Vaunat, J. & Casini, F. (2017). *Géotechnique* **67**, No. 7, 631–636 [http://dx.doi.org/10.1680/jgeot.15.T.016]

TECHNICAL NOTE

A procedure for the direct determination of Bishop's χ parameter from changes in pore size distribution

J. VAUNAT* and F. CASINI†

Most of the recent works relating to the concept of effective stress in unsaturated soils focus on the proposal by Bishop, and, more particularly, on the search for suitable relationships between Bishop's χ parameter and the main controlling variables. These relationships are generally formulated by theoretical derivations and back-analyses of the dependency of mechanical parameters on hydraulic variables such as suction or saturation. In this note, a new procedure is proposed to evaluate directly, and without any a priori assumptions, values for Bishop's χ parameter. In the first part, a general derivation based on the definition of work conjugated variables allows the χ parameter to be defined as the ratio of the change of water volume over the change in pore volume during a process at constant suction. This definition is further exploited to evaluate Bishop's parameter from the changes suffered by material pore size distribution during loading. The method is applied to data obtained by mercury intrusion porosimetry tests on low-plasticity silt (Jossigny silt), low-plasticity sandy clay (lean clay) and highly plastic clay (Febex clay). Values obtained for these materials show that the χ parameter is close to the effective degree of saturation rather than the total degree of saturation.

KEYWORDS: clays; fabric/structure of soils; partial saturation; silts; suction

INTRODUCTION

It is now well established that the behaviour of unsaturated soils should be described using two stress-like variables defined as a combination of total stress, $\boldsymbol{\sigma}_{ij}$, pore water pressure, p_w, and pore air pressure, p_a (Fredlund & Morgenstern, 1977; Houlsby, 1997; Tarantino *et al.*, 2000; Pereira *et al.*, 2005; Coussy *et al.*, 2010). Among all the possible combinations, the pair (suction $s = p_a - p_w$, net stresses $\boldsymbol{\sigma}''_{ij} = \boldsymbol{\sigma}_{ij} - \delta_{ij} p_a$ where δ_{ij} is the Kronecker symbol) have historically played a central role because of their experimental convenience, while the Bishop proposal $\boldsymbol{\sigma}''_{ij} = \boldsymbol{\sigma}_{ij} - \delta_{ij}[\chi p_w - (1-\chi) p_a]$ took relevance in constitutive modelling, because it provides a natural and smooth extension of Terzaghi's effective stress. The value of the χ parameter has been essentially explored by looking at relationships with volumetric variables measuring the amount of water in the soil: the degree of saturation (Jommi, 2000; Loret & Khalili, 2000; Khalili *et al.*, 2004; Pereira *et al.*, 2005), the Lagrangian degree of saturation (Coussy *et al.*, 2010) and the degree of saturation of the macrostructure (Alonso *et al.*, 2010). As pointed out by Coussy *et al.* (2010), all of these concepts are theoretically justified and differ uniquely by the assumption considered for the relationship between changes in porosity, water content and air content under straining.

The aim of this note is to put forward an objective procedure that relaxes the latter assumption and allows the χ parameter to be assessed directly from the $\delta e - \delta e_w$ relationship obtained from the change in pore size distribution (PSD) measured in mercury intrusion porosimetry (MIP) experiments. The method is supported by experimental results obtained for three compacted materials: a low-plasticity sandy clay, a low-plasticity silt and a highly active clay.

GENERAL DERIVATION OF BISHOP'S χ PARAMETER

According to Gens (1995), a combination of total stresses, water and air pressures is an effective stress if any changes in these variables that cause the same change in effective stress result in the same mechanical response of the material. A preliminary condition is that the work performed by the effective stress must be equal to that produced by the total stresses and the fluid pressures. Houlsby (1997) showed that the total work input into an unsaturated soil sums up components related to the mechanical work, the work necessary to change water and air contents in the soil, and the work of water and air filtration forces (equation (16) in Houlsby (1997)). Under isotropic loading conditions, neglecting the work done by filtration forces, and identifying the divergences of water and air Darcy's velocities with the changes in air and water volumes per unit volume of the material, the expression provided by Houlsby (1997) reduces to

$$\delta w = -p\frac{\delta V}{V} + p_w\frac{\delta V_w}{V} + p_a\frac{\delta V_a}{V} \quad (1)$$

where w is the work input per unit volume of material; V is the material total volume; p is the total mean stress (positive in compression); p_a is the air pressure; p_w is the water pressure; V_w and V_a are the air and water volumes; and δ denotes the increment of the variable. δV is the change of volume of the sample, whereas δV_w and δV_a are the volumes of air and water expelled by it. Under the restriction of solid

Manuscript received 5 June 2015; revised manuscript accepted 20 December 2016. Published online ahead of print 23 January 2017.
Discussion on this paper closes on 1 December 2017, for further details see p. ii.
* Department of Geotechnical Engineering and Geosciences, Universitat Politècnica de Catalunya – BarcelonaTech, Barcelona, Spain.
† Department of Civil Engineering and Computer Science Engineering, Università degli Studi di Roma Tor Vergata, Rome, Italy.

