Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii))... cos(thetaL)*(-Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii))... sin(thetaL)*(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii))... ]<0];
end
sdpoptions = sdpsettings('showprogress',1,'solver','sedumi','sedumi.eps',1e-10,'sedumi.maxiter',300);
diagnosticsL = solvesdp(F,[],sdpoptions);
temp = double(XL); clear XL;
XL = double(temp);
for k=1:vertices
    W{k} = double(W{k});
    Kvertex(:,:,k) = W{k}*inv(XL);
    PolesK(:,k) = eig(Avertex(:,:,k)+Bvertex(:,:,k)*Kvertex(:,:,k));
end
toc

display('The LMIs for designing the state feedback controller are:')
if diagnosticsL.problem == 0
disp('Feasible')
elseif diagnosticsL.problem == 1
disp('Infeasible')
else
disp('Something else happened')
end
eigtest = zeros(3*vertices+1,1);
k = 1;
eigtest(k) = max(eig(XL));
for ii = 1:vertices
    k = k+1; eigtest(k) = max(eig(Avertex(:,:,ii)*XL+Bvertex(:,:,ii)*W{ii}+XL*ATvertex(:,:,ii)+W{ii}'*BTvertex(:,:,ii)+2*lambdaL*XL <0]);
end
toc
BTvertex(:,i:i+2+lambdaL*XL));

k = k+1; eigtest(k) = max(eig([-rL*XL Avertex(:,i:i)+XL*Bvertex(:,i:i)+W{ii} ; ...
XL*ATvertex(:,i:i)+W{ii}]*BTvertex(:,i:i) -rL*XL]));

k = k+1; eigtest(k) = max(eig([sin(thetaL)*(Avertex(:,i:i)+XL*Bvertex(:,i:i)+W{ii})+cos(thetaL)*(Avertex(:,i:i)+XL*Bvertex(:,i:i)+W{ii})+cos(thetaL)*(Avertex(:,i:i)+XL*Bvertex(:,i:i)+W{ii})+sin(thetaL)*(Avertex(:,i:i)+XL*Bvertex(:,i:i)+W{ii})]*BTvertex(:,i:i) ...]

end

%%
clear W

figure(1);
plot(real(PolesK),imag(PolesK),'.b'); title('Pole clustering of the controller');
hold on;
plot([-lambdaL -lambdaL],[-2*rL 2*rL],'.-r',[-rL*cos(linspace(0,2*pi,200)),rL*sin(linspace(0,2*pi,200))],'.-r'
,qL-qL+qLrqL+qLtthetaL+qL+qLtthetaL+qL+qLtthetaL, '.-r');
xlabel('Real(s)'); ylabel('Imag(s)');
figure(2);
if(eigtest(1)>0)
plot(eigtest(2:end));
else
plot(eigtest);
end

title('Eigenvalues test for the design of the controller');
save datacontroller.mat thetarange Kvertex PolesK Avertex Bvertex

