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# Use of Hydrodynamic Cavitation for Volatile Removal Compound

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## Abstract

Hydrodynamic cavitation and its feasibility for volatile compound removal in enclosed channels is discussed in this paper. Very high Reynolds numbers are needed to rupture liquid by decreasing its pressure below its saturated vapor pressure. Hence, a simple stratified flow, at which the two phases separate, is precluded in vertical and horizontal tubes, where turbulence stresses will be much larger than the buoyant forces. The most probable flow regime at this high turbulence regime is a bubble- or annular flow, where the volatile matter tends to concentrate in the centre of the pipe because of the lift force resulting from the unequal flow of the viscous liquid around the bubbles in the presence of the pipe wall. Therefore, boiling the volatile matter for volatile compound removal is not enough if hydrodynamic cavitation is pursued. The attainable efficiency must also be assessed. An expression for the volatile removal efficiency and the main parameters affecting this efficiency were derived by utilising a simplified geometrical and physical model. The efficiency was found to approximate a power law as a function of the volatile concentration and its strong dependence on the size of the volatile bubble reasonably well. This result implied the need of bubble growth and the limitation of the process for highly concentrate compounds to a few percent concentrations. With regard to energetic requirements, both thermal and hydrodynamic cavitations are quantitatively similar. Furthermore, the choice of one or another corresponds more to the kind of energy source available.

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## 1. Introduction

Volatilisation by heating methods is generally characterised by great simplicity and ease of operation, except when high temperatures or highly corrosion-resistant materials are needed. A compound can be volatilised by heating to increase its vapor pressure. Heating methods include the volatilisation of water, mercury, or arsenic trichloride to separate these substances from interfering elements. However, an alternative physical method for volatilisation that does not rely on liquid heating is possible. Indeed, volatilisation occurs when a liquid at a constant temperature is subjected to a decreasing pressure below the saturated vapor pressure. This phenomenon is called hydrodynamic cavitation.

There has been a revival in interest in hydrodynamic cavitation during the last few years on numerical (Boris et al 2017, Yin et al, 2016) as well as theoretical studies including the basic mechanism of pollutant degradation, modeling of pressure distribution in the cavitation reactor, and bubble dynamics models coupled with chemical reactions are evaluated (Arroyo and Benito, 2008). New results of laboratory experiments on the application of cavitation effects to decompose selected organic compounds which hardly undergo biodegradation have been presented, (Janusz 2012), application of hydrodynamic cavitation to wastewater treatment (Yuequn et al, 2016., Seechi Wqashio 2014) or in microbial cell disruption (Save and Joshi, 1997), are some of the most promising fields. However, the use of hydrodynamic cavitation as a sole technique or in combination with other techniques such as ultrasound has only recently been suggested and employed, ( Matevz et al. 2016).

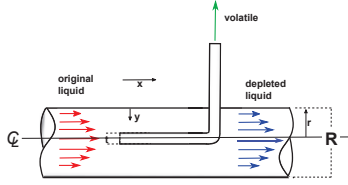
Despite that hydrodynamic cavitation processes are known to lead to very effective liquid degassing (Gogate and Pandit, 2011; Iben et al., 2015), to such a degree that it can be used for removal of undesired volatile compounds and therefore boiling becomes unessential, (Albanese et al, 2017). Nevertheless the volatile removal efficiency has not been addressed as far as the author knows. This study aims to analyse the theoretical efficiency of hydrodynamic cavitation as an attractive alternative method of thermal vaporisation.

By an analogy with heating volatilisation, it could be thought that boiling volatiles by decreasing the liquid pressure after increasing its velocity and relying on buoyancy forces (if horizontal pipes or channels are used) would be enough. Nevertheless, the situation is not that simple. First, in contrast with thermal volatilisation, hydrodynamic cavitation implies a very high Reynolds number depending on the degree of compound volatilisation (i.e. on its saturated vapor pressure). The situation is even worse if open channels are used. For the sake of illustration, a velocity profile around  $\left[\frac{2(p_o-p_s)}{\rho_l}\right]^{\frac{1}{2}} \simeq 14\text{m/s}$  and a Reynolds number of  $\mathbf{Re} \approx 1.4 \times 10^5 D$ , where  $D$  is the pipe diameter in centimetres, would be necessary if a volatile compound initially dissolved into water at a room temperature of  $T = 300\text{K}$ , an atmospheric pressure of  $p_o = 10^5$  Pa with a density  $\rho_l = 10^3\text{kg/m}^3$  and a saturation pressure of  $p_s = 3 \times 10^3$  Pa (i.e. higher than the saturation pressure of water at this temperature) is desired to be removed. This simple illustrative example shows that we are in a very high turbulent regime for practical pipes with diameters around a few centimetres. This turns out in precluding a simple stratified flow, at which the two phases are separated, followed by straightforward removal. The flow regime at this high turbulence regime is a bubble or an annular flow, where the volatile bubbles tend to concentrate in the centre of the pipe because of the lift force resulting from the unequal flow of the viscous liquid around the bubbles in the presence of the pipe wall. The exact profile of the radial distribution function for the bubbles at that turbulent regime is not easy to obtain, and in principle, cannot be determined by pure theoretical treatment. Nevertheless, some insights into the relevant parameters affecting the efficiency of hydrodynamic cavitation volatile removal may be obtained by a simplified analysis. The real situation can also be improved using empirical modifications.

## 2. Theoretical Background

We first need to know the radial distribution function for the bubbles inside a pipe in the flowing liquid to calculate the efficiency of the volatile removal by hydrodynamic cavitation.

For the sake of generality, let us consider a pipe (Fig. 1), where the pressure declines to some value below the saturated vapor pressure of the volatile compound desired to be removed because of the liquid velocity. The con-



**Fig. 1.** Physical model for the hydrodynamic cavitation extraction of volatiles.

centration profile in a pipe could be approximated as follow as a power law distribution (Bankoff 1960):

$$c^* = s^{\frac{1}{n}} \quad (1)$$

70 where  $c^* = \frac{c}{c_m}$  with  $c_m$  is the maximum concentration of the gas at the centreline;  $s = \frac{y}{R}$  being  $y$  is the coordinate from the wall (Figure 1);  $R$  is the pipe radius; and  $n$  is a positive constant depending on the liquid velocity profile,  $n \rightarrow \infty$  when  $u_l \rightarrow 0$  and  $n \rightarrow 0$  when  $u_l \rightarrow \infty$  with  $u_l$  as the liquid velocity. This constant will be discussed in a later section.

75 The minimum velocity in the pipe should be the critical velocity, in which cavitation occurs (for the volatile compound), to produce hydrodynamic cavitation through the pipe. The velocity profile in the pipe is provided as follows by the well-known logarithmic velocity law:

$$u_l = u_m \left[ 1 - \frac{1}{\kappa} \sqrt{\frac{f}{2}} \ln \frac{R}{y} \right] \quad (2)$$

80 where  $u_l$  is the liquid velocity in the  $x$ -direction at a distance  $y$  from the wall;  $u_m$  is the velocity of the two-phase mixture at the tube centreline;  $\tau$  is the wall shear stress in the two-phase system;  $\kappa$  is a universal constant; and  $f$  is the friction factor.

The velocity (i.e.  $u_c$ ) through the entire pipe must be at least the critical velocity, at which volatile cavitation occurs, to produce the total cavitation of the volatile. An induced cavitation generally implies working with high velocities and at the turbulent regime. Hence, a very thin velocity boundary layer  $\delta_v$  would exist. This layer is defined as the distance from the wall, at which the velocity is 99% of the velocity found at the centre. The thickness of the very thin boundary layer is approximately in micrometres. Hence, for practical purposes, we assume that all the volatiles compound will boil when cavitation is imposed at this tiny distance from the wall. The condition for full cavitation may be approximated by imposing the following:

$$u_m \simeq 1.01u_c \quad (3)$$

The velocity at the centreline is 101% higher than the critical velocity. The critical velocity is provided when the following hydrodynamic condition is accomplished:

$$\frac{\rho_l u_c^2}{2} = p_o - p_v \quad (4)$$

where  $p_o$  is the initial pressure of the liquid and  $p_v$  is the vapor pressure of the volatile to be separated.

The volatile removal efficiency may be defined as follows: the maximum extraction of the volatile occurs when the gas concentration attains its maximum theoretical value. Let us call this as  $\rho_g$ , which is directly related with the gas density. The total amount of gas could be located in a minimum volume at a radial distance  $t$  from the centreline (Fig. 1) calculated as follows:

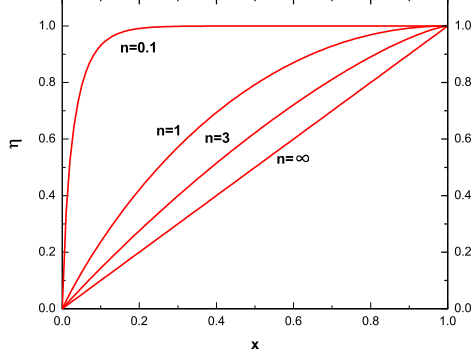
$$\bar{c}\pi R^2 = \rho_g \pi t^2 \quad (5)$$

where  $\bar{c}$  is the average gas concentration across the cross-section. Thus, the radial distance is calculated as follows:

$$t = \sqrt{\frac{\bar{c}}{\rho_g}} R \quad (6)$$

Meanwhile, the total amount of gas  $Z_g$  inside the volume  $t$  for a given concentration profile is given by:

$$Z_g = 2\pi R^2 c_m \int_{1-\frac{t}{R}}^1 c^*(1-s) ds \quad (7)$$



**Fig. 2.** Plot of  $\eta$  as a function of  $x$  calculated from Eq.(15 for various values of  $n$  calculated from Eq.(15)

The efficiency  $\eta$  is obtained by dividing this expression by its maximum theoretical value (i.e.  $Z_g = \pi t^2 \rho_g$ ) yielding:

$$\eta = \frac{2R^2 c_m}{t^2 \rho_g} \int_{1-\frac{t}{R}}^1 c^*(1-s) ds \quad (8)$$

For practical purposes, the concentration at the centreline can be assumed  
 110 to attain its maximum permissible (i.e.  $c_m = \rho_g$ ). One obtains the following  
 equation by inserting Eq.(1) into Eq.(8) and after integration:

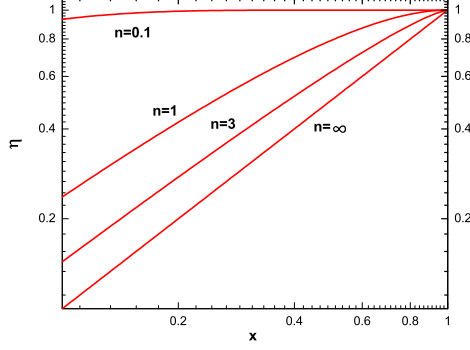
$$\eta = \frac{2c_m \Psi^2 n^2}{\rho_g (1+n)(1+2n)} \left[ 1 - \left( \frac{1+2n}{n} \right) \Phi^{1+\frac{1}{n}} + \left( \frac{1+n}{n} \right) \Phi^{2+\frac{1}{n}} \right] \quad (9)$$

where,

$$\Psi = \sqrt{\frac{\rho_g}{\bar{c}}} \quad (10)$$

and

$$\Phi = 1 - \sqrt{\frac{\bar{c}}{\rho_g}} \quad (11)$$



**Fig. 3.** Logarithmic plot of  $\eta$  as a function of  $x$  calculated from Eq.(15 for various values of  $n$

Eq.(9) may be simplified by considering that the average concentration  
 115 of the volatiles is:

$$\bar{c} = 2c_m \int_0^1 c^*(1-s)ds \quad (12)$$

The following equation is obtained after integration:

$$\frac{\bar{c}}{c_m} = \frac{2n^2}{(1+n)(1+2n)} \quad (13)$$

The following formula is obtained when the above equation is inserted into Eq.(9):

$$\eta = \left[ 1 - \left( \frac{1+2n}{n} \right) \Phi^{1+\frac{1}{n}} + \left( \frac{1+n}{n} \right) \Phi^{2+\frac{1}{n}} \right] \quad (14)$$

or

$$\eta(x) = \left[ 1 - \left( \frac{1+2n}{n} \right) (1-x)^{1+\frac{1}{n}} + \left( \frac{1+n}{n} \right) (1-x)^{2+\frac{1}{n}} \right] \quad (15)$$

120 As will be discussed in the next section, the value of parameter  $n$  is dependent on the liquid velocity, physical properties, geometrical factors, pipe radius, and bubble size. The efficiency of the volatile separation by hydrodynamic cavitation will only be dependent on the volatile concentration when these parameters are fixed.



125 The curve shape predicted by Eq.(15) is plotted in Fig. 2 as a function  
of the initial concentration of the volatile  $\frac{\bar{c}}{\rho_g}$  and some values of  $n$ . A loga-  
rithmic plot of the same curves (Fig. 3) shows that they are reasonably well  
approximated by straight lines. The efficiency may be approximated by a  
power law as follows:

$$\eta \simeq x^{\frac{1}{m}} \quad (16)$$

130 where  $m$  is a positive constant.

### 3. Calculation of parameter $n$

A power law for the concentration profile of the volatile is assumed in  
the preceding section (Eq.(1)). The justification and the quantification of  
parameter  $n$  are discussed in this section.

135 Let us assume small volatile bubbles of radius  $a$  located in a liquid of  
infinite extent at a distance  $y$  from a vertical wall. The liquid streams past  
the bubble if the co-ordinate axes are fixed at the centre of the bubble. The  
viscous liquid flows less rapidly around the bubble on the side closest to the  
wall. Hence, a lift force is developed in addition to the drag force, which  
140 propels the bubble away from the wall. With these conditions, the bubble  
concentration is provided as follows by the relationship found by (Bankoff  
1960):

$$\ln \frac{c^*}{c_m} = C_1 \left( \frac{s-1}{s} \right) + \frac{C_2 \ln s}{s} \quad (17)$$

where,

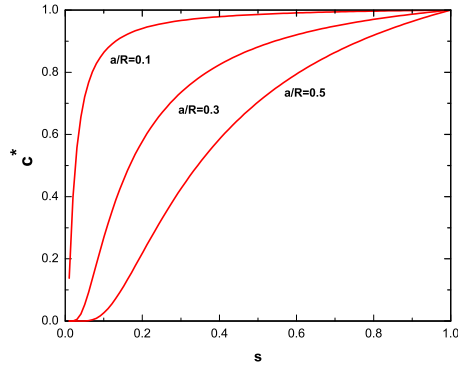
$$C_1 = \frac{Au_m}{R} \left[ 1 - \frac{1}{\kappa} \sqrt{\frac{f}{2}} \right] \quad (18)$$

and

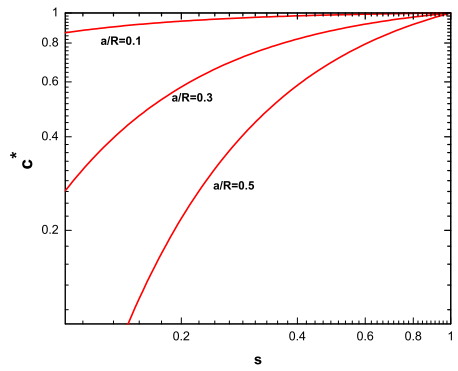
$$C_2 = \frac{A}{R} \sqrt{\frac{f}{2}} u_m \quad (19)$$

145 Parameter  $A$  is presented by:

$$A = \frac{a^2}{3\kappa^2\nu_l} \quad (20)$$



**Fig. 4.** Radial compound concentration  $c^*$  from Eq.(20) considering various values of the bubble radius



**Fig. 5.** Logarithmic plot of the radial compound concentration  $c^*$  from Eq.(20) considering various values of the bubble radius

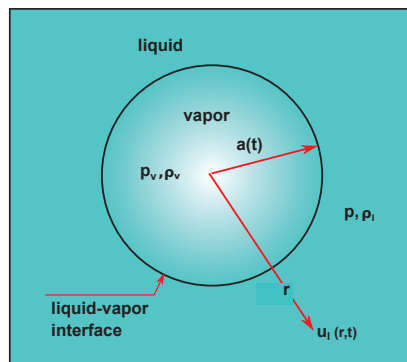
where  $a$  is the bubble radius;  $\nu$  is the kinematic velocity of liquid; and  $\kappa$  is a non-dimensional universal constant.

We assume a hypothetical case to obtain some idea on the curve shape predicted by Eq.(17). In this case, extracting a certain compound mixed in water by hydrodynamic cavitation is desirable. The compound and water properties are as follows: for water  $\kappa = 0.4$  (Bankoff 1960);  $f = 0.005$ ;  $\nu = 0.016 \text{ m}^2/\text{s}$ ;  $\rho = 10^3 \text{ kg}/\text{m}^3$ ;  $\sigma = 7.0 \times 10^{-2} \text{ N}/\text{m}$ ; initial pressure  $p_o = 1.0 \times 10^5 \text{ Pa}$  ( $\approx 1 \text{ atm}$ ); and vapor pressure for the compound of  $p_v = 0.95 \times 10^5$  ( $\approx 0.5 \text{ atm}$ ), in which we obtain  $u_c \simeq 3.2 \text{ m}/\text{s}$  by applying Eq.(4) and from Eq.(3)  $u_m = 3.1 \text{ m}/\text{s}$  with a reasonable pipe radius  $R = 1.0 \text{ cm}$ . Fig. 4 shows the resulting curves for several values of the bubble radius. The very strong dependence of the curves on the bubble radius and the plot of the same figure in a logarithmic scale as in Fig. 5 show that the approximation of the power law is reasonable.

#### 4. Bubble radius

The very strong dependence on the bubble size in the efficiency of the separative process is discussed in the preceding section. The bubble size should be a small fraction of the pipe size to obtain reasonable efficiencies (Fig. 2). Therefore, considering that the initial bubbles have a micron size, it is easy to see that the growth of the bubble and the velocity is necessary.

Let us consider a bubble at an instant  $t$ , a radius  $a(t)$ , and a growth velocity  $\frac{da}{dt}$  schematically depicted in Fig. 6. Let us also assume that thermal effects are negligible (i.e. the gas temperature and the surrounding liquid is constant). This kind of bubble dynamic behaviour is termed as inertially controlled, and is a good approximation when the time does not exceed a certain critical time  $t_c$  (Appendix). Meanwhile, the volume rate of vapor production must be equal to the rate of the bubble size increase,  $4\pi a^2 \frac{da}{dt}$ . Thus, the evaporation mass rate must be equal to  $\rho_g 4\pi a^2 \frac{da}{dt}$ . This mass flow must be equal to the mass flow of the liquid inward relative to the interface. The inward velocity of the liquid relative to the interface is given by  $\frac{\rho_v}{\gamma \rho_l} \frac{da}{dt}$ , where factor  $\gamma$  is the volatile liquid fraction of the liquid considering that only this fraction is being evaporated. The radial outward velocity  $u(a, t)$  is



**Fig. 6.** Schematic of a spherical bubble in an infinite liquid volume.

given as follows:

$$u(a, t) = \left[ 1 - \frac{1}{\gamma} \frac{\rho_v}{\rho_l} \right] \frac{da}{dt} \quad (21)$$

180 The radial outward growth requires  $\rho_v < \gamma\rho_l$ . In many practical cases and in concentrations up to a small fraction of the phase desired to be separated, the assumption  $\rho_v \ll \gamma\rho_l$  is valid.

$$u(a, t) = \left[ 1 - \frac{1}{\gamma} \frac{\rho_v}{\rho_l} \right] \frac{da}{dt} \quad (22)$$

where  $u(a, t) \simeq \frac{da}{dt}$  when  $\rho_v \ll \gamma\rho_l$  which is an allowable assumption. With the abovementioned assumptions and being confined to the inviscid case, the asymptotic growth rate of the bubble is provided as follows by the solution  
185 of the Rayleigh–Plesset equation (Brennen 1995) :

$$\frac{da}{dt} \rightarrow \left[ \frac{2}{3} \frac{(p_v - p)}{\rho_l} \right]^{\frac{1}{2}} \quad (23)$$

The following is obtained after a period of acceleration:

$$t_a = \left[ \frac{2\rho_l a_o^2 (p_v - p)}{3(p_o - p)^2} \right]^{\frac{1}{2}} \quad (24)$$

where  $a_o$  is the initial bubble radius (around micrometres), and  $p_o$  is the initial pressure before being decreased to  $p$ .

190 It is easy to see that the bubble growth may be controlled by parameter  $\Delta p = p_v - p$  (i.e. tension), which can be done by increasing or decreasing the liquid velocity, then decreasing or increasing  $p$ . Therefore, the bubble velocity will be on 0.8 m/s if a volatile liquid initially diluted in water is desired to be separated with  $\rho_l = 10^3 \text{kg/m}^3$  and work in a water tunnel with  
195 a tension (e.g.  $10^3 \text{ Pa}$ ). Accordingly, above  $6 \times 10^{-3} \text{ s}$  would be required to reach a radius above 5 mm.

## 5. Energetic comparison with thermal evaporation

It would be interesting to perform a comparative study on the energy requirements for the volatile separation by using the studied hydrodynamic  
200 cavitation and the classical thermal evaporation or ebullition.

To begin with, the liquid temperature must be increased up to the temperature at which the vapor is formed if  $M$  is the amount of volatile initially dissolved into liquid at temperature  $T$  and pressure  $p_o$ , and it desires to boil first the volatile compound by heating the liquid. This temperature is termed as the supercritical temperature  $T_c$ . The required thermal energy is presented as follows:

$$W_T = (c_{pl}\Delta T + L) M \quad (25)$$

where  $\Delta T = T_c - T$ , and the latent heat is considered because of the phase transition.

It is convenient for our purpose to express  $\Delta T$  as a function of the critical superheat defined as follows:

$$\Delta T_c = T_c - T_s \quad (26)$$

where  $T_c$  is the critical temperature at which vapor is formed, and  $T_s$  is the normal saturation temperature. Subsequently, we acquire the following equation:

$$\Delta T = \Delta T_c + T_s - T \quad (27)$$

Eq.(25) then becomes:

$$W_T = [c_{pl}(\Delta T_c + T_s - T) + L] M \quad (28)$$

We need to perform mechanical work  $W_w$  to move a volume of the liquid through the pipe, which is nothing else than the pumping energy given as follows, if the volatilisation of the same amount  $M$  is desirable using hydrodynamic cavitation:

$$W_W = \Delta p^* A_c u t \quad (29)$$

where  $\Delta p^*$  is the pressure loss over the length of the pipe  $l$ ;  $A_c$  is the pipe area cross-section; and  $t$  is the time needed to move the liquid mass  $M$ .

Meanwhile, time is provided as  $t = \frac{l}{u}$  and  $A_c l \rho_l = M$ . Hence, Eq.(29) becomes:

$$W_W = \left[ \frac{\Delta p^*}{\rho_l} + L \right] M \quad (30)$$

where the latent heat considers the phase transition. The pressure loss over the pipe can be written as follows:

$$\Delta p^* = f \frac{l}{D} \frac{\rho_l u_l^2}{2} \quad (31)$$

225 where  $D$  is the pipe diameter. It is subjected to a decreasing pressure by increasing the velocity up to  $u_l$  (i.e. hydrodynamic cavitation) if the initial liquid pressure is  $p_o$ . The final pressure at which the cavitation occurs  $p_c$  is termed as the critical pressure. The difference between the pressure, at which rupture occurs (cavitation), and the saturated vapor pressure  $\Delta p_c = p_v - p_c$  230 is called the tensile strength of the liquid (Brennen 1995). We know by Bernoulli's theorem that:

$$p_o - p_c = \frac{\rho_l u_l^2}{2} \quad (32)$$

Then,

$$\Delta p_c = p_v - p_o + \frac{\rho_l u_l^2}{2}$$

$$\frac{\rho_l u_l^2}{2} = \Delta p_c + (p_o - p_v) \quad (33)$$

Finally, the tensile strength can be expressed as follows as a function of the superheat, except at that close to the critical point (Brennen 1995) :

$$\Delta T_c = \Delta p_c \frac{T}{L \rho_v} \quad (34)$$

235 Inserting Eq.(34) into Eq.(33) yields:

$$\frac{\rho_l u_l^2}{2} = \frac{\Delta T_c L \rho_v}{T} + (p_o - p_v) \quad (35)$$

Meanwhile, one obtains the following equation if Eq.(35) is inserted into Eq.(31):

$$\Delta p^* = f \frac{l}{D} \left[ \frac{\Delta T_c L \rho_v}{T} + (p_o - p_v) \right] \quad (36)$$

When inserted into Eq.(30), the above formula yields:

$$W_W = \left[ f \frac{l}{D} \left[ \frac{\Delta T_c L \rho_v}{\rho_l T} + \frac{(p_o - p_v)}{\rho_l} \right] + L \right] M \quad (37)$$

Finally, a  $\Gamma$  term may be defined as the ratio of the required energy for hydrodynamic cavitation provided by Eq.(A.2) and by thermal volatilisation provided by Eq.(37) as:

$$\Gamma = \frac{W_W}{W_T}$$

$$W_T = [c_{pl}(\Delta T_c + \Delta T_s) + L] M \quad (38)$$

$$\Gamma = \left[ f \frac{l}{D} \left( \frac{\Delta T_c \rho_v}{\rho_l T} + \frac{p_o - p_v}{\rho_l L} \right) + 1 \right] \left[ \frac{c_{pl}(\Delta T_c + T_s - T)}{L} + 1 \right]^{-1} \quad (39)$$

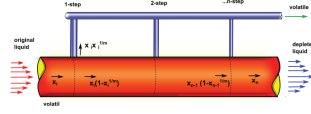
The terms of pressure and temperature inside the brackets are negligible for practical cases. The energy requirements are the same.

In other words, an assessment of the better suitability of thermal or hydrodynamic extraction for a given volatile should be presented and evaluated in terms of the quality of energy rather than the quantity (i.e. the ability to produce mechanical work) and instead of the thermal energy or vice versa. For instance, hydrodynamic extraction should be more attractive for sources as marine or wind energy if renewable energy is the source. Meanwhile, volatilisation by heating is the choice if the source is solar energy.

## 6. N-step separation

The hydrodynamic cavitation for volatile extraction could also be applied as a cascade of separation consisting of several similar stages or steps with each extraction of the output from the previous stage. As previously mentioned, the efficiency of the hydrodynamic cavitation separation follows a power law with the concentration. Each step will have a smaller efficiency than the previous ones provided that the volatile concentration decreases with each step. The final volatile concentration in the liquid after N-steps may be calculated as follows: if the concentration in a given step is  $x_i$ , the next





**Fig. 7.** Schematic of the N-step hydrodynamic separation of the volatiles.

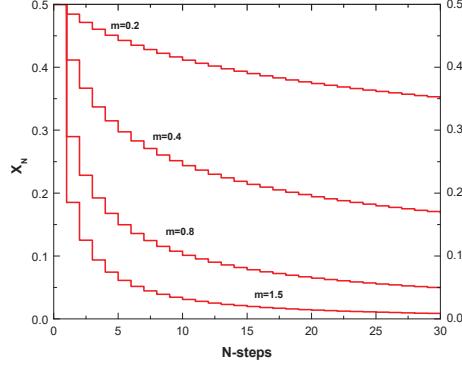
step after extraction will have a volatile concentration of  $x_{i+1} = x_i(1 - \eta_i)$ , which can be rewritten as  $x_{i+1} = x_i(1 - x_i^{\frac{1}{m}})$  considering Eq.(16). More importantly, the radius of the pipe  $t$ , where the volatile is being evacuated in each step  $t$ , must be fixed considering the concentration decrease according to Eq.(6). Fig. 5 shows the sketch of a possible cascade of separation. The calculation for the N-steps is provided as follows:

$$\begin{aligned}
 x_o &= x_o \\
 x_i &= x_o(1 - x_o^{\frac{1}{m}}) \\
 x_{i+1} &= x_i(1 - x_i^{\frac{1}{m}}) \\
 &\vdots \\
 x_{n-1} &= x_{n-2}(1 - x_{n-2}^{\frac{1}{m}}) \\
 x_n &= x_{n-1}(1 - x_{n-1}^{\frac{1}{m}})
 \end{aligned} \tag{40}$$

The abovementioned equations can be easily iteratively calculated. Fig. 8 presents the final volatile concentration in the liquid as a function of the number of steps  $N$  and considering an initial concentration  $x = 0.5$ .

## 7. Summary of results and conclusions

This study reported on hydrodynamic cavitation and its feasibility for volatile compound removal. Some of the following interesting observations were recorded:



**Fig. 8.** Concentration of the volatile  $x$  as a function of the number of steps  $N$  and considering an initial concentration  $x = 0.5$ .

- (a) The efficiency reasonably well approximated with a power law as a function of the volatile concentration.
- (b) The efficiency is strongly dependent on the size of the volatile bubble. The removal is only efficient when the volatile bubble is approximately 10% of the pipe diameter.
- (c) Bubble growth is necessary, and the method is only feasible from volatile concentrations in the range of a few percent or higher.
- (d) The energetic requirement for the volatile removal using hydrodynamic cavitation is similar to thermal volatilisation. The attractiveness of either one mostly relies on the capability to produce mechanical energy (for hydrodynamic cavitation) or thermal energy (for thermal vapourisation).

## Appendix A.

A bubble growth can be considered as initially controlled rather than thermally controlled when the thermal effect on its growth can be neglected. This condition requires the growth time not exceeding a certain critical time  $t_c$  defined as, (Brennen 1995)

$$t_c = \frac{\Delta p}{\rho_l} \cdot \frac{1}{\Sigma^2} \quad (\text{A.1})$$

where  $\rho_l$  is the liquid density.  $\Delta p$  is the tension given by  $\Delta p = p_v - p$  with  $p_v$ , and  $p$  as the vapor and the liquid pressure, respectively. The term  
 295  $\Sigma$  is a thermodynamic parameter given as:

$$\Sigma(T) = \frac{L^2 \rho_v^2}{\rho_l^2 c_{pl} T \alpha_l^{\frac{1}{2}}} \quad (\text{A.2})$$

where  $L$  and  $c_{pl}$  are the latent heat and the heat capacity, respectively;  $T$  is temperature and  $\alpha_l$  is the thermal diffusivity of the liquid. For the water system, at 20°C., has a value on  $\Sigma \simeq 1 \text{ m/s}^{\frac{3}{2}}$ . The critical time is of the order of 10 s. We obtain  $\Sigma \simeq 10^3 \text{ m/s}^{\frac{3}{2}}$  for the water working  
 300 close to the boiling temperature  $T \simeq 100 \text{ C}^\circ$ . The critical time is reduced to 10 $\mu$ ms (Brennen 1995) .

## NOMENCLATURE

- $a$  = volatile bubble radius (m)
- 305  $A_c$  = pipe area cross-section (m<sup>2</sup>)
- $c$  = local bubble concentration (m<sup>-3</sup>)
- $\bar{c}$  = average gas concentration across the cross-section (m<sup>-3</sup>)
- $c_m$  = concentration at the tube centreline (m<sup>-3</sup>)
- $c^* = \frac{c}{c_m}$
- 310  $c_{pl}$  = heat capacity of liquid(J kg<sup>-1</sup> K<sup>-1</sup>)
- $D$  = pipe diameter (m)
- $s = \frac{y}{R}$  dimensionless distance from the wall
- $f$  = dimensionless friction factor
- $L$  = latent heat of the volatile (J kg<sup>-1</sup>)
- 315  $n$  = dimensionless exponent
- $m$  = dimensionless exponent
- $M$  = mass of gas to be removed(kg)
- $u$  = velocity (ms<sup>-1</sup>)
- $u_c$  = critical velocity at which cavitation