Simulation of lightning currents in CFRP components of wind turbines and aircraft by means of the FDTD method.

MEMORIA

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ABSTRACT

This TFM starts with an introduction to the state of art of finite different time domain, where all publications of different fields related to FDTD subject are analyzed. Once the method is examined, the parameters that should be taken into account to set up the FDTD software in a wind turbine blade are defined. Afterwards, the methods used on the analysis of the transitory effects of the lightning stokes in a wind turbine blade are evaluated. Furthermore, it is showed a brief part of the code development of FDTD jointly with some simulations of software operation.
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1. Case Introduction

The aim of this study is trying to understand the operation of the finite differences in time domain (FDTD) and analyzing the possibility of applying FDTD method to the protection of wind turbines against lightning strokes.

Nowadays, lighting stokes are producing several damages on wind turbine blades and even in aircrafts. For this reason, it is important to study how these damages are produced and examine different possibilities to solve them. To do this, an accurate method should be chosen, such as Finite Difference Time Domain (FDTD). This method is able to work at high frequency and take into account all the electromagnetic properties of the materials involved in the performance. This method allows not only simulating the materials, but also all the variables affecting the operation, such as the surrounding air, boundaries, etc.

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1.1. Objectives

In this study we will try to achieve different objectives related to the case, which are the following:

- Acknowledge the FDTD method regarding all the publications in different fields. Specially, those ones related to CFRP materials, wind turbines and aircrafts.
- Analyze the composition of elements for the protection of wind turbine blades against lightning stokes.
- Simulate a lightning surge and develop a code able to resolve electromagnetic problems in wind turbines using the FDTD method.

2. State of Art Finite Difference Time Domain

2.1. Introduction

Among all the methods to solve Maxwell’s equations FDTD method is the technique that continues growing due to availability of numerical codes and increased computer capabilities.

In the Fig. 1 it is possible to observe the trend line that ISI Web of Science predicted in 1995 about all the publications regarding “FDTD” or “finite difference time domain” in either the title, abstract or key words.
These publications have been classified in the following lightning topics:

- Lightning electromagnetic fields at close and far distance
- Lightning surges on wind turbine generators
- Lightning effects on aircrafts
- Lightning effects on electric vehicle
- Surge on grounding electrodes
- Lightning effects on buildings
- Military protection device against lightning struck

2.2. Lightning effects on buildings

Taking into account the remarkable damage that a building may suffer if it is struck by a lightning there are many recent publications (e.g., [1],[2],[3],[4],[5]) analyzing the current, voltage and electromagnetic field occasioned for this phenomena.

The increase of the internal lightning damage on buildings may be caused by the following factors:

1) Degraded immunity of IT equipment against lightning surge.
2) Complicated and versatile lightning invading passages generated by widened network and complicated wiring in buildings

Fig. 2 shows a communication terminal board struck by a lightning.
Wang et al [6] have analyzed the potential distribution in reinforced steel frames of a building under direct lightning strike.

The field approach i.e. MoM (method of moment) [7] has been applied to calculate the electromagnetic fields inside of the building in the frequency domain. Although the electromagnetic coupling of the steel frames could be automatically taken into account, it has been replaced by a perfect conducting plane and the grounding resistance of each column has been assumed to be zero. This way, the effects of grounding conductivity are not taken into consideration.

For this reason this paper applied the FDTD method, because it is carried out entirely in the time domain. Furthermore, almost all the electromagnetic characteristics of a construction are considered automatically.

The model of a three-floor is shown in the Fig.3 and represents a building suffering a direct lightning strike. The reinforced steel bars of the structure are with diameter of 10mm and conductivity of 7.69x10^6 S/m. The simulated circuit is shown in the Fig.4, where the distances between objects can be observed.
The impulse was simulated with a bi-exponential waveform (2.6/40µs) with a current amplitude of 10609 A.

Finally, from the calculation’s results, they obtained the following conclusions:

- The higher the floor is, the larger the peak potential and uneven potential distribution.
- On the same floor, the peak potential at corner nodes is always greater than those adjacent nodes.
- The intensity of the oscillations is higher on peripheral nodes, and the front edge of the waveforms at peripheral nodes is steeper.

The graphically results are shown in Fig.5 and Fig.6.

Hajime et al [8] carried out a similar study where they made experiments to validate the FDTD model using a building scale model. This model was built on 1/4 scale of a building.

However, they applied a surge waveform of 100 kA 10/350µs for the input. In the Fig.6 it is shown how the electromagnetic field is distributed when a lightning struck the body of the building. The deeper blue color area indicates lower generating electromagnetic field intensity while the deeper red color area, higher generating electromagnetic field intensity.
Once the simulation was done, they compared the results of the simulation to the results measured in the scale of building, as example Fig.7 published the comparison between both results in column B, and the error was 0.7%.

2.3. Lightning effects on aircrafts

The interaction of lightning with an aircraft induces voltages and currents in the onboard wire harnesses, which may damage critically the electronic equipment, compromising the flight safety.

Nowadays, aeronautical companies are trying to made the aircraft’s with composite materials, like Boeing 787 or Airbus 380, for this reason they are investing on the research of relevant aspects related to the modeling and simulation of innovative advanced composite materials with improved protection against a lightning electromagnetic pulse stroke (e.g., [9],[10],[11],[12]).
Mauricio et al [13], published a report in 2008 where reviewed the computational tool (VAM-LIFE) developed for the complete analysis of the electromagnetic fields inside and outside a medium-sized aircraft.

All the entry and exit points of the lightning strike on an aircraft Fig.8 are defined by the Federal Aviation Regulations (FARs), who define the performance protection requirements.

VAM-LIFE predicts the indirect effect of the lightning interaction with an aircraft using the block diagram shown in Fig.9 that it was approved by the Italian Aeronautic Authority and was used to achieve the certification of the transport aircraft C-27J.

The impulse simulated was a peak value of nearly 250 A, rise time of 500 ns and semi-value time of 49µs
The results obtained demonstrate the distribution of magnetic field during the lightning strike in all entry-exit combinations, as example Fig.10a display the space distribution of the Hz component in entry-exit configuration A1 or Fig10.b shows the space distribution of the Hy component with and entry-exit configuration A2.

![Fig 10a. Space distribution Hx of A1 entry-exit distribution [13]](image)

![Fig 10b. Space distribution Hy of A2 entry-exit distribution [13]](image)

2.4 Lightning effects on electric vehicles

One of the sectors that should contribute to reduce carbon dioxide to achieve EPA proposals is the transport sector, for this reason ecological electric vehicles and hybrid cars are being popularized rapidly.

Taking this into account, many publications related with this topic were published few years ago (e.g.[14],[15],[16],[17],[18]).

Yamamoto et al.[19], in 2012 studied the transient magnetic fields and the current distributions in an electric vehicle produced by a lightning struck using the FDTD method.

They modelled 3 possibilities of injection points as it can be shown in Fig.11 (a-c)

![Fig.11a Simulation Setup [19]](image) ![Fig.11b Top view[19]](image) ![Fig.11c Side view[19]](image)
For example, in Case 1, where Antenna was the current injected point and the discharge points were all wheels, the distribution of $\frac{dH}{dt}$ was comparatively larger around the wheels and the axle at the rear of electric vehicle than around other parts of the car, as Fig12a-b shows.

Taking the same $\frac{dH}{dt}$ graphic distribution but changing the discharge point to the right back wheel, they obtained the “Case 3”. As it can be observed in the Fig13a-b, larger $\frac{dH}{dt}$ appears not around wheels but also in the middle of the vehicle body because current was discharged from only one wheel and current passed partially throughout the vehicle body.

2.5. Lightning protection systems for military applications

With the aim of protecting the rocket launchers, in 1994 Marvin et al [20] developed a FDTD code using Fortran with the requirements of the derivate of the driving current in the electromagnetic coupling process and testing lightning effects in full-scale structures Fig.14.
The test consisted on measuring in 24 locations the current value when a lightning of 37 kA struck in this structure.

The conclusions of this paper show that if a direct-strike lightning appears it will not be safe to stored explosives inside the structure. In addition to this, the use of triggered lightning as a practical testing tool for certain special applications was demonstrated.

2.6. Lightning overvoltages on transmission lines

In this kind of installations, it is important to evaluate the lightning outage risk and the lightning protection specifications. For example, to calculate the lightning outage rate of a transmission or distribution line and to design the lightning protection, the overvoltage generated due to the transient response of the tower at lightning must be precisely determined.

The accurate way to analyze the surge that propagates through the tower in a vertical direction, a dynamic electromagnetic field analysis based on Maxwell’s equations is needed instead of analysis based on circuit theory with the quasi-stationary state of the electromagnetic field as a precondition.

However, many studies have also been conducted on the impedance of a stand-alone transmission line (e.g.[21-23]) through a circuit approach. The problem of this technique is that cannot handle the surge phenomenon at the top of the tower.

Furthermore, in the lightning protection design of a power station or substation, a lightning strike to the top of the first tower is commonly assumed as a standard condition where the incoming lines from the first tower to the substation are inclined conductors.

Analyses based on electromagnetic theory (e.g.[24-26]) have various issues, such as the difference in experimental results and the ability to handle only very simply-shaped models.
Due to FDTD can precisely reproduce the electromagnetic field in space because it solves Maxwell equations in the time domain, is a really smart tool to solve this kind of problems.

Marvin et al [27], show the analytical accuracy of FDTD technique base on the experimental results using a reduce scale model of the transmission line, Fig.15.

The voltage rated of the circuits is 500 kV and are two overhead ground wires, which are connected to the top of the towers and gantry.

On the other hand, conductors were laid out to reproduce the experiment using a reduce-scale model and the current source was installed on the top of the tower, as shown in Fig.16.
The comparison between experimental measured values and values performed through FDTD clearly show the core lance between both methods, as shown Fig. 17a-d.

![Fig. 17a Injected current waveforms][27]. ![Fig. 17b Voltage of the upper phase arm of the tower][27].

![Fig. 17c Voltage of the insulator strings][27]. ![Fig. 17d Voltage at the top of gantry tower][27].

In addition to this study, they also realized the influence of the inclined angle of incoming lines to the substation. The angle of the incoming lines to the substation shown in Fig. 18 was changed from 0 to 69 degrees to calculate the voltage generated. Although a zero-degree angle does not apply to actual incoming lines, the conditions with this power line layout have generally been used for conventional measurement and calculation of the tower impedance.

![Fig. 18 Parameters of the inclined angle of incoming lines to the substation][27].

The voltage of the insulators strings is likely to decline with increasing inclined angle. The voltage of the insulator string is 40.5 in the case of an inclined angle of 14.6 degrees, and 36.7 V in the case of an inclined angle of 69.3 degrees. Those voltages decline by 14 and 24 % respectively, from 48.2 V for a zero-degree angle, Fig.19. Furthermore the power line voltage rises with increasing inclined angle.
They also published an interesting conclusion: all the insulators strings with an angle of 37.5 degrees had a 20% less of voltage than the insulators with zero-degree incoming line angle.

A similar report was published in 2014, where Takami et al [28] focused in the lightning surge into a substation at a back-flashover.

The lightning protection design of a power station and substation [29,30] is based on standard lightning stroke conditions, where lightning stroke to the top of the first tower occurs to generate a back-flashover at the upper phase and a surge intrudes into a power station, this propagation occurs in a non-uniform line.

The analysis is shown in Fig.20 and 21 where 500 kV incoming lines are represented. The vertical arrow at the top of the tower represents the lightning discharge.
The performance consisted on measuring the power line voltage and current at different phases when the tower is struck by a lightning, finding out the influence of the inclined angle of incoming lines and, finally, showing the relationship between maximum overvoltage and the incoming line span length, using the FDTD method and the conventional circuit analysis measuring.

The following Figure 22 shows the voltage distribution in every single phase, where a high value is induced in the upper phases.

According to Fig.22, the power line voltage rises more gradually than the waveform of the current of the Fig.23, flowing into the top of the tower and forming a waveform similar to the voltage waveform observed for vertical conductors [31,32].

Taking the report [27] as a reference, they also tried to find the relationship between the maximum voltage and current through the power line where a back-flashover occurred and the incline angle of the incoming line. The results after the simulations are shown in Fig.24, where was published that, as inclination incoming line angle increased, the voltage declined at any position and the voltage of the lightning surge intruding into the substation also decreased.
Finally, the evaluation of the lightning protective level was done using an impulse of 200 kA, comparing the FDTD to circuit analytical results, with a lightning path impedance of 1 kΩ. Table 1 shows the lightning overvoltages values obtained, based on both techniques for the standard lightning impulse waveform.

<table>
<thead>
<tr>
<th>Table 1. Lightning overvoltages based on FDTD and circuit analysis and their comparison [kV] [28].</th>
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<tbody>
<tr>
<td><strong>Incoming line span length</strong></td>
</tr>
<tr>
<td>FDTD</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Circuit analysis</td>
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<tr>
<td></td>
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<tr>
<td>Relative ratio (1) / (2)</td>
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</table>

Taking the comparison of the analytical circuit, it was indicated that lightning protective level could potentially be reduced by about 30% to 50% through highly accurate analysis using the FDTD simulation. For this reason, the publication ended with the following citation: “The need to promote the study on an actual application of FDTD simulation as well develop a highly accurate model usable for circuit analysis are issues to address in future.”
2.7. Lightning surges in wind turbine generator

Nowadays, lightning protection of wind turbines is becoming an important public issue due to the growing up of the power capacity installed around the worldwide. Wind turbines are often struck by lightning because of their open air locations, normally are installed at the top of mountains and also for the high longitude of the blades, which are the place where the lightning normally struck.

Although some reports (including IEC TR 61400-24:2002[33]), have indicated a methodology for protection against such accidents, a standardized solution remains to be established. This problem has recently surfaced as an important issue.

According to IEC TR 61400-24, a Type A arrangement “vertical or/and horizontal electrode” and Type B arrangement “ring earth electrode” are recommended for wind turbine earthing. However, generally, wind turbines are constructed onto circular or polygonal foundations made of a reinforced concrete, for this reason, wind turbines normally use type B arrangement.

Among several elements of lightning protection, an earthing system is one of the most important points to be considered. For this reason, many investigations and reports [35-36] have been published during these years.

As example, Yasuda et al [34] performed an electromagnetic analysis on an earthing system of a wind turbine to confirm the effect of a ring earth electrode with the aim of answering, for example, the questions about what is the best combination of the ring electrode or vertical rods to realize the effective lightning protection for a wind turbine.

The FDTD simulation was performed to resolve the transient analysis on wind turbine earthing system, assuming a simplified ideal foundation with the following assumptions.

(i) A tower is not considered and the lightning current is assumed to be strike at the top surface of the foundation.

(ii) The foundation is made of rectangular solid block and area of the base is 4.5 m x 4.5 m.

(iii) A reinforced bar in the foundation is simulated by a copper frame.
In the Table 2, is shown the parameter for FDTD calculation and Fig. 25 is the model of a wind turbine base.

<table>
<thead>
<tr>
<th>Table 2. Parameters for FDTD calculation[34].</th>
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<tr>
<td><strong>domain of space</strong></td>
</tr>
<tr>
<td><strong>step size of space</strong></td>
</tr>
<tr>
<td><strong>step size of time</strong></td>
</tr>
<tr>
<td><strong>relative permittivity</strong></td>
</tr>
</tbody>
</table>
| air | 1  
| soil | 10  
| concrete | 6  
| conductor (copper) | 1  |
| **conductivity** |  
| soil | 5.0×10⁻³ S/m  
| concrete | 58×10⁻³ S/m  
| conductor (copper) | 50×10⁶ S/m  |
| **permeability** |  
| air | 4π×10⁻⁷ H/m  
| soil | 1  
| conductor (copper) | 1  
| cast iron | 60  |
| **velocity of electromagnetic wave** | 3×10⁸ m/s |
| **lightning: lump wave** |  
| crest width | 1 µs  
| wave tail | 70 µs  
| crest peak | 30 kA  |
| **boundary condition** | the second-order Laue’s absorbing condition[10] |

They prepared three cases to compare the effect of combination of ring earth and vertical rods:

- Case I: Four vertical rods are installed on the bottom of the foundation’s four corners
- Case II: Four rods are installed on the bottom of the four corners of the outer ring earth electrode
- Case III: Eight rods are installed on both of the foundation and the ring electrode.

As example of the results obtained, Fig. 26 represents the distribution of electrical field in Case II, in different planes.

![Fig. 26 Distribution electric field Case II[34].](image)
The results and conclusions that they published were pretty interesting, due to the comparison between FDTD and the minimum length of vertical rods according to IEC did not match. For example, if Case I is selected, the individual length of vertical rods is required more than 70 m to achieve 10 Ω earth resistance, this length goes far beyond the required minimum defined in IEC. Case II seems to barely satisfy the IEC requirements and Case II stays within the recommended area, Fig. 27.

![Fig. 27 Comparison of FDTD method and minimum length defined by IEC [34].](image)

The conclusions of this analysis, discussed the effect of a ring earth electrode and auxiliary vertical rods for wind turbine as recommended in IEC TR 61400-24:2002. Moreover, there is a case where the earth resistance would not become reduced enough even if the case satisfied the recommendation by IEC standard.

A similar ground study was approached in 2011 by Raju et al [37]. The publication was about the electromagnetic analysis of lightning surge response of interconnected wind turbine grounding system.

Because of IEC TR61400-24[33] suggest interconnection of the grounding system of wind turbines to achieve low steady-state resistance, the complete earth system has to be studied with the aim of finding out the influence of interconnection on the lightning surge.

Three cases were investigated on the effect of interconnection of wind turbine groundings. The analytical setup for FDTD calculation is shown in Fig. 28.

- Case 1: Single turbine grounding.
- Case 2: Two groundings interconnected by an insulated cable of conductor radius 10mm.
- Case 3: Two groundings interconnected by a bare conductor of 10 mm radius.
This report studied this problem by means of two different numerical methods which solves Maxwell’s equations in frequency and time domain, one of them was using (NEC-4)[38], based on MoM and the other one was the VSTL software[39], based on FDTD simulation code. The results for the three cases can be observed in table 3.

They concluded that FDTD method turns out to be more reliable in evaluating the transient behavior, the transient grounding impedances calculated by FDTD method shows 10 to 20% lower values after several thousands of time steps irrespective of the resistivity of soil.

Finally, the published that the interconnection of wind turbine groundings was effective for reducing the maximum voltage rise at the wind turbine struck by lightning, when the current has a rise time of more than few μs, and is more effective for soil of high resistivity. Nearly half of the current invades into the grounding of an adjacent turbine when interconnection is made by insulated cable. On the other hand, the employ of a bare earth conductor for the interconnection, reduces the invading current and also the transferred overvoltage.
2.8. Lightning current analysis on CFRP materials

Nowadays, most applications are manufactured from CFRP, for example the main structure of the aircraft or the body of the wind turbine blade. Since they still have unknown electrical characteristics in lightning and electrostatic charges, the aircraft manufactures have to do test and redesign the aircraft again and again.

For this reason aeronautical companies as well as wind turbine manufactures are promoting the investigation for knowing the electrical and magnetic properties.

One of these researches was carried out by Takayuki et al [40] with the target of developed and simulated the behavior of lightning current and predicted the locations where ignition sources were likely to occur.

For validating the current distribution of FDTD simulation, they simulated a fuel tank. Fig. 29 structure with all the structures (skin, rib, stringer, spar, fuel pipe, fuel quantity indicating system).

![FDTD Mesh and Real Structure](image)

Figure 30 shows the simulation results of current distribution along the fuel tank, as is shown the results obtained from FDTD and analysis results were practically the same.

![Simulation Results of Current Distribution](image)

One of the mismatch that they found it was the flowing current through the center of the tank, as is shown in Fig. 31 the blue arrow represents the simulation results and the red arrow the experimental results.
Finally, for confirming the current concentration of the fibers they recorded with a thermography camera the heat transferred when a 50 kA impulse current was applied, the snapshot of the camera is shown in Fig.32.

A similar approach was made this year for Smorgonskiy et al [41], they analyzed two cases, when CFRP elements are covered by the copper mesh and without such protection.

As we have seen before modeling of lightning current interaction with a wind turbine is a challenging problem [42,43] because of one of the main difficulties is relate to the presence of many different materials within the same structure. One of this materials is carbon fibers, that are electrically conductive (x and y axis Fig. 33) however the polymers are not conductive in the transverse direction (z axis).
They assume that the blade was a uniform cross-section, did not consider the variation dimensions along the blade and the blade tip, and was set to 30 kA.

The first study where the CFRP has a cooper mesh, results that 47% of the total lightning current impulse passes through the cooper mesh. Relatively small part of the lightning current flows through the lightning down conduction (LPS), as well as the top and bottom blade spar caps made of CFRP as shown in Fig. 34.

![Fig. 34 Portion of lightning current flowing in the lightning down conductor (LPS) vs two CFRP spar caps as function of frequency[41].](image)

On the other hand, the second analysis where CFRP did not have the copper mesh resulted that this material carried with the majority of high-frequency current while lightning down conductor serves as a lightning path at low frequencies.

Due to such current carried by CFRP the temperature of the carbon fiber raises rapidly which leaded to the vaporization of the surrounding polymer and delamination of the material[43], the proportion of energy is shown in Fig. 35.

![Fig. 35 Portion of lightning current in the LPS and CFRP spar caps (without cooper mesh) as a function of frequency[41].](image)
3. **COMPUTATIONAL ELECTROMAGNETICS**

3.1. Introduction

Several electromagnetic problems like electromagnetic scattering, electromagnetic radiation or modeling waveguides, are not analytically calculable, for the multitude of irregular geometries found in actual devices. Computational numerical techniques can overcome the inability to derive closed form solutions of Maxwell's equations under various constitutive relations of median and boundary conditions.

This makes computational electromagnetics (CEM), important to the design, and modeling of antenna, radar, and high speed silicon electronics, medical imaging and high frequency applications as could be a lightning struck.

CEM typically solves the problem of computing the $E$ (Electric), and $H$ (Magnetic) fields across the problem domain. CEM models may or may not assume symmetry, simplifying real world structures to idealized cylinders, spheres and other regular geometrical objects. CEM models extensively make use of symmetry, and solve for reduced dimensionality from 3 spatial dimensions to 2D and even 1D.

3.2. Methods

Choosing the right technique to solve a problem is really important, because choosing the wrong one can give incorrect results, or results which take excessively long to compute.

In Fig.45, it is possible to observe two main groups to resolve CEM problems. These groups are *Differential Equations Methods* or *Integral Equations methods*.
Nowadays, the most commercial software to resolve CEM problems are shown in Fig. 46.

![Commercial Software Packages](image)

**Figure 46: Commercial software packages [50]**

As the main used techniques to resolve CEM problems are Method of Moments (MOM) and Finite Difference frequency domain (FDTD), will be describe briefly.

### 3.2.1. Method of Moments (MoM)

The Method of Moments (MoM) or Boundary Element Method (BEM) is a numerical method of solving linear partial differential equations which have been formulated as integral equations.

In the electric field is applicable to problems involving currents on metallic and dielectric structures and radiation in free space. The structures are electrically small and are typically made of metals, although special extensions allow the inclusion of dielectrics.

This technique makes the analysis of very low frequency, decomposing the problem space with special basis functions, nowadays the commercial software’s computed the current distribution on the object under test for frequencies as low as 0.001 Hz.

The drawback of this method is the resolution under high frequency shapes as could be the lightning waveform, for this reason FDTD technique was selected.

### 3.2.2. Finite Difference Time Domain (FDTD)

The FDTD solution has gained popularity in computational electromagnetics (CEM) over the past decade due to its relatively straightforward and efficient formulation.

The FDTD method is well suited to modeling inhomogeneous materials Fig. 47 and simulation of wide band antennas and high frequency waveforms.
The FDTD method uses a volume meshing technique that employs voxels to accurately mesh the computational space. Voxels are calculated on a non-uniform rectilinear mesh: the mesh can be locally refined in regions of geometric detail or in areas where high field gradients are expected.

The method is conditionally stable and the time step must be calculated from the smallest mesh cell to guarantee stability. Including small geometrical features will reduce the time step, resulting in longer computational time.

4. Introduction to FDTD Method

4.1. Yee cell

In 1966, Yee originated a set of finite-difference equations for the time dependent Maxwell’s curl equations system. These equations can be represented in discrete form, both in space and time, employing the second order accurate central difference formula. This technique consists on divide the geometry that we want to study into cells to form a grid in a three dimensional geometry.

In the following Fig.36 on the left side, there are shown two bricks and one sphere into a grid, and, on the right side, those objects are divided into cells. This allows using Yee technique.
The complete grid is composed of \((N_x \times N_y \times N_z)\) cells. A unit cell of this grid is called Yee cell. Using rectangular Yee cells, a quite approximation of the surface and internal geometry of the structure of interest is made with a space resolution set by the size of the unit cell.

The discrete spatial positions of the field components have a specific arrangement in the Yee cell, as demonstrated in Fig. 37. The electric field vector components are placed at the center of the edges of the Yee cells and orientated parallel to the respective edges, and the magnetic field vector components are placed at the centers of the faces of the Yee cells and are oriented normal to the respective faces.

Figure 37 shows the indices of the field components, which are indexed as \((l,j,k)\) associated with a cell indexed as \((i,j,k)\). For a computational domain composed of uniform Yee cells having dimensions \(\Delta x\) in the \(x\) direction, \(\Delta y\) in the \(y\) direction and \(\Delta z\) in the \(z\) direction, the actual positions of the field components with respect to an origin coinciding with the position of the node \((1,1,1)\) can be easily calculated as:
\[ E_x(i,j,k) = ((i-0.5) \Delta x, (j-1) \Delta y, (k-1) \Delta z) \]
\[ E_y(i,j,k) = ((i-1) \Delta x, (j-0.5) \Delta y, (k-1) \Delta z) \]
\[ E_z(i,j,k) = ((i-1) \Delta x, (j-1) \Delta y, (k-0.5) \Delta z) \]
\[ H_x(i,j,k) = ((i-1) \Delta x, (j-0.5) \Delta y, (k-0.5) \Delta z) \]
\[ H_y(i,j,k) = ((i-0.5) \Delta x, (j-1) \Delta y, (k-0.5) \Delta z) \]
\[ H_z(i,j,k) = ((i-0.5) \Delta x, (j-0.5) \Delta y, (k-1) \Delta z) \]

The FDTD algorithm samples and calculates the fields at discrete time instant, however the electric and magnetic field component are not sampled at the same time instants. The magnetic field components are sampled at time 0.5\( \Delta t \), when electric field components are sampled at \( \Delta t \). Therefore, the electric field components are calculated at integer time steps, and magnetic field components are calculated at half-integer time steps.

The material parameters (permittivity, permeability, electric and magnetic conductivities) are distributed over the FDTD grid and are associated with field components.

Maxwell’s equations that are given in scalar forma can be expressed in terms of finite differences. For instance:

\[
\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_x} \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma_x E_x - j \xi \right)
\]

The derivatives in this equation can be approximated by using the central difference formula with the position of \( E_x(i,j,k) \) being the center point in time. Considering the field component positions given in Fig. 38 the following expression can be written.

**Fig. 38 Field components around \( E_x(i,j,k) \)**
\[ E_{x}^{n+1}(i,j,k) = E_{x}^{n}(i,j,k) - \frac{\Delta t}{\varepsilon_{x}(i,j,k)} \left( \frac{H_{y}^{n+0.5}(i,j,k)-H_{y}^{n+0.5}(i,j-1,k)}{\Delta y} - \frac{H_{y}^{n+0.5}(i,j,k)-H_{y}^{n+0.5}(i,j,k-1)}{\Delta z} \right) \]
\[ + \frac{1}{\varepsilon_{x}(i,j,k)} \frac{\Delta t}{\mu_{x}(i,j,k)} \left( \frac{E_{y}^{n+0.5}(i,j,k+1)-E_{y}^{n+0.5}(i,j,k)}{\Delta z} - \frac{E_{y}^{n}(i,j+1,k)-E_{y}^{n}(i,j,k)}{\Delta x} \right) \]

\[ H_{x}^{n+0.5}(i,j,k) = H_{x}^{n-0.5}(i,j,k) + \frac{\Delta t}{\mu_{x}(i,j,k)} \left( E_{y}^{n}(i,j,k+1) - E_{y}^{n}(i,j,k) \right) \]

These expressions express the future components in terms of the past components in X direction, it will be remains the same study in Y and Z direction for resolve the total electric and magnetic field.

### 5. FDTD ADJUSTMENT FOR SIMULATING WIND TURBINE BLADES

#### 5.1. Introduction

As it has seen before, FDTD is able to solve the most part of ECM problems. However, one of the most drawbacks is the difficulty for understanding and implementing the software.

In the following chapter, it will be explained all the adaptation that a normal FDTD code should has for simulating wind turbines blades. These differences are shown in the following points.

- Lightning surge
- Blade’s longitude
- Internal down conductor
- Boundaries
- Materials

#### 5.2. Lightning surge

There are two possibilities for simulating the lightning current, both of them proposes a linear combination of exponential functions in the time domain, which is afterward analytically transformed into the frequency domain.

The most simple model is using a Bi-exponential function, one exponential with their alpha coefficient controls the waveform rise time whereas the other exponential and beta coefficient control the fall-down time, term \( I_{0} \) defines the amplitude of waveform, Fig. 36.

\[ I(t) = I_{0} \cdot \left( e^{-\alpha \cdot t} - e^{-\beta \cdot t} \right) \]

**Fig. 36 Biexponential function, with \( I_{0} = 33.7 \cdot 10^{5} \) A, \( \alpha = 9.23 \cdot 10^{3} \) and \( \beta = 4 \cdot 10^{6} \)**
The second method for simulating the lightning current surge is using Heidler’s functions, this method is more accurate than biexponential function, although this time the coefficients for control the rise and falling-down time are called \( \tau_{11} \) (rise time) and \( \tau_{21} \) (fall-down time), Fig. 37.

![Fig. 37 Heidler’s distribution simulation with different \( T_{11} \) and \( T_{21} \) (44) and formula.](image)

\[
I(0, t) = \frac{I_0}{\eta_1} \cdot \frac{\left( \frac{t}{\tau_{11}} \right)^n}{1 + \left( \frac{t}{\tau_{21}} \right)^n} \cdot e^{-\left( \frac{t}{\tau_{21}} \right)}
\]

\[
\eta_1 = -e^\left[ \frac{\tau_{11}}{\tau_{21}} (n \cdot \tau_{21})^{\frac{1}{n}} \right]
\]

Were:

- \( I_0 = I_s \) the peak amplitude
- \( \tau_{11} y \tau_{21} \) are the coefficients for controlling the rise and fall-down time.

Depending on the type of waveshape, IEC 60060-1 defines the front time \( (T_1) \) and the time to half value \( (T_2) \) as it can be shown in Fig. 38.

![Fig. 38 Impulses defined by IEC 60060-1.](image)

<table>
<thead>
<tr>
<th>waveshape</th>
<th>Front time ( T_1 )</th>
<th>Time to half value ( T_2 )</th>
<th>Peak value</th>
<th>Polarity reversal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/20</td>
<td>1 ( \mu s ) ( \pm 10% )</td>
<td>20 ( \mu s ) ( \pm 10% )</td>
<td>( \pm 10% )</td>
<td>20%</td>
</tr>
<tr>
<td>4/10</td>
<td>4 ( \mu s ) ( \pm 10% )</td>
<td>10 ( \mu s ) ( \pm 10% )</td>
<td>( \pm 10% )</td>
<td>20%</td>
</tr>
<tr>
<td>8/20</td>
<td>8 ( \mu s ) ( \pm 10% )</td>
<td>20 ( \mu s ) ( \pm 10% )</td>
<td>( \pm 10% )</td>
<td>20%</td>
</tr>
<tr>
<td>30/80</td>
<td>30 ( \mu s ) ( \pm 10% )</td>
<td>80 ( \mu s ) ( \pm 10% )</td>
<td>( \pm 10% )</td>
<td>20%</td>
</tr>
</tbody>
</table>
5.2.1 Current source

Modeling FDTD voltage sources is straightforward due to the electric field $E$ appears explicitly in the standard FDTD equations. However, modeling FDTD current sources is not so easily to understand because the current is usually not explicitly included.

In this case, is necessary to start with the integral form of the:

$$\int H \cdot dl = \int J \cdot dS + \int \frac{\varepsilon dE}{dt} \cdot ds$$

The FDTD representation of the integral form of Maxwell’s equations is shown in Fig. 60. The current enclosed by the loop of $H$ fields is equal to the current density $J_s$ multiplied by the area $\delta x$, $\delta y$ and $\delta z$. The FDTD average current density $J_s$ is used to represent the current element $I \cdot dl$ average over the volume.

![Fig. 60 FDTD representation of Maxwell’s equations [52]](image)

The Figure 61, illustrates a current sources with a magnitude of $I_s$ and the internal resistance is representate with $R_s$. This time the relation between voltage and current can be written as

$$I = I_s + \frac{\Delta V}{R_s}$$

![Fig. 61 Current source in FDTD models][45]
The equations this time is the same that with the voltage source replacing the $V_s(i,j,k)$ term in by $R_s \Delta n^{0.5}(i,j,k)$.

With the aim of decrease the FDTD error, Dale.N et al [52] shows that the best way for including the current source will be on the edge of a Yee cell as is illustrate in Fig. 60.

5.2.2 Voltage source

Voltage sources are typically modeled in FDTD formulations by either of two methods: 1 replacing the calculated electric field $E$ on a Yee-cell by the source $E$ at every time step, or the second way is adding the source to the FDTD. The first method appears to be more commonly used taking into account which kind of source to use is important due to their effects on the system are quite different, for example replacing system may cause reflection of waves propagating back to the source location while the second method can be transparent to these incoming waves.

The way to implement voltage sources in the FDTD method is considering the following equation:

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_x} \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma_x E_x - jI_x \right)$$

This equation is the relation between the current density flowing in the X direction and the electric and magnetic field vector components. Applying of the central difference method and space derivatives is obtained the temporal value of the electric field in the X direction.

$$E_x^{n+1}(i,j,k) = E_x^n(i,j,k) - \frac{\Delta t}{\varepsilon_x(i,j,k)} \left( \frac{H_z^{n+0.5}(i,j,k) - H_z^{n+0.5}(i,j-1,k)}{\Delta y} - \frac{H_y^{n+0.5}(i,j,k) - H_y^{n+0.5}(i,j,k-1)}{\Delta z} \right) - \frac{1}{\varepsilon_x(i,j,k)} jI_x^{n+1/2}(i,j,k)$$

As shown it the figure 59, the FDTD voltage sources has to be implemented the internal resistances of the voltage with the aim of ensure an accurate simulation. The variable $V_s$ is a time-varying function with a predetermined waveform while $\Delta V$ is the voltage difference between the nodes that the source is connected.

![Fig. 59: Voltage source in a FDTD model [45]](image)

Applying Ohm’s law, the following formulation can be written as:

$$I = \frac{\Delta V + V_s}{R_s}$$
Although, the formulation that FDTD uses has to be changed in a discrete form at time step $\Delta t$, for this reason the current flowing “in this case in X direction” through the surface enclosed by the magnetic field components can be expressed in terms of

$$I^{n+0.5}(i,j,k) = \Delta y \Delta z J_{ix}^{n+\frac{1}{2}}(i,j,k)$$

Updating these 3 equations at every time step is how the FDTD software is able to simulate a voltage source in the X direction.

5.3. Blade’s longitude

The FDTD algorithm samples the electric and magnetic field at discrete points both in time and space. The choice of the period os sampling ($\Delta t$ in time, $\Delta x$, $\Delta y$, $\Delta z$ in space) must comply with certain restrictions to guarantee the stability of the solution.

The numerical stability of the FDTD method is determined by the CFL condition “Courant-Friedrich-Lewy”, this factor determine the duration of the time step. This control of the time step duration is due to the result error from a numerical truncation of a real number, in Fig.39 it can be observe the time-step domain grid with error $\varepsilon$ propagating with $\lambda=1$, where $\lambda$ is $\Delta t/\Delta x$, in one dimensional grid.

![Fig. 39: The time-domain grid with error $\varepsilon$ propagating with $\lambda=1$](image)

The error will keep propagating in this time domain algorithm, when $\lambda=2$, the propagation of error as shown in Fig.40.

![Fig. 40: The time-space domain grid with error $\varepsilon$ propagating with $\lambda=2$](image)
As it is easily to understand, finally the error will be large enough and will destroy the model. As a result, the time-domain algorithm will not give an accurate result due to a very small initial error.

For a cubical spatial grid, the CFL condition reduces to:

\[ \Delta t \leq \frac{\Delta x}{c\sqrt{3}} \]

With this relation, we are able to calculate the Time-Step of the FDTD software for the wind turbine blades with the following input data for every different lightning impulse, as example it will be shown the calculation for a 1.2/50 \( \mu \)s impulse.

The frequency of this impulse is the order of 1 Mhz although it will be use 10 Mhz due to the risk of missing information is lower when the software frequency is 10 times greater than the real one.

\[ F = 10 \text{ MHz} \]

\[ T = \frac{1}{F} = \frac{1}{10 \times 10^6} = 0.0000001 \text{ seg} \]

Using the formula for a cubical spatial grid, we obtain the maximal longitude of simulating (x):

\[ \Delta x \geq \Delta T \cdot c \cdot \sqrt{3} = 0.0000001 \cdot 3 \cdot 10^8 \cdot \sqrt{3} = 51.96 \text{ meters} \]

This result shows that \( \Delta t \) should be 51.96 m for simulating 10 MHz, with this numbers is impossible to simulate the blade. Notice that, whether \( \Delta t \) is 0.01 m that is a reasonable value for simulating the blade, the number of steps for simulating 100 \( \mu \)s will be:

\[ \Delta t \leq \frac{\Delta x}{c\sqrt{3}} \]

\[ \Delta t \leq \frac{0.01}{3 \times 10^6 \sqrt{3}} = 1.92 \times 10^{-11} \text{ seg} \]

Total simulation time = 100 \( \mu \)s

\[ N. \text{ of time steps} = \frac{\text{Total simulation}}{\Delta t} = \frac{100 \times 10^{-6}}{1.92 \times 10^{-11}} = 5.208.333 \text{ iterations} \]

5.4. Internal down conductor

The protection of the wind turbines blades against lightning struck is done by a receptor at the top of the blade Fig 41, and a down conductor, that connect the receptor with the grounding electrodes at the bottom of the wind turbine.
Fig. 41: Typical segmented lightning diverter configuration [46]

The presence of CFRP for reinforcing the blades introduces a new set of problems to be dealt with in the design of the lightning wind turbine protection, because of when a blade wind turbine is struck by a lightning a huge amount of energy is transferred for the internal down conductor and for induction in CFRP laminates cause important energy dissipation [47] Fig. 42, which might result in mechanical stresses or even destroy de laminates.

Fig 42. Geometry for the evaluation of eddy currents in a CRP laminates [47]

For this reason, it is strongly recommended to simulate the current flowing for this conductor, Fig. 43 and also the induced current that can be received the CFRP laminates. This simulation can be done with the thin wire modeling, resolving the Faraday’s law given in integral form as:

\[-\mu\oint_s \frac{dH}{dt} \cdot ds = \oint_l E \cdot dl\]
5.5. Boundaries

Due to computational storage space is finite, the FDTD method problem space size is finite and needs to be truncated by special boundary conditions, the most used termination is called (PEC) Perfect Electric Conductor. However, many applications, such as scattering and radiation problems, require the boundaries simulated as open space.

In addition to this, another problem appears because of the imperfect truncation, the problem will create numerical reflection which will corrupt the computational results in the problem developed [48, 49]

With the aim of resolving this issue, the Perfectly Matched Layer (PML) was created by Berenger. PML is a finite-thickness special medium surrounding the computational space based on fictitious constructive parameters to create a wave impedance matching condition, which is independent of the angles and frequencies of the wave incident on this boundary. The PML boundaries can be attached to FDTD problems for every single face of the cell, as shown in Fig.44.

The problem starts because of every magnetic field needs the four surrounding co-plane electric field values, as it can be observed in fig.51
It is shown in Fig. 61 that every magnetic field can be calculated by the values of the electric field that already belong in the same cell, although the need of implementing boundaries starts when there are more than one cell due to it cannot be calculate de electric field, this problem is exposed in Fig. 62.

The only way that can be resolve this problem, is calculating all the electric values that can be calculated and enforce boundaries on those remaining.
The Fig. 62 demonstrated this problem, the electric field $E_x$ surrounded by a purple circle, is calculated by means of the magnetic field surrounded by green circles, but the electric field surrounded by an orange circle cannot be calculated due to remains one value of the magnetic field in the Z axis.

The two most common boundary conditions, as it has been introduced before are Dirichlet and Periodic Boundary conditions. Dirichlet boundary conditions just assume fields outside the grid are all zero. This is "sort of" the boundary condition you would implement if didn't do anything. Forcing the fields to zero outside the grid makes the outside of the grid a perfect electric conductor "PEC" or a perfect magnetic conductor "PEM" depending on what field type you forced to be zero. Obviously, you would get reflections from a PEC or PMC so this is why you get reflections in FDTD if you do nothing about boundaries.

For periodic boundary conditions, you assume the fields outside the left side of your grid are the exact same as the fields at the far right side of your grid. Thus, you just use the fields at the right side of the grid in place of the fields outside the left side of the grid, and also the other way around.

The Perfectly Matched Layer is a way of "building in" loss into the outer 10 or so layers of your grid to absorb outgoing waves. This prevents their reflection, but doesn't tell you what to do at the very outside of your grid. Typically Dirichlet boundary conditions are used in conjunction with the PML.

As Mur reported [51], absorbing boundaries must be carefully located to keep the model as small as possible to maximize efficiency while maintaining acceptable accuracy.

5.6. Materials

Another problem that FDTD has to deal is with the materials, it has been defined all the material properties as, relative permittivity $\varepsilon_r$, relative permeability $\mu_r$, electric conductivity $\sigma^e$, and magnetic conductivity $\sigma^m$ of every isotropic and homogeneous materials, and also the properties of the air.

The problem can be done when the material to simulated is not homogeneous and isotropic as could be CFRP composite, in this case the properties in all directions ($x, y, z$) has to be increasing the difficulty of the problem.

The following values are the typical conductivity values of composite materials.

<table>
<thead>
<tr>
<th>Conductivity Range (S/m)</th>
<th>$\sigma_x = 40000$</th>
<th>$\sigma_y = 200$</th>
<th>$\sigma_z = 10$</th>
</tr>
</thead>
</table>

Experimental approach for electrical characterization has two main drawback. First, the knowledge of the conductivity $\sigma_x$, $\sigma_y$, and $\sigma_z$ is most of the time insufficient to predict the resistance of different materials because these conductivities are imply in a complex manner.
Many works [54-56] deal with the modelling of the electrical and electromechanical behavior of CFRP materials due to carbon fibers are strained and broken gradually under mechanical loading inducing an increase of the electrical resistance. In this context, Park [54] introduces the concept of “electrical ineffective length” which is the typical length over which a broken fiber remains the capability of carrying current. Park uses a Monte Carlo technique to handle the random distribution of the contact points. Replacing them with contact resistors, Fig. 64 the composite is modelled by a DC network solved with Kirchhoff’s rules.

Finally, Zhang [57] studies the effective electrical properties of a conductive fibrous network consisting of short coated fibers studies the effective electrical properties of a conductive fibrous network consisting of short coated fibers. The methodology proposed can be used to predict electrical behaviour of fibrous materials including nanotubes. Short fibers are modelled as randomly distributed cylinders with thin conductive coating layers. The effective conductivity is deduced from a finite element discretization scheme. Monte Carlo simulations are performed to quantify the impact of the different material settings (fiber ratio, thickness of the coating layer, fibers orientation).

Multilayer composite materials are made of different plies of carbon fibers with various orientations to achieve the required thickness and mechanical strength. The most common sequence of plies in aeronautic industry is 0°/45°/90°/-45°, the composite being then considered as quasi-isotropic (QI).
In the Fig 65, it can be observe the principle of the wired.

5.7. Code development

Once all the parameters have been explained a time-marching algorithm can be constructed as illustrated Fig.48. The first step in this algorithm is setting up the problem space, including the objects, material types and sources, afterwards the program has to calculate de the electric and magnetic field in all the directions (x,y,z) and stored as arrays before the iteration started. Every iteration the magnetic are updated for instant \((n+0.5\Delta t)\), then the convolutional perfectly matched layer coefficients are calculated for the magnetic field. The following step is updating the electric field components every \((n+1)\Delta t\), in the problem space using regular updating equations derived and as the magnetic field recalculate this values applying the boundaries (CPML). Finally the program has to increment the time step in \(n+1\) until the last iteration arrives.
5.8. Simulations

In the following point, an example is simulated using the software developed. This example is a simulation of a resistor excited by a voltage source. Moreover, the basic software code for implementing this example will be explained.
5.8.1. Define problem space parameters

In this subroutine the users has to define the following points:

- Number of time steps
- Courant factor: This factor determines the duration of time step
- Dimensions of the unit cell: In X, Y and Z direction meters
- Boundaries: Has to define the boundary conditions parameters (PEC, PMC or PML)
- Material types: Has to define the materials which will be implemented and their electric and magnetic conductivity

As example, Fig. 50 shows the a part of the subroutine “Defined problem space parameters”

```
disp('defining the problem space parameters');

% maximum number of time steps to run FDTD simulation
number_of_time_steps = 5000;

% A factor that determines duration of a time step
% wrt CFL limit
courant_factor = 0.9;

% A factor determining the accuracy limit of FDTD results
number_of_cells_per_wavelength = 20;

% Dimensions of a unit cell in x, y, and z directions (meters)
dx = 1e-3;
dy = 1e-3;
dz = 1e-3;

% CPHL extends inside of the domain rather than outwards

boundary.type_xn = 'cpml';
boundary.air_buffer_number_of_cells_xn = 10;
boundary.cpml_number_of_cells_xn = 0;

boundary.type_xp = 'cpml';
boundary.air_buffer_number_of_cells_xp = 10;
boundary.cpml_number_of_cells_xp = 0;

% ===material types===
% Here we define and initialise the arrays of material types
% ep_x : relative permittivity
% mu_x : relative permeability
% sigma_x : electric conductivity
% sigma_m : magnetic conductivity

% air
material_types(1).ep_x = 1;
material_types(1).mu_x = 1;
material_types(1).sigma_x = 0;
material_types(1).sigma_m = 0;
material_types(1).order = [1 1 1];

% PMC: perfect electric conductor
material_types(2).ep_x = 1;
material_types(2).mu_x = 1;
material_types(2).sigma_x = 10000;
material_types(2).sigma_m = 0;
material_types(2).sigma_e = 0;
material_types(2).order = [1 0 0];

% PMC: perfect magnetic conductor
material_types(3).ep_x = 1;
material_types(3).mu_x = 1;
material_types(3).sigma_x = 0;
material_types(3).sigma_m = 10;
material_types(3).sigma_m = 0;
material_types(3).order = [1 0 0];
```

Fig. 50 Part of subroutine “Defined problem space parameters”
The Fig. 50 shows how the physical properties of the materials (relative permittivity, relative permeability, electric and magnetic conductivity) have been implemented.

Due to carbon fiber is characterized for being an anisotropic material, the program has to take this into account. For this reason, the user can adjust the electric conductivity in the three axes directions.

The part of the program responsible of configuring these parameters is presented in Fig 63 and 64.

In this subroutine the first step was defined the variables sigma_e_y and sigma_e_z, which regards the value that the user defined for every electric conductivity and material,

Afterwards, these variables have to be treated, as well as the electrical conductivity in X direction. The part of the subroutine which makes the calculations for every time step and cell is shown in the Fig 64.

5.8.2. Define problem geometry

In this section it will show how the user can define the objects that it will be simulated. This FDTD software is able to build three different kinds of objects (brick, prisms and sphere) in the three-dimensional Yee Grid. The user has to define the kind of object to simulate the position in Cartesian coordinate axes, if it is a sphere it will be to implement the radius or the second corner of the brick in Cartesian coordinates and finally the material type of the object.

In Fig. 51, a thin wire and also the PEC plates are defined.
5.8.3. Define sources and lumped elements

In this part of the main program, the user can define how many sources and lumped elements their system will be. The position of the elements works like the previous point, the elements has to be distributed over the cell using Cartesian coordinate system. Besides the parameters defining the positioning, some additional parameters as well are needed to specify the properties of these components. For voltage or currents source there are six directions in which the user can define it: xn, xp, yn, yp, zn or zp. Here p refers to positive direction and n refers to negative direction. The other lumped elements can be defined with the directions x, y and z. The other parameters needed to specify the elements are magnitude, resistance and in the case of sources the type of the waveform.

In this example, with the aim to simulate a lightning surge a biexponential waveform was simulated. The parameters of the voltage source and resistor are shown in Fig. 52.

```
% define a PEC plate
bricks(2).min_x = 0;
bricks(2).min_y = 0;
bricks(2).min_z = 4e-3;
bricks(2).max_x = 8e-3;
bricks(2).max_y = 2e-3;
bricks(2).max_z = 4e-3;
bricks(2).material_type = 2;

% define a thin wire
thin_wires(1).min_x = 0;
thin_wires(1).min_y = 3e-3;
thin_wires(1).min_z = 0;
thin_wires(1).max_x = 8e-3;
thin_wires(1).max_y = 3e-3;
thin_wires(1).max_z = 0;
thin_wires(1).radius = 0.25e-3;
```

**Fig. 51 Definition of PEC plate and thin wire**
5.8.4. Define waveform type

As it can be shown in the previous point, the user is able to define the waveform type. The subroutine “initializing source waveforms” is the place where the waveform is set, in the Fig.52 it can be observed two kinds of waveforms: sinusoidal and biexponential.

![Fig. 52 Defining the source waveforms](image)

These are the basics parameters that the user should defined for simulating any problem; afterwards the program can be executed.

5.8.4. Define output parameters

Obviously, the user also has to define what kind of results should the FDTD program shows. This can be done in the subroutine “define_output_parameters Fig 63”. Inside this part the electric and magnetic field, voltages and currents can be defined as output parameters.

The way this parameter is set up is exactly the same as the user defines the geometry, is necessary introduce the coordinates “X, Y and Z axes” that has to be evaluated, choosing between a point, surface or volume depending on the number of coordinates changes.
In the previous subroutine, it can be observed how the script in this subroutine. The part where the voltage is sample is defined with the index number 1, for this reason is between branches, this index configure the number of samples that the user defines.

In addition to this, in this case is sample a volume because every coordinate is different, if the users need to define a surface choosing the X axes as reference for example, min_x and max_x must has de same value.

Finally, it can be selecting the direction of the magnitude that the users' needs to represent. This is the same concept that the polarity of a source, in this case as example is selected the Z axes and positive way ‘zp’, the negative polarity in Z direction will be ‘zn’.

5.8.5. Simulation example
The program results gives different solutions, for example show the FDTD problem space geometry where the user can observed the disposition of the objects inside the Yee grid, Fig.53.
In this example it can be observed how the voltage source ($V_{s_1}$) is connecting the resistor ($R_1$) with two PEC plates, then a thin wire ($TW_1$) is modeled.

The current is measured with the sensor $SC_1$ “orange arrow” and the voltage with $SV_1$, Fig.54.

In addition to this, the software also show the distribution on relative permittivity, relative permeability, electric conductivity and magnetic conductivity in “k, i, j,” direction over the materials, the configuration of the display is done with the console show in Fig.55.
Fig. 55 Console for displaying the material mesh

Fig. 56 demonstrates the electric conductivity over the PEC plates.

Finally, it can be obtained the sampled voltage Fig. 57, the voltage source Fig. 57 and the sampled current Fig. 58.
Fig. 58 Sampled current of the system
6. CONCLUSIONS

Along this document the fundamental theory of the FDTD has been reviewed, including the most representative applications in which this method is applied showing the effectiveness and the accuracy of the solutions obtained by the FDTD method.

In addition to this, the most important techniques for resolving electromagnetic problems have been introduced, identifying the advantages and disadvantages of every method.

If a FDTD method is needed to be carried out in wind turbines, the most important points to take into account in this particular case are lightning surge, blade’s longitude, internal down conductor, boundaries and materials, all of them clearly affecting the result.

Particularly in the case of wind turbines, it will be a great challenge to model all the components of the turbine (blade, nacelle, earthing system and LPS) with FDTD method, with the aim of studying the induction current that CFRP material suffers during a lightning stroke, obtaining the maximal induced current that this material may resist.

Finally, the implemented FDTD code has been showed. Before displaying the specific code, the basic concepts to take into account when writing it have been explained, just to understand how the program works.

6.1. Accomplished objectives:
All the objectives have been accomplished along this thesis, the main goals were:

- Understanding the FDTD method
- Implementing the 3 conductivities into the software
- Realizing a FDTD software with accuracy results
- Implementing the display of the electric field

6.2. Not accomplished objectives:
In this thesis all the objectives have been accomplished

6.3. Possible improvements:
As possible improvements could be:

- Improvement in the CPU calculations: One of the main drawbacks of this method is the time that the system needs for realizing the calculations. Along this thesis, it can be observed that during the Matlab simulation the CPU use was around 35%. It will be a good improvement the increase of the percentage use during the calculations. This will reduce of the total time simulation.

This could be performed completing the core of the program due to is the part were realizes the calculations, in other computer language more efficient as could be Fortran or C. If this is performed, the modelation of the case should remain in Matlab, then the calculations will be transferred and processed in Fortran on C and finally transferred again in Matlab code for realize de Post-processing program “show the results”.
- Implementing the Current distribution: Another good improvement will be introduced the displaying of the current distribution over the geometry. This will be performed taking as a reference the electric field distribution and the electric properties of the material. With this implementation the user could observe how is the current distribution over the geometry, detecting quickly which part of the CFRP will be delaminated or whether affects the modification of the geometry.

- Modelation of the CFRP taking into account the spatial distribution of the fibers. The idea will be using Montecarlo’s method, fill the geometry with fibers of CFRP modeling also imperfections with high resistivity bricks.

- Modelation of the whole blade turbine taking into account the laminations of the CFRP and their assembly forming stacks. Paying attention in the connections with the internal down conductor.
7. ANNEX I

In the following annex I, is possible to observe different simulations done with the FDTD software, with the aim of validating the program whether the measures are correct.

7.1. Current source, thin wire and resistance. Current measurement

The first simulations made with the FDTD software were performed with very simple circuits, because of the purpose was to know very well that the results obtained were correct.

In the following screen capture, it can be observe a circuit compose of a current source with a resistance connected by means of two thin wires. The source and the resistor have a PEC surface which connects the element with the thin wire.

The results of the program shows the waveforms generated in a red graph. The waveform generated is define for the user and could not be the same that the waveform implemented “blue graph” in the circuit. For this reason, is a good practice compare the waveform generated with the waveform implemented.

As example, in the figure 66 can be observed the biexponential generated with a peak of 0.7 amperes.
Afterwards, were implemented two different samples of the current in the source. The difference between them was the volume that was measuring. Figure 67 displays the current flowing through the source taking as a reference the “X” plane and the “Z” direction.

For contrasting this value, was measured the current through all the source’s volume Fig. 67 instead of “X plane “, obtaining as a result practically the same waveform than in the previous simulation.

Another sampled current was performed along the thin wire, Fig. 68 and 69 represents the current flowing through the thin wire taking as a reference two different points. These figures show that the current in both points are exactly the same and equals in amplitude and time respecting the source as a reference.

*Fig. 67 Current sample in source “X axes and Z direction”*

*Fig. 67 Current flowing through source volume*

*Fig. 68 and 69 Sampled current through the thin wire*
Finally, only remains to simulate the current flowing through the resistance and their PEC connections. The different between Fig 70 and 71 is the surface that the program is sampling. Fig. 70 was performed with the aim of evaluating the current that was flowing through the resistance in the Z direction and “positive way”, taking the plane X as a reference. On the other hand, Fig. 71 represents the current flowing in the Z direction and “positive way” taking all the resistance volume as a reference. Due to the resistance is not performed as an anisotropic material, the conductivity in all directions have the same value, for this reason both samples are identically.

The following graph “Fig.72 and 73” represents the current in the resistor’s PEC connections.
7.2. Current source, thin wire and resistance. Voltage measurement

Another fundamental point of this program is also guarantee that the measurement of the voltage is done correctly. For this reason was simulated the same circuit than in the previous case but sampling the voltage in some points, Fig.74.

In the same way than in the previous case, it was select two points for evaluating the voltage measurement in the current source. The value of this measurement is related with the internal impedance of the source as it was explained in the point 5.2.2.

The first point for evaluating the voltage was taking as reference all the current’s volume Fig.75 and it can be compared with the voltage measure in the vertical thin wire Fig.76

Due to both graphs shows the same voltage response was validated the way of obtaining the voltage in the source. In the following pictures is display the voltage in the thin wire Fig.77 and also the voltage in the resistance Fig.78, in both charts it can be observe a drop voltage in amplitude an attenuation of the frequency at the peak value corresponding to the voltage drop in the line.
7.3. Three Axes conductivity implementation.

Already was implemented the 3 axes conductivity in the software as it was explained in the point 5.8.1, the way of testing this modification was decomposing the current waveform through the brick in 3 axes direction and then changed the electric conductivity and observed how was the current flowing after these modifications, in the Fig. 79 is display the current conductivity through a brick which is not anisotropic material.
Obviously, the high module of current has to flow in Z direction Fig.80 “vertical way” because of the geometry of this circuit forces the current to flow in this direction due to is the easy way for ensuring the electrical conductivity of the circuit. Figures 81 and 82 displays the current flowing into the brick in Y and X direction.

![Sampled current](image)

**Fig. 81** Current flowing in Z direction inside an Anisotropic brick

![Sampled current](image)

**Fig. 82** Current flowing in Y direction inside an Anisotropic brick

Firstly, it was defined the electric conductivity in Z axes with 0 value, afterwards was displayed the graphs 81, 82 and 83 again Fig. 83, 84 and 85.
The results clearly showed that the current flowing in the Z direction was nearly 0 (Fig. 83), as a consequence the currents was flow using X way direction (Fig. 85) and in the major part Y way direction (Fig. 84).

### 7.4. Longitude of the thin wires

Another interesting point that it was evaluated was how affects the longitude of the thin wires in the electrical circuit, with impulses with a quick inrush.

A 200 volt biexponential waveform was simulated using the circuit display in the Fig. 74 although the thin wires were extended 10 time their normal longitude.

In the figure 86, is shown the sampled voltage in the current source. It can be noticed the electrical reflexions that appears at time step 3.5 ns and double the amplitude of the normal waveform.
Figure 87 display the voltage that appears in the resistance, it denotes nearby 2 ns of delay respecting the voltage source, and the amplitude voltage has decrease considerably.

Observing the current in this same circuit and positions, when appears the reflexion an overcurrent peak is display at 3.7 ns Fig.88, the delay is also appreciate in the resistance, although this the current amplitude in the resistance is two time the current appears in the source, Fig.89.

7.5. Impulse with a duration of ms

All the impulses realized in the previous points were about a few ns, taking as consideration that the lightning impulses has a duration of 50 ms, it was considered do this simulation. The duration of the waveform is related with the number of Time-steps and also the size of the cell.

The simulation was performed with a simple circuit as it is demonstrated in Fig.90, the green brick are use as PEC connections and the red brick is normal brick.
The results of this test are shown in the following figures:

- Figure 91: Current source generated
- Figure 92: Current source implemented
- Figure 93: Current in PEC connection of the source (X direction)
- Figure 94: Current in thin wire
- Figure 95: Current in PEC connection of the brick (X direction)
As it is demonstrate all the figures has the same waveform like the waveform generated, except the Fig. 93 that must be a waveform with amplitude 0, due to the way of the current is in Z direction.

7.6. Impulse simulating a real CFRP

The final geometry simulation was performed with real dimensions of a CFRP plate “60 cm length, 21 cm width and 3 cm high” Fig. 98 and 99. Simulating two PEC electrodes installed at the top connecting the current source with the CFRP plate.
7.7. Display the electric field over the geometry simulation

Finally, it was implemented a code which is able to show the distribution electric field in magnitude or in “Z,X and Y” direction taking as a reference a plane. This is a very useful tool for evaluating the voltage different potential that can be suffering a specific region of the geometry which can originated the delamination of the material.

As example, in the Fig.100 it can be observed a red brick which is connected to a current source by means of PEC connectors (green bricks) and twin wires.

It was configured to the program that shows the module of the electric field at the top face of the brick, and in the figure 101 and 102 are the results.
The result clearly demonstrate how the electric field are distributed over the brick connecting both PEC connections, and it is easily to representate where is the current source due to it is obtained most of the electric field part.
8. REFERENCES