MATLAB code for TS observer design

% TS model for observer design
clear all; clc; close all;
load AeroDynamics.mat
[ANGLE,LAMBDA] = meshgrid(Angle,Lambda);
ANGLE = ANGLE(:,11:end);
LAMBDA = LAMBDA(:,11:end);
Cq = Cq(:,11:end);
qLAMDA = Cq./ANGLE;
% wind turbine parameter
omega_n=11.1; xi=0.6; rho=1.225; R=57.5; J_r=55e6; B_dt=775.49; B_g=45.6;
B_r=7.11; N_g=95; K_dt=2.7e9; eta_dt=0.97; J_g=390; v_max = 25; tau_g = 20e3;
\[
\begin{align*}
\text{thetamin} &= \rho \pi R^3 v_{\text{max}}^2 \min(\min(CqLAMBDA))/(6J_r); \\
\text{thetamax} &= \rho \pi R^3 v_{\text{max}}^2 \max(\max(CqLAMBDA))/(6J_r); \\
\text{thetarange} &= [\text{thetamin} \ \text{thetamin} \ \text{thetamin} \ ; \\
&\quad \text{thetamax} \ \text{thetamax} \ \text{thetamax}]; \\
\text{amatcaixa} &= \text{pvec('box',thetarange)}; \\
\text{caixavertex} &= \text{polydec(amatcaixa)}; \\
nx &= 10; \ ny = 6; \\
\text{Avertex} &= \text{zeros(nx,nx,size(caixavertex,2))}; \\
\text{ATvertex} &= \text{zeros(nx,nx,size(caixavertex,2))}; \\
\text{Cvertex} &= \text{zeros(ny,nx,size(caixavertex,2))}; \\
\text{CTvertex} &= \text{zeros(nx,ny,size(caixavertex,2))}; \\
\text{obsv\_UNFAULTY} &= \text{zeros(size(caixavertex,2),1)}; \\
\text{obsv\_LOSS1} &= \text{zeros(size(caixavertex,2),1)}; \\
\text{obsv\_LOSS2} &= \text{zeros(size(caixavertex,2),1)}; \\
\text{obsv\_LOSS3} &= \text{zeros(size(caixavertex,2),1)}; \\
\text{obsv\_LOSS4} &= \text{zeros(size(caixavertex,2),1)}; \\
\text{obsv\_LOSS5} &= \text{zeros(size(caixavertex,2),1)}; \\
\text{obsv\_LOSS6} &= \text{zeros(size(caixavertex,2),1)}; \\
\text{a11} &= -(B_{\text{dt}}+B_r)/J_r; \ a12 = B_{\text{dt}}/(N_g*J_r); \ a13 = -K_{\text{dt}}/J_r; \\
a21 &= \text{eta}_{\text{dt}}B_{\text{dt}}/(N_g*J_g); \ a22 = -(\text{eta}_{\text{dt}}B_{\text{dt}}/(N_g^2*J_g)+B_g/J_g); \ a23 = \text{eta}_{\text{dt}}K_{\text{dt}}/(N_g*J_g); \\
a24 &= -1/J_g; \ a32 = -1/N_g; \ a44 = -1/\tau_g; \ b41 = 1/\tau_g; \ a88 = -2xi\omega_n; \\
a65 &= -\omega_n^2; \ a66 = -2xi\omega_n; \ b62 = \omega_n^2; \ a87 = -\omega_n^2; \\
b83 &= \omega_n^2; \ a109 = -\omega_n^2; \ a1010 = -2xi\omega_n; \ b104 = \omega_n^2; \\
\text{for } k=1:size(caixavertex,2) \\
\text{Avertex(i,:,k)} &= [a11 \ a12 \ a13 \ 0 \ \text{caixavertex}(1,k) \ 0 \ \text{caixavertex}(2,k) \ 0 \ \text{caixavertex}(3,k) \ 0; \\
&\quad a21 \ a22 \ a23 \ a24 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \\
&\quad 1 \ a32 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \\
&\quad 0 \ 0 \ 0 \ a44 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \\
&\quad 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0; \\
&\quad 0 \ 0 \ 0 \ 0 \ a66 \ a66 \ 0 \ 0 \ 0 \ 0; \\
&\quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0; \\
&\quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ a87 \ a88 \ 0 \ 0; \\
&\quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1; \\
&\quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ a109 \ a109 \ a1010]; \\
\text{ATvertex(i,:,k)} = \text{Avertex(i,:,k)}; \\
\text{Bvertex(i,:,k)} &= [0 \ 0 \ 0 \ b41 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \\
&\quad 0 \ 0 \ 0 \ 0 \ b62 \ 0 \ 0 \ 0 \ 0 \ 0; \\
&\quad 0 \ 0 \ 0 \ 0 \ 0 \ b83 \ 0 \ 0 \ 0 \ 0; \\
&\quad 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ b104]; \\
\text{Cvertex(i,:,k)} &= [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \\
&\quad 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0; \\
&\quad 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0];
\end{align*}
\]
62 \[
0 0 0 0 1 0 0 0 0 ;
0 0 0 0 0 1 0 0 0 ;
0 0 0 0 0 0 0 1 0 ;
\]
CTvertex(:,:,k)= Cvertex(:,:,k)';
63
64
65 obsv.UNFAULTY(k) = rank(obsv(Avertex(:,:,k),Cvertex(:,:,k)));
66 obsv.LOSS1(k) = rank(obsv(Avertex(:,:,k),Cvertex([2 3 4 5 6],:,:,k)));
67 obsv.LOSS2(k) = rank(obsv(Avertex(:,:,k),Cvertex([1 3 4 5 6],:,:,k)));
68 obsv.LOSS3(k) = rank(obsv(Avertex(:,:,k),Cvertex([1 2 4 5 6],:,:,k)));
69 obsv.LOSS4(k) = rank(obsv(Avertex(:,:,k),Cvertex([1 2 3 5 6],:,:,k)));
70 obsv.LOSS5(k) = rank(obsv(Avertex(:,:,k),Cvertex([1 2 3 4 6],:,:,k)));
71 obsv.LOSS6(k) = rank(obsv(Avertex(:,:,k),Cvertex([1 2 3 4 5],:,:,k)));
72
73 end
74
75 A = [a11 a12 a13 0 ; a21 a22 a23 a24 ; 1 a32 0 0 ; 0 0 0 a44];
76 C = [1 0 0 0 ; 0 1 0 0 ; 0 0 0 1];
77 obsv.reduced1 = rank(obsv(A,C));
78 obsv.reduced2 = rank(obsv(A,C));
79 vertices = size(Avertex,3);
cos(\theta_L) \cdot (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii} - (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii})') ; \ldots
\cos(\theta_L) \cdot (-\{AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii} + (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii})\}) \ldots
\sin(\theta_L) \cdot (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii} + (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii})') \end{Bmatrix} < 0;\]