incompressibility, the change in volume δV is equal to the change in pore volume δV_{V}, in turn equal to the sum of the change of water and air volume contained in the pores

$$\delta V = \delta V_{\mathrm{V}} = \delta V_{\mathrm{w}} + \delta V_{\mathrm{a}} \tag{2}$$

Although not explicitly stated in the initial proposals, the different concepts for Bishop's type effective stress can all be formally derived by considering that δV_{w} can be split into two independent components δV_{w1} and δV_{w2}, one of them (δV_{w1}) proportional to the change in pore volume

$$\delta V_{\mathrm{w}} = \delta V_{\mathrm{w1}} + \delta V_{\mathrm{w2}} = \chi \delta V_{\mathrm{V}} + \delta V_{\mathrm{w2}} \tag{3}$$

where χ is a proportionality coefficient between δV_{w1} and δV_{V}. Substitution of equations (3) and (2) into equation (1) leads to the following expression for the work input

$$\delta w = -[p - \chi p_{\mathrm{w}} - (1-\chi) p_{\mathrm{a}}] \frac{\delta V_{\mathrm{V}}}{V} - (p_{\mathrm{a}} - p_{\mathrm{w}}) \frac{\delta V_{\mathrm{w2}}}{V} \tag{4}$$

or, in terms of volumetric strain increment, $\delta \varepsilon_{\mathrm{V}}$

$$\delta w = [p - \chi p_{\mathrm{w}} - (1-\chi) p_{\mathrm{a}}] \delta \varepsilon_{\mathrm{V}} - (p_{\mathrm{a}} - p_{\mathrm{w}}) \frac{\delta V_{\mathrm{w2}}}{V} \tag{5}$$

Equation (5) states that Bishop's effective stress, with Bishop's parameter equal to the proportionality coefficient introduced in equation (3), is work-conjugated to $\delta \varepsilon_{\mathrm{V}}$ provided that suction ($p_{\mathrm{a}} - p_{\mathrm{w}}$) is the intensive variable associated to changes in $-\delta V_{\mathrm{w2}}/V$.

The physical meaning of the partition stated by equation (3) is illustrated in Fig. 1 for an incremental path involving a generic change of suction (δs) and void ratio (δe) that may represent, depending on the magnitude and sign considered for δs and δe, all the possible combinations between drying, wetting, swelling, compression and collapse. Water volume change is expressed in the figure by the change in water ratio $\delta e_{\mathrm{w}} = \delta V_{\mathrm{w}}/V_{\mathrm{s}}$ where V_{s} is the volume of solid phase, which is considered incompressible for the sake of simplicity.

According to the figure, the total change in water volume $V_{\mathrm{s}} \delta e_{\mathrm{w}}$ can be decomposed into the sum of two components

(*a*) the change of water volume ($V_{\mathrm{s}} \delta e_{\mathrm{w2}}$) due to application of the suction increment at constant volume (path OA). During this process, the material state moves along the e_{w}–s curve at a fixed void ratio, e, and, since δV_{V} is equal to zero in equation (3), all the change in water volume is imputable to the change in δV_{W2}.

(*b*) the change of water volume ($V_{\mathrm{s}} \delta e_{\mathrm{w1}}$) due to the application of an increment in void ratio at constant suction (path AB). During this process, the e_{w}–s curve shifts to a new position at $e + \delta e$, and the water volume changes to maintain constant suction. This change is caused only by deformation and corresponds to the term $\chi \delta V_{\mathrm{V}} = \chi V_{\mathrm{s}} \delta e$ in equation (3).

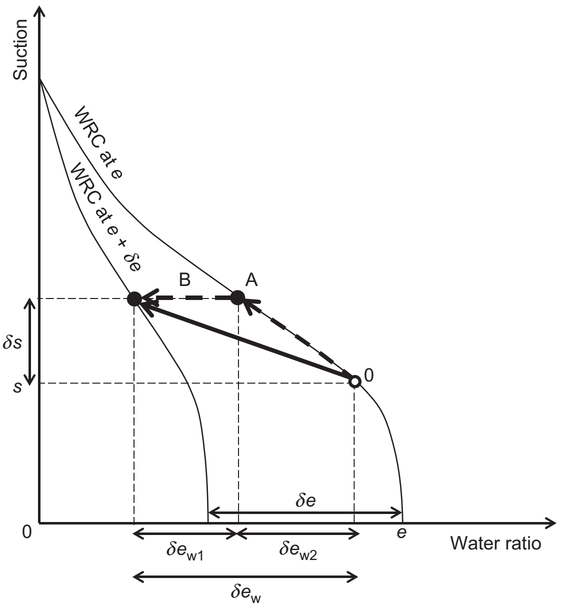

Fig. 1. Partition of total water volume change into components due to suction and deformation only for a path going from the water retention curve (WRC) at *e* to WRC at *e* + δ*e*

According to this decomposition, the χ coefficient is defined as the ratio

$$\chi = \frac{\delta V_{\mathrm{w1}}}{\delta V_{\mathrm{V}}} = \frac{\delta e_{\mathrm{w1}}}{\delta e} \tag{6}$$

where δe_{w1} is the water ratio exchanged during straining while maintaining constant the suction at which χ is computed and δe is the change in void ratio resulting from the straining process. Equation (6) emphasises that Bishop's parameter has to be related to the change with strain of the soil-water storage curve rather than to averaging variables of gas and liquid phase volumes. In a comparable approach, Mašín (2013) proposed using Bishop's parameter to describe the strain dependency of the water retention curve.

Another expression for Bishop's parameter is obtained by substituting water ratio by degree of saturation (S_{r}) in equation (6)

$$\chi = \left.\frac{\delta e_{\mathrm{w}}}{\delta e}\right|_{\mathrm{s=Cnst}} = \left.\frac{\delta(e S_{\mathrm{r}})}{\delta e}\right|_{\mathrm{s=Cnst}} = S_{\mathrm{r}} + e \left.\frac{\delta S_{\mathrm{r}}}{\delta e}\right|_{\mathrm{s=Cnst}} \tag{7}$$

Equation (7) provides evidence that the common assumption that Bishop's parameter is equal to degree of saturation (Jommi & di Prisco, 1994; Houlsby, 1997; Lewis & Schrefler, 1998; Jommi, 2000; Sheng *et al.*, 2004; Pereira *et al.*, 2005; Casini, 2008) only applies to materials provided with S_{r}–s curves independent of void ratio. Fig. 2(a) depicts schematically such a curve expressed in terms of water ratio plotted against suction. Since, for any suction, $e_{\mathrm{w0}}/e_0 = S_{\mathrm{r}} = e_{\mathrm{w1}}/e_1$, the curve at e_1 is simply obtained by scaling the whole curve at e_0 by factor e_1/e_0. On the other hand, $-\delta V_{\mathrm{w2}}/V = -(\delta V_{\mathrm{w}}/V - \chi \mathrm{d}V/V) = S_{\mathrm{r}} \mathrm{d}V/V - \delta V_{\mathrm{w}}/V$ can be readily shown to be equal to $-n\delta s_{\mathrm{r}}$. The work-conjugated variables proposed by Houlsby (1997) are thus valid for S_{r}–s curves independent of void ratio.

Alonso *et al.* (2010) partly relax the latter assumption by considering that the material may have a fixed volume of water trapped in the microstructure (V_{wm}) and propose to equate Bishop's coefficient with the effective degree of saturation (S_{rM}) defined as

$$S_{\mathrm{rM}} = \frac{e_{\mathrm{w}} - e_{\mathrm{wm}}}{e - e_{\mathrm{wm}}} \tag{8}$$

where $e_{\mathrm{wm}} = V_{\mathrm{wm}}/V_{\mathrm{S}}$ is a material constant giving the microstructural water ratio. The basis for such a proposal can be explored by substituting equation (8) into equation (6), which leads to the expression

$$\chi = S_{\mathrm{rM}} + (e - e_{\mathrm{wm}}) \left.\frac{\delta S_{\mathrm{rM}}}{\delta(e - e_{\mathrm{wm}})}\right|_{\mathrm{s=Cnst}} \tag{9}$$

According to equation (9), the assumption $\chi = S_{\mathrm{rM}}$ is thus valid for S_{r}–s curves that are independent of strain over the range of effective degree of saturation. A schematic example of an e_{w}–s curve leading to Alonso *et al.*'s proposal for Bishop's coefficient is shown in Fig. 2(b). The e_1-curve is identical to the e_0-curve for water ratio below e_{wm}. For

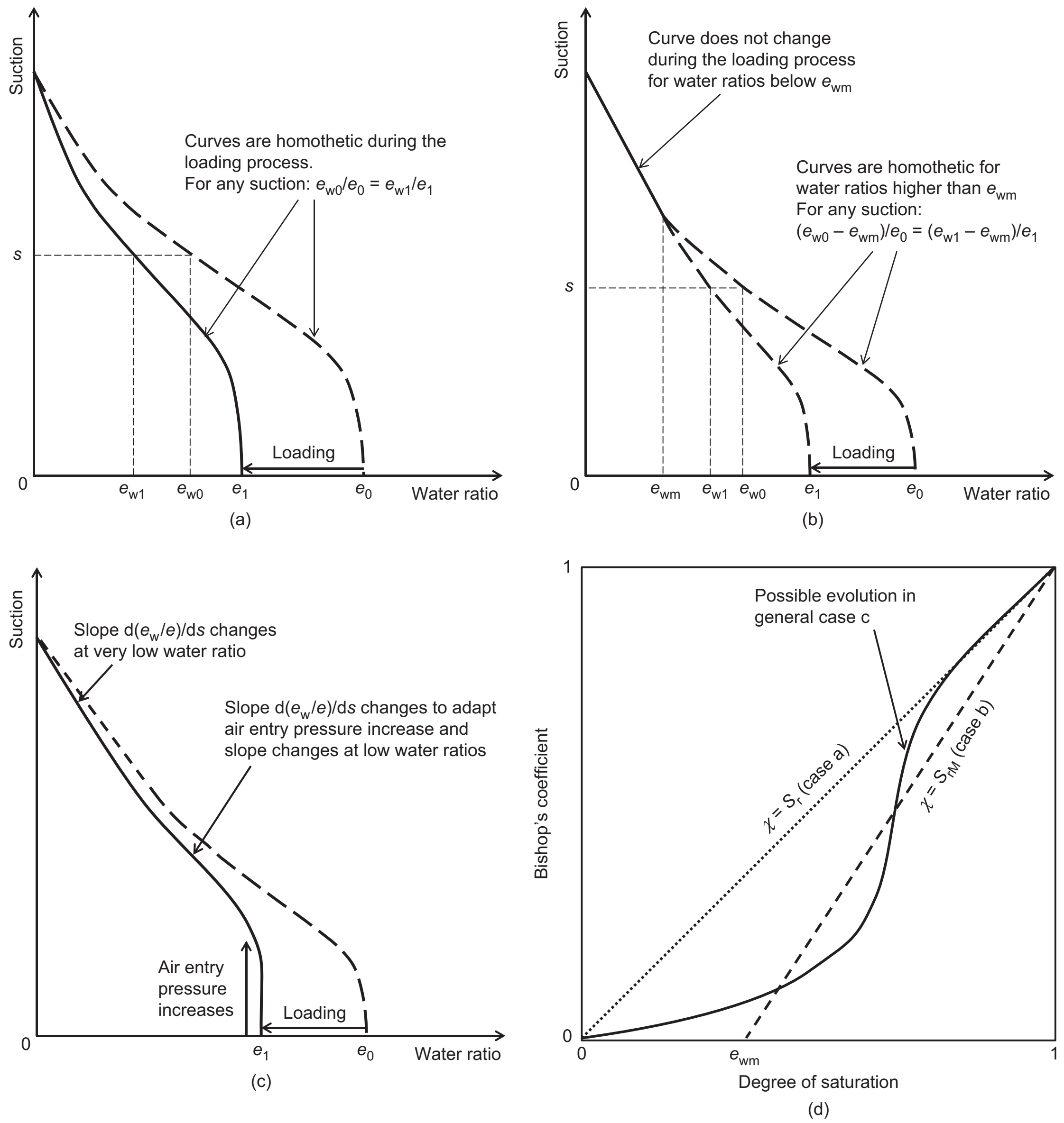

Fig. 2. Schematic variation of Bishop's coefficient for different types of water retention curve strain dependency: (a) water ratio–suction curve that preserves its shape over the whole range of degree of saturation during straining; (b) water ratio–suction curve that preserves its shape over the range of macrostructural degree of saturation and remains constant in the microstructural range; (c) water ratio–suction curve that changes in shape during straining; (d) variation of Bishop's coefficient with degree of saturation for cases a, b and c

higher water ratio, it is obtained as the e_0-curve scaled by $(e_1 - e_{wm})/(e_0 - e_{wm})$.

In the most general case, S_r–s curve experiences a change in shape during straining, often governed by the change of air entry pressure with density and the ability of the smallest pores to undergo volume changes. A schematic example is presented in Fig. 2(c).

Figure 2(d) summarises the variation of Bishop's coefficient obtained by the present approach for the different types of water retention curve. It is equal to the degree of saturation in case a, the effective degree of saturation in case b and takes other values in case c. In this last case, the χ coefficient needs to be evaluated by a direct procedure without any a priori assumption about possible dependency on degree of saturation.

DIRECT EVALUATION OF BISHOP'S PARAMETER

Procedure of evaluation

Determination of water retention curves at different void ratios may result in a long process if addressed through the classical methods based on water transfer. An alternative way consists in obtaining them from MIP, by recognising that the intrusion of a non-wetting fluid such as mercury obeys the same principles as the water drying process (Durner, 1994; Romero & Simms, 2008; Casini *et al*., 2012). According to this possibility, the χ coefficient can be directly estimated from the change in PSD under straining.

The procedure consists of the following steps.

(*a*) Obtain the PSDs at two different void ratios e and $e + \Delta e$. Each distribution must have been scaled such that the area computed below it from the lowest (R_{min}) to the highest (R_{max}) pore entrance radii gives directly the value of the void ratio at which it has been determined. The difference between both areas thus gives the change in void ratio Δe.

(*b*) Select a value of suction s. According to Laplace and Wasburn laws, s is inversely proportional to the radius (R) of the pore able to sustain it, through a proportionality coefficient μ (0·484 N/m at 25°C) that includes the effects of water and mercury contact angles

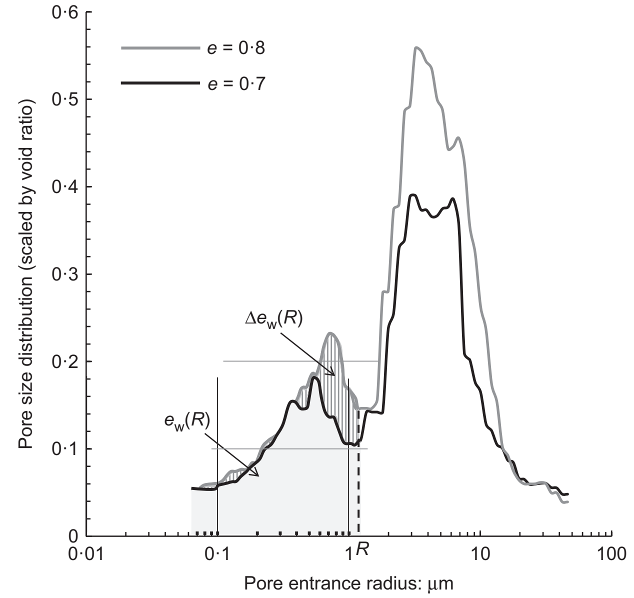

Fig. 3. Pore size distribution of two samples of Jossigny silt compacted at void ratios equal to 0·8 and 0·7 (the PSD has been scaled by the respective value of void ratios)

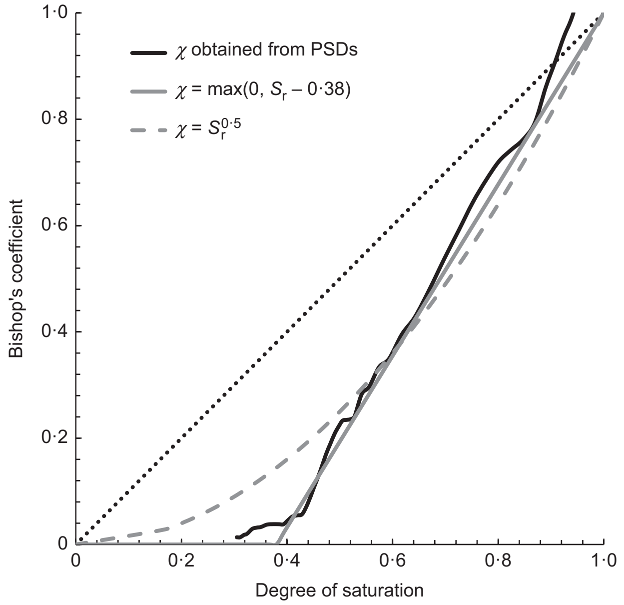

Fig. 4. Variation of χ with degree of saturation for samples of Jossigny silt compacted at void ratio equal to 0·8

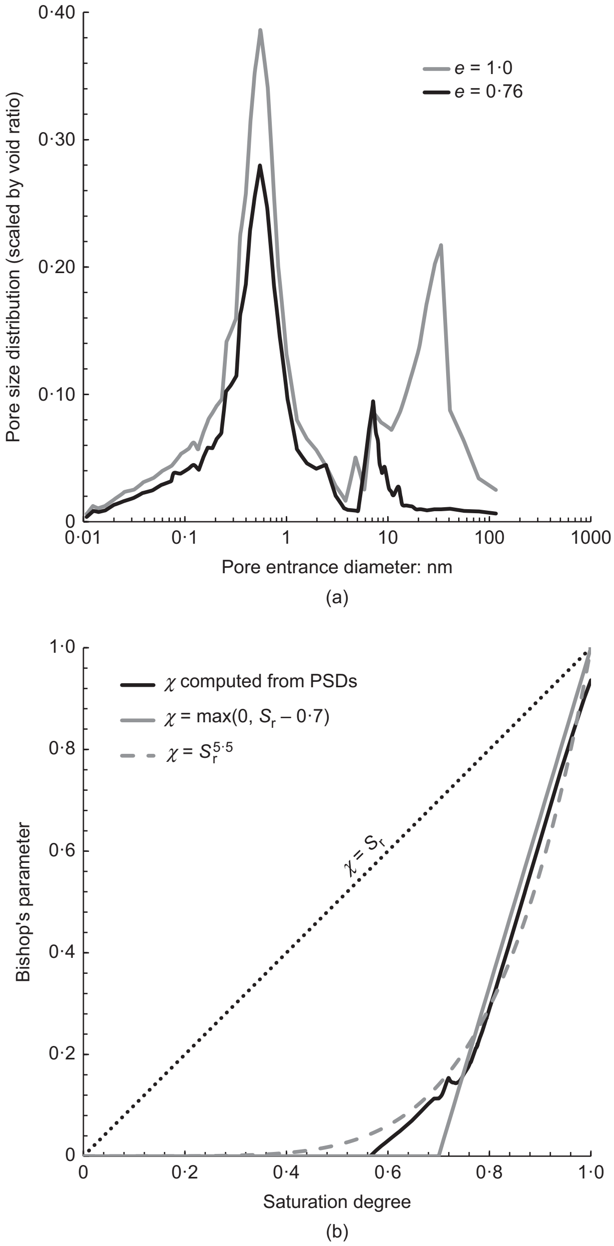

Fig. 5. Determination of Bishop's coefficient for low-plasticity sandy clay (lean clay): (a) PSD (after Li & Zhang, 2009); (b) variation of Bishop's coefficient with degree of saturation

and surface tensions. Thus, during application of suction s, all pores with entrance radius lower than R are filled by water and the area below the PSD between $R_{\min}$ and R gives the water ratio e_w of the material at suction s. The change in water ratio Δe_w during application of a change in void ratio Δe can thus be computed as the difference of the PSDs at e and $e + \Delta e$ from $R_{\min}$ to R.

(*c*) Compute χ as the ratio Δe_w over Δe.

Example of computation

The procedure is tested on MIP tests obtained on three materials: a sandy clay (lean clay, CF = 11%, $w_L = 48\%$, IP = 18%; Li & Zhang, 2009); a low-plasticity silt (Jossigny silt, CF = 25%, $w_L = 32\%$, IP = 15%; Vicol, 1990; Cui, 1993; Cui & Delage, 1996; Casini *et al*., 2012) and an active clay (Febex bentonite, CF = 67%, $w_L = 102\%$, IP = 49%; Villar, 2002; Lloret & Villar, 2007) composed of 90% montmorillonite and provided by a cation exchange capacity equal to 111 meq/100 g.

Figure 3 shows the PSDs obtained on two samples of Jossigny silt compacted at different void ratios ($e_1 = 0{\cdot}8$ and $e_1 = 0{\cdot}7$) and at a constant water content equal to 15%. They show bi-modal distributions typical of materials compacted in a relatively dry state. According to the procedure described in the last section, the change in water ratio during a compaction process at constant suction s is computed as the area of the hatched zone in Fig. 3

$$\Delta e_w(R) = \sum_{r_i = R_{\min}}^{R} [\mathrm{PSD}_1(r_i) - \mathrm{PSD}_2(r_i)]\delta \log_{10}(r_i) \qquad (10)$$

where R is equal to μ/s. The value of χ at suction s is thus computed as the ratio $\Delta e_w(R)/\Delta e$.

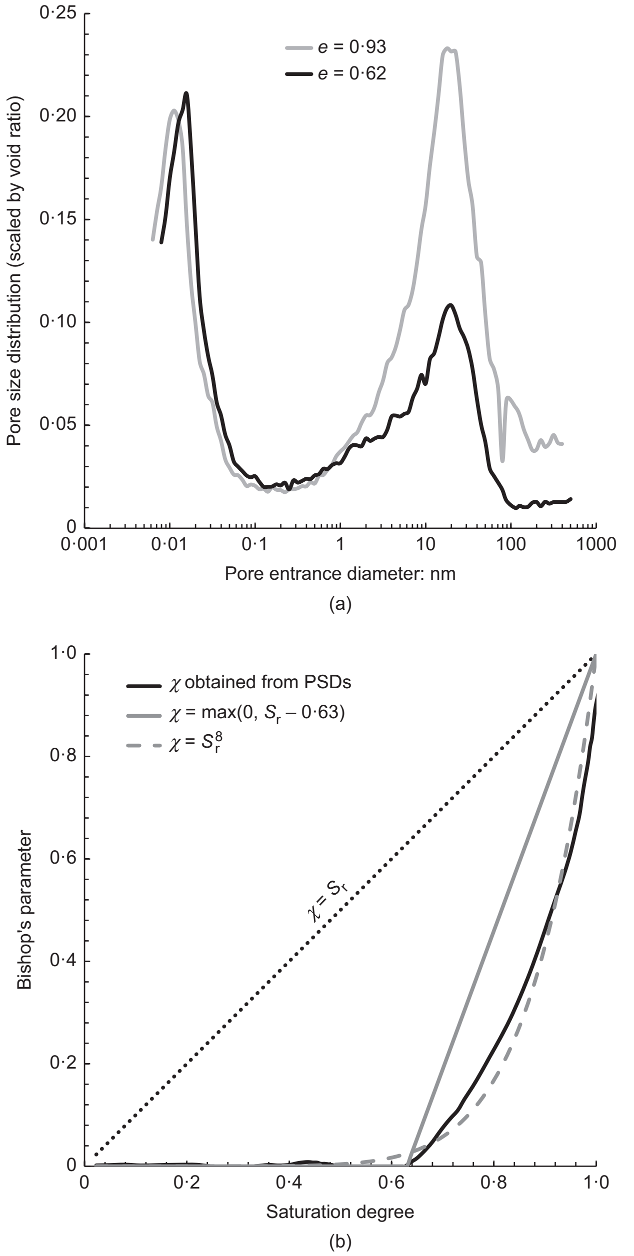

Fig. 6. Determination of Bishop's coefficient for highly active clay (Febex clay): (a) PSD (after Lloret & Villar, 2007); (b) variation of Bishop's coefficient with degree of saturation

Figure 4 show the values of Bishop's coefficient obtained from the change in PSD depicted in Fig. 3. It is expressed as a function of degree of saturation, computed by convention at the reference state, $e = 0{\cdot}8$. S_r is equal to e_w/e, where e_w is obtained as the area below the PSD for $e = 0{\cdot}8$ and target radius R (shaded area in Fig. 3). χ is close to zero at a low degree of saturation, when water is stored only in the smallest pores. For this range of pore radii, the PSD appears to suffer little change under the applied compaction load and there is thus only a small amount of expelled water. For degrees of saturation between 0·3 and 0·42, χ exhibits a low increase associated with pore volume reduction for the lowest mode of the PSD (entrance radii between 0·4 and 1 μm). For degrees of saturation above 0·42, a strong increase of χ is observed, owing to the fact that there is a significant amount of water expelled during compaction by pores with entrance radii between 1 and 15 μm. χ reaches a value of 1 for a degree of saturation equal to 0·95, as no volume change is detected for pore entrance radii above 15 μm. The χ parameter obtained in this manner matches quite well the effective degree of saturation computed by considering $e_{wm} = 0{\cdot}3$ and proposed by Alonso *et al.* (2010) from back-analyses of the variation of elastic stiffness with suction.

Similar trends can be observed for the variation of Bishop's coefficient obtained on lean and Febex clays (Fig. 5 and Fig. 6). PSDs of both materials show a pronounced double structure characterised by a low deformability of the micropores. As a result, most of the volume change experienced by the samples during compaction is imputable to the deformation of the largest pores and Bishop's parameter takes null values for degrees of saturation typically below 60%. For higher degree of saturation, χ lies close to the effective degree of saturation for the lean clay. For the Febex clay, values obtained using the direct procedure match quite well to the power law $\chi = S_r^{\alpha}$ proposed by Alonso *et al.* (2010).

CONCLUSIONS

A novel and general derivation of Bishop's parameter has been presented and lends a new significance to this parameter: it provides, for any saturation state of the material, the ratio between changes in water volume and total volume during a loading process at constant suction. This ratio is shown to be equal to the total or effective degree of saturation only for retention curves with a specific strain dependency.

Accordingly, a procedure has been proposed to estimate the change in water content from the PSD of samples compacted at two different densities. The method was tested on results obtained on a low-plasticity silt by MIP. Values of χ appear to be comparable with published results obtained by back-analysis of the change in elastic stiffness with suction.

ACKNOWLEDGEMENT

The financial support of MIUR (Ministero dell'Istruzione, dell'Università e della ricerca) for the second author through the 'Programma per Giovani Ricercatori – Rita Levi Montalcini' is acknowledged.

NOTATION

e	void ratio
e_w	water ratio
e_{wm}	microstructural water ratio
n_M	porosity of macrostructure
p	mean total stress
p_a	air pressure
p_w	water pressure
R	pore entrance radius
R_{max}	highest pore entrance radius of PSD
R_{min}	lowest pore entrance radius of PSD
r_i	pore entrance radius of ith family
S_r	degree of saturation
S_{rM}	degree of saturation of macrostructure
s	suction
V	soil volume
V_a	pore air volume
V_S	solid grain volume
V_V	pore volume
V_w	pore water volume
V_{wm}	volume of water trapped in soil microstructure
w	work per soil unit volume
α	power coefficient in the power law relating χ to degree of saturation
Δe, δe	change in void ratio
Δe_w, δe_w	change in water ratio

δ_{ij} Kronecker symbol
δs_r change in degree of saturation
δV change in soil volume
δV_a change in pore air volume
δV_V change in pore volume
δV_w change in pore water volume
δV_{w1} change in pore water volume at constant suction
δV_{w2} change in pore water volume at constant void ratio
μ proportionality coefficient between suction and pore entrance radius
$\boldsymbol{\sigma}_{ij}$ total stress tensor
$\boldsymbol{\sigma}''_{ij}$ net stress tensor
χ Bishop's parameter

REFERENCES

Alonso, E. E., Pereira, J. M., Vaunat, J. & Olivella, S. (2010). A microstructurally based effective stress for unsaturated soils. *Géotechnique* **60**, No. 12, 913–925, http://dx.doi.org/10.1680/geot.8.P.002.

Casini, F. (2008). *Effetti del grado di saturazione sul comportamento meccanico di un limo*. PhD thesis, Universitá degli Studi di Roma 'La Sapienza', Rome, Italy (in Italian).

Casini, F., Vaunat, J., Romero, E. & Desideri, A. (2012). Consequences on water retention properties of double porosity features in a compacted silt. *Acta Geotechnica* **7**, No. 2, 139–150.

Coussy, O., Pereira, J. M. & Vaunat, J. (2010). Revisiting the thermodynamics of hardening plasticity for unsaturated soils. *Comput. Geotech.* **37**, No. 1–2, 207–215.

Cui, Y. J. (1993). *Etude du comportement d'un limon non saturé et de sa modélisation dans un cadre élasto-plastique*. PhD thesis, Ecole Nationale des Ponts et Chaussées, Paris, France (in French).

Cui, Y. J. & Delage, P. (1996). Yielding and plastic behavior of an unsaturated compacted silt. *Géotechnique* **46**, No. 2, 291–311, http://dx.doi.org/10.1680/geot.1996.46.2.291.

Durner, W. (1994). Hydraulic conductivity estimation for soils with heterogeneous pore structures. *Water Resources Res.* **30**, No. 2, 211–223.

Fredlund, D. G. & Morgenstern, N. R. (1977). Stress state variables for unsaturated soils. *J. Geotech. Engng Div.* **103**, No. GT5, 447–466.

Gens, A. (1995). Constitutive modelling: application to compacted soils. In *Unsaturated soils* (eds E. E. Alonso and P. Delage), vol. 3, pp. 1179–1200. Rotterdam, the Netherlands: Balkema.

Houlsby, G. T. (1997). The work input to an unsaturated granular material. *Géotechnique* **47**, No. 1, 193–196, http://dx.doi.org/10.1680/geot.1997.47.1.193.

Jommi, C. (2000). Remarks on the constitutive modelling of unsaturated soils. In *Experimental evidence and theoretical approaches in unsaturated soils* (eds A. Tarantino and C. Mancuso), pp. 139–153. Rotterdam, the Netherlands: Balkema.

Jommi, C. & di Prisco, C. (1994). A simple theoretical approach for modelling the mechanical behaviour of unsaturated granular soils. In *Il ruolo dei fluidi in ingegneria geotecnica* (ed. Gruppo di Coordinamento per gli Studi di Ingegneria Geotecnica), vol. 1, No. II, pp. 167–188. Rome, Italy: Consiglio Nazionale delle Ricerche (in Italian).

Khalili, N., Geiser, F. & Blight, G. (2004). Effective stress in unsaturated soils: review with new evidence. *Int. J. Geomech.* **4**, No. 2, 115–126.

Lewis, R. W. & Schrefler, B. A. (1998). *The finite element method in the static and dynamic deformation and consolidation in porous media*. Chichester, UK: J. Wiley.

Li, X. & Zhang, L. M. (2009). Characterization of dual-structure pore-size distribution of soil. *Can. Geotech. J.* **46**, No. 2, 129–141.

Lloret, A. & Villar, M. V. (2007). Advances on the knowledge of the thermo-hydro-mechanical behaviour of heavily compacted FEBEX bentonite. *Phys. Chem. Earth* **32**, No. 8, 701–715.

Loret, B. & Khalili, N. (2000). A three phase model for unsaturated soils. *Int. J. Numer. Analyt. Methods Geomech.* **24**, No. 11, 893–927.

Pereira, J. M., Wong, H., Dubujet, P. & Dangla, P. (2005). Adaptation of existing behaviour models to unsaturated states: application to CJS model. *Int. J. Numer. Analyt. Methods Geomech.* **29**, No. 11, 1127–1155.

Romero, E. & Simms, P. H. (2008). Microstructure investigation in unsaturated soils: a review with special attention to contribution of mercury intrusion porosimetry and environmental scanning electron microscopy. *Geotech. Geol. Engng* **26**, No. 6, 705–727.

Sheng, D., Sloan, S. W. & Gens, A. (2004). A constitutive model for unsaturated soils: thermomechanical and computational aspects. *Comput. Mech.* **33**, No. 6, 453–465.

Tarantino, A., Mongiovì, L. & Bosco, G. (2000). An experimental investigation on the independent isotropic stress variables for unsaturated soils. *Géotechnique* **50**, No. 3, 275–282, http://dx.doi.org/10.1680/geot.2000.50.3.275.

Vicol, T. (1990). *Comportement hydraulique et mécanique d'un sol fin non saturé. Application à la modélisation*. PhD thesis, Ecole Nationale des Ponts et Chaussées, Paris, France (in French).

Villar, M. V. (2002). *Thermo-hydro-mechanical characterization of a bentonite from Cabo de Gata. A study applied to the use of bentonite as a sealing material in high level radioactive waste repositories*, Technical Publication ENRESA 01/2002. Madrid, Spain: ENRESA.