end

sdoptions = sdpsettings('showprogress',1,'solver','sedumi','sedumi.eps',1e-10,'sedumi.maxiter',300);
diagnosticsL = solvesdp(F,[],sdoptions);
temp = double(XL); clear XL;
XL = double(temp);
for k=1:vertices
    W{k} = double(W{k});
    Lvertex(:,:,k) = (W{k}/XL)';
    PolesL(:,:,k) = eig(Avertex(:,:,k)+Lvertex(:,:,k)*Cvertex(:,:,k));
end

toc

display('The LMI for designing the state observer are:')
if diagnosticsL.problem == 0
    disp('Feasible')
elseif diagnosticsL.problem == 1
    disp('Infeasible')
else
    disp('Something else happened')
end

eigtest = zeros(3*vertices+1,1);
k = 1;
eigtest(k) = max(eig(XL));
for ii = 1:vertices
    k = k+1; eigtest(k) = max(eig(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii}+(ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})'+2*lambdaL*XL));
    k = k+1; eigtest(k) = max(eig([-rL*XL qL*XL+ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii}; ...](qL*XL+ATvertex(:,:,ii)*XL+CTvertex(:,:,ii)*W{ii})' -rL*XL]));
    k = k+1; eigtest(k) = max(eig([\sin(\theta_L) \cdot (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii} + (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii})') ... \cos(\theta_L) \cdot (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii} - (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii})') ; \ldots
\cos(\theta_L) \cdot (-\{AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii} + (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii})\}) \ldots
\sin(\theta_L) \cdot (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii} + (AT_{\text{vertex}}(:,:,ii) \cdot XL + CT_{\text{vertex}}(:,:,ii) \cdot W{ii})')]);
end

figure(1);
The Matrix A of Wind Turbine T-S model

\[
A_1 = \begin{bmatrix}
-1.4229 \times 10^{-5} & 1.4842 \times 10^{-7} & -49.0909 & 0 & -0.0304 & 0 & -0.0304 & 0 & -0.0304 & 0 \\
0.0203 & -0.1171 & 7.0688 \times 10^4 & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
-1.4229 \times 10^{-5} & 1.4842 \times 10^{-7} & -49.0909 & 0 & 0.0873 & 0 & -0.0304 & 0 & -0.0304 & 0 \\
0.0203 & -0.1171 & 7.0688 \times 10^4 & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
$A_3 = \begin{bmatrix}
-1.4229 \times 10^{-5} & 1.4842 \times 10^{-7} & -49.0909 & 0 & -0.0304 & 0 & 0.0873 & 0 & -0.0304 & 0 \\
0.0203 & -0.1171 & 7.0688 \times 10^4 & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-123.4321 & -13.332 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$

$A_4 = \begin{bmatrix}
-1.4229 \times 10^{-5} & 1.4842 \times 10^{-7} & -49.0909 & 0 & 0.0873 & 0 & 0.0873 & 0 & -0.0304 & 0 \\
0.0203 & -0.1171 & 7.0688 \times 10^4 & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-123.4321 & -13.332 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$

$A_5 = \begin{bmatrix}
-1.4229 \times 10^{-5} & 1.4842 \times 10^{-7} & -49.0909 & 0 & -0.0304 & 0 & -0.0304 & 0 & 0.0873 & 0 \\
0.0203 & -0.1171 & 7.0688 \times 10^4 & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-123.4321 & -13.332 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$
\[
A_6 = \begin{bmatrix}
-1.4229 \times 10^{-5} & 1.4842 \times 10^{-7} & -49.0909 & 0 & 0.0873 & 0 & -0.0304 & 0 & 0.0873 & 0 \\
0.0203 & -0.1171 & 7.0688 \times 10^4 & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 \\
\end{bmatrix}
\]

\[
A_7 = \begin{bmatrix}
-1.4229 \times 10^{-5} & 1.4842 \times 10^{-7} & -49.0909 & 0 & -0.0304 & 0 & 0.0873 & 0 & 0.0873 & 0 \\
0.0203 & -0.1171 & 7.0688 \times 10^4 & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 \\
\end{bmatrix}
\]

\[
A_8 = \begin{bmatrix}
-1.4229 \times 10^{-5} & 1.4842 \times 10^{-7} & -49.0909 & 0 & 0.0873 & 0 & 0.0873 & 0 & 0.0873 & 0 \\
0.0203 & -0.1171 & 7.0688 \times 10^4 & -0.0026 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -123.4321 & -13.332 \\
\end{bmatrix}
\]
CHAPTER 5. CONCLUSIONS

The Matrix $B$ of Wind Turbine T-S model

$$B = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
50 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 123.4321 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 123.4321 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 123.4321 \\
\end{bmatrix}$$

The Controller Gain of Wind Turbine T-S model

$$K_1 = \begin{bmatrix}
-6.6081 \times 10^5 & -235.4576 & -235.5610 & -235.5351 \\
7.5821 \times 10^3 & 2.5520 & 2.5532 & 2.5529 \\
3.8556 \times 10^7 & -878.9696 & -877.3409 & -877.2067 \\
-0.5291 & -1.4289 \times 10^{-5} & -1.4377 \times 10^{-5} & -1.4365 \times 10^{-5} \\
152.3250 & -9.1843 & -0.1076 & -0.1081 \\
-14.8008 & -0.4547 & 6.7634 \times 10^{-4} & 6.7468 \times 10^{-4} \\
152.2279 & -0.1079 & -9.1843 & -0.1080 \\
-14.8008 & 6.7544 \times 10^{-4} & -0.4547 & 6.7469 \times 10^{-4} \\
152.2559 & -0.1080 & -0.1081 & -9.1843 \\
-14.7992 & 6.7379 \times 10^{-4} & 6.7472 \times 10^{-4} & -0.4547 \\
\end{bmatrix}^T$$
\[
K_2 = \begin{bmatrix}
-6.4775 \times 10^5 & 301.8732 & -256.2100 & -256.1775 \\
7.4389 \times 10^3 & -3.6483 & 2.7718 & 2.7714 \\
3.8140 \times 10^7 & 4.5556 \times 10^3 & -1.0690 \times 10^3 & -1.0690 \times 10^3 \\
-0.5117 & -3.5147 \times 10^{-5} & -1.2809 \times 10^{-5} & -1.2787 \times 10^{-5} \\
-3.0018 \times 10^3 & -9.5430 & -0.6342 & -0.6342 \\
-18.7857 & -0.4638 & -0.0128 & -0.0128 \\
204.1472 & 0.0521 & -9.2606 & -0.1845 \\
-13.7177 & -0.0016 & -0.4553 & 3.1784 \times 10^{-5} \\
204.1747 & 0.0521 & -0.1844 & -9.2607 \\
-13.7181 & -0.0016 & 3.3548 \times 10^{-5} & -0.4553
\end{bmatrix}^T
\]

\[
K_3 = \begin{bmatrix}
-6.4775 \times 10^5 & -256.1874 & 301.99112 & -256.2039 \\
7.4390 \times 10^3 & -2.7715 & -3.6496 & 2.7717 \\
3.8140 \times 10^7 & -1.0698 \times 10^3 & 4.5540 \times 10^3 & -1.0695 \times 10^3 \\
-0.5117 & -1.2776 \times 10^{-5} & -3.5065 \times 10^{-5} & -1.2784 \times 10^{-5} \\
204.1122 & -9.2606 & 0.0525 & -0.1884 \\
-13.7188 & -0.4553 & -0.0016 & 3.3218 \times 10^{-5} \\
-3.0018 \times 10^3 & -0.6341 & -9.5431 & -0.6342 \\
-18.7858 & -0.0128 & -0.4638 & -0.0128 \\
204.1072 & -0.1844 & 0.0524 & -9.2607 \\
-13.7179 & 3.1949 \times 10^{-5} & -0.0016 & -0.4553
\end{bmatrix}^T
\]

\[
K_4 = \begin{bmatrix}
-6.5441 \times 10^5 & 289.2313 & 289.1852 & -287.8942 \\
7.5187 \times 10^3 & -3.4788 & -3.4783 & 3.1342 \\
3.7992 \times 10^7 & 3.8168 \times 10^3 & 3.8172 \times 10^3 & -1.2454 \times 10^3 \\
-0.5088 & -1.6466 \times 10^{-5} & -1.6499 \times 10^{-5} & -1.7288 \times 10^{-5} \\
-2.9996 \times 10^3 & -9.3801 & -0.3039 & -0.6121 \\
-19.3131 & -0.4603 & -0.0050 & -0.0127 \\
-2.9996 \times 10^3 & -0.3039 & -9.3803 & -0.6120 \\
-19.3130 & -0.0050 & -0.4603 & -0.0127 \\
202.1078 & -0.0012 & -0.0012 & -9.3055 \\
-13.4734 & -0.0030 & -0.0030 & -0.4554
\end{bmatrix}^T
\]
$K_5 = \begin{bmatrix}
-6.4779 \times 10^5 & -256.2578 & -256.3059 & 302.0925 \\
7.4394 \times 10^3 & 2.7724 & 2.7730 & -3.6509 \\
3.8141 \times 10^7 & -1.0689 \times 10^3 & -1.0687 \times 10^3 & 4.5528 \times 10^3 \\
-1.5117 & -1.2831 \times 10^{-5} & -1.2861 \times 10^{-5} & -3.4975 \times 10^{-5} \\
204.0164 & -9.2607 & -0.1843 & 0.0524 \\
-13.7205 & -0.4553 & 3.7125 \times 10^{-5} & -0.0016 \\
203.9850 & -0.1843 & -9.2606 & 0.0524 \\
-13.7193 & 3.4116 \times 10^{-5} & -0.4553 & -0.0016 \\
-3.0021^3 & -0.6342 & -0.6343 & -9.5429 \\
-18.7911 & -0.0128 & -0.0128 & -0.4638
\end{bmatrix}^T$

$K_6 = \begin{bmatrix}
-6.5431 \times 10^5 & -289.1592 & -287.8585 & 289.1112 \\
7.5176 \times 10^3 & -3.4779 & 3.1338 & -3.4774 \\
3.7991 \times 10^7 & 3.8180 \times 10^3 & -1.2461 \times 10^3 & 3.8189 \times 10^3 \\
-0.5087 & -1.6545 \times 10^{-5} & -1.7249 \times 10^{-5} & -1.6584 \times 10^{-5} \\
2.9994 \times 10^3 & -9.3803 & -0.6119 & -0.3043 \\
-19.3082 & -0.4603 & -0.0127 & -0.0050 \\
202.2692 & -0.0012 & -9.3054 & -0.0013 \\
-13.4725 & -0.0030 & -0.4554 & -0.0030 \\
-2.9994^3 & -0.3042 & -0.6120 & -9.3806 \\
-19.3067 & -0.0050 & -0.0127 & -0.4603
\end{bmatrix}^T$

$K_7 = \begin{bmatrix}
-6.5417 \times 10^5 & -287.7359 & 288.9450 & 288.9411 \\
7.5159 \times 10^3 & 3.1324 & -3.4755 & -3.4754 \\
3.7991 \times 10^7 & -1.2472 \times 10^3 & 3.8202 \times 10^3 & 3.8206 \times 10^3 \\
-0.5086 & -1.1715 \times 10^{-5} & -1.6681 \times 10^{-5} & -1.6689 \times 10^{-5} \\
202.6746 & -9.3050 & -0.0018 & -0.0020 \\
-13.4656 & -0.4554 & -0.0030 & -0.0030 \\
-2.9997 \times 10^3 & -0.6120 & -9.3800 & -0.3038 \\
-19.3152 & -0.0127 & -0.4603 & -0.0050 \\
-2.9997^3 & -0.6121 & -0.3037 & -9.3800 \\
-19.3137 & -0.0127 & -0.0050 & -0.4603
\end{bmatrix}^T$
The Observer Gain of Wind Turbine T-S model

\[
K_8 = \begin{bmatrix}
-6.6936 \times 10^5 & 283.3040 & 283.2523 & 283.2774 \\
7.6992 \times 10^3 & -3.3850 & -3.3844 & -3.3847 \\
3.7898 \times 10^7 & 3.0947 \times 10^3 & 3.0946 \times 10^3 & 3.0947 \times 10^3 \\
-0.5117 & 1.6220 \times 10^{-6} & 1.6020 \times 10^{-6} & 1.6185 \times 10^{-6}
\end{bmatrix}^T
\]

\[
L_1 = \begin{bmatrix}
-1.6351 \times 10^3 & -547.8591 & -476.0504 & 104.0550 & -310.4509 & 66.7191 \\
-8.6364 & -2.0073 & -1.7948 & 0.7026 & -1.6 & 0.4723 \\
0.3972 & 0.0098 & -0.5319 & 0.0950 & 0.0702 & -0.2058 \\
4.4347 & -0.6755 & -46.4516 & -177.8392 & -3.5263 & -0.0018 \\
173.5854 & -53.9899 & -2.9764 \times 10^3 & -6.6615 \times 10^3 & -182.9723 & -29.7156 \\
-6.5259 & 0.4094 & 12.6860 & -1.8113 & -168.4083 & -0.4079 \\
-148.0912 & 48.1471 & 83.9713 & -198.4434 & -6.4346 \times 10^3 & 68.5189 \\
5.8482 & 0.9137 & 1.7098 & -0.1267 & 2.3324 & -172.2188 \\
258.1297 & 36.5226 & 83.2163 & -3.9799 & 144.8436 & -6.86 \times 10^3
\end{bmatrix}
\]

\[
L_2 = \begin{bmatrix}
-1.6174 \times 10^3 & -548.5564 & -342.1237 & 291.6040 & -260.0630 & 42.1194 \\
-8.5620 & -2.0151 & -1.3090 & 1.4244 & -1.4161 & 0.3755 \\
0.3836 & 0.0144 & -0.4801 & 0.0834 & 0.0731 & -0.2005 \\
-2.7396 & -1.2344 & -3.7865 & -182.0787 & 0.0469 & 0.3345 \\
-282.5246 & -75.4089 & -257.3216 & -6.8845 \times 10^3 & 39.4311 & -15.9691 \\
-0.7699 & 0.1088 & -16.2923 & 5.4132 & -172.9995 & -1.3064 \\
233.0251 & 39.3652 & -1.0074 \times 10^3 & 211.0560 & -6.6884 \times 10^3 & 18.8557 \\
5.3453 & 0.7321 & -2.9040 & 1.3258 & 0.4436 & -168.9640 \\
\end{bmatrix}
\]
\[ L_3 = \begin{bmatrix}
-111.1250 & -8.8792 & -8.3852 & -0.0866 & 0.2484 & 8.0118 \\
-1.7058 \times 10^3 & -550.5897 & -455.7737 & -72.7657 & 114.5116 & 242.2445 \\
-8.8884 & -2.0611 & -1.7441 & 0.0235 & 0.0269 & 1.1479 \\
0.3936 & 0.0148 & -0.4821 & 0.0851 & 0.0596 & -0.2124 \\
3.4701 & 0.8554 & -16.8637 & -185.9632 & 0.0267 & 0.3237 \\
137.1636 & 16.0830 & -1.0974 \times 10^3 & -7.1771 \times 10^3 & 41.7604 & 13.1356 \\
-2.1533 & -2.3220 & -1.4028 & 0.6235 & -169.3548 & -2.5547 \\
91.3365 & -61.7626 & -54.7692 & -65.0241 & -6.5388 \times 10^3 & -72.9122 \\
4.8400 & -0.4825 & 3.2434 & -1.0478 & 1.1018 & -170.4604 \\
164.0856 & -26.2823 & 183.05044 & -63.0097 & 111.1238 & -6.8330 \times 10^3
\end{bmatrix} \]

\[ L_4 = \begin{bmatrix}
-105.4526 & -7.9962 & -0.2105 & -0.6779 & -0.4861 & 4.6861 \\
-1.3389 \times 10^3 & -553.2440 & -37.3995 & -125.0797 & 74.1805 & 75.8004 \\
-7.5244 & -2.0432 & -0.1249 & -0.1555 & -0.1283 & 0.5032 \\
0.3867 & 0.0110 & -0.5257 & 0.1028 & 0.0603 & -0.2070 \\
-0.3065 & -0.0667 & 4.6760 & -178.4511 & 6.4975 & -1.7869 \\
-79.6761 & -34.8378 & 262.1044 & -6.6427 \times 10^3 & 432.0387 & -145.0900 \\
-83.4520 & -91.3365 & 1.3566 \times 10^3 & 106.8009 & -6.4836 \times 10^3 & 173.4844 \\
4.7563 & 1.0182 & -7.6826 & -0.7090 & 0.8058 & -170.5892 \\
218.6711 & 40.1636 & -503.3398 & -46.2311 & 61.2126 & -6.7996 \times 10^3
\end{bmatrix} \]

\[ L_5 = \begin{bmatrix}
-104.7293 & -6.4917 & 24.0376 & 7.0548 & 2.4146 & 3.9324 \\
-1.3555 \times 10^3 & -543.2514 & 1.2564 \times 10^3 & 302.6711 & 223.6370 & 41.8943 \\
-7.5587 & -1.9626 & 4.8024 & 1.4568 & 0.4466 & 0.3750 \\
0.3870 & 0.0353 & -0.4843 & 0.0799 & 0.0468 & -0.2117 \\
1.6193 & -2.1645 & 3.9241 & -180.3373 & -0.6355 & -0.2503 \\
40.0312 & -120.0279 & 304.7652 & -6.8308 \times 10^3 & 23.5521 & -52.1217 \\
-2.6792 & -1.8969 & 3.9508 & 2.5382 & -169.4048 & -0.4300 \\
57.2773 & -48.7051 & 93.2176 & 36.1387 & -6.535 \times 10^3 & 68.0529 \\
3.2066 & 0.4190 & 14.3290 & -0.4677 & -0.6618 & -171.8501 \\
100.0895 & 12.1342 & 1.0014 \times 10^3 & -33.9605 & -8.1962 & -6.9250 \times 10^3
\end{bmatrix} \]
\[
\begin{bmatrix}
-1.7188 \times 10^3 & -549.0893 & -332.3252 & -275.2274 & -161.1051 & 227.0177 \\
-8.9500 & -2.0231 & -1.2835 & -0.7522 & -1.0234 & 1.0784 \\
0.3991 & 0.0113 & -0.4464 & 0.1040 & 0.0612 & -0.2174 \\
3.9544 & 1.5892 & -16.9282 & -183.5102 & -1.8489 & 1.0338 \\
179.6454 & 38.8406 & -1.0875 \times 10^3 & -7.0217 \times 10^3 & -79.1611 & 31.6258 \\
-6.0559 & -0.2800 & -5.5666 & -1.4296 & -167.0782 & 1.1970 \\
-114.8826 & 21.9083 & -341.4645 & -133.5394 & -6.3844 \times 10^3 & 144.0781 \\
4.3025 & 0.3519 & 14.3059 & 1.6999 & 0.6438 & -172.1566 \\
157.9751 & 12.4279 & 915.3956 & 70.4329 & 61.2577 & -6.9344 \times 10^3 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-1.5263 \times 10^3 & -546.7569 & -534.3626 & 327.4700 & 45.2978 & 253.0217 \\
-8.2192 & -2.0028 & -2.0338 & 1.5653 & -0.2470 & 1.1782 \\
0.3890 & 0.0088 & -0.4474 & 0.0768 & 0.0656 & -0.2123 \\
-0.1912 & -1.3068 & -3.7625 & -183.4477 & 4.2944 & -0.0096 \\
-99.7616 & -80.7704 & -265.9834 & -6.9850 \times 10^3 & 301.3019 & -18.5448 \\
-3.7186 & -0.1513 & 16.9524 & 4.2745 & -166.8024 & -0.8920 \\
-11.7978 & 34.1723 & 1.1577 \times 10^3 \times 4 \times 0.7052 & -6.3681 \times 10^3 & 14.3286 \\
4.7414 & 0.3204 & 10.3315 & -0.3334 & 1.1943 & -171.7243 \\
200.1382 & 10.6690 & 651.1185 & 1.0255 & 106.8290 & -6.8470 \times 10^3 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-1.5833 \times 10^3 & -546.7527 & -209.0207 & -74.8028 & -362.9247 & 18.1625 \\
-8.4310 & -2.0021 & -0.7810 & 0.0192 & -1.8085 & 0.2815 \\
0.3930 & 0.0090 & -0.4829 & 0.0879 & 0.0743 & -0.2084 \\
-0.2199 & 0.3541 & 4.3510 & -186.0633 & -0.7371 & -0.3030 \\
-2.2270 & 0.7792 & 12.3329 & 0.5793 & -168.2786 & 0.1182 \\
122.1867 & 69.0153 & 820.7416 & -35.2306 & -6.3964 \times 10^3 & 98.3385 \\
5.1005 & 1.0760 & 0.9720 & 0.2994 & 1.8858 & -171.3168 \\
226.8578 & 41.5065 & 49.0076 & 2.6549 & 107.9132 & -6.8893 \times 10^3 \\
\end{bmatrix}
\]
Bibliography


[29] Urs Giger, Patrick Kühne and Horst Schulte. *Fault Tolerant and Optimal Control of Wind Turbines with Distributed High-Speed Generators*. 2017 by the authors; licensee MDPI, Basel, Switzerland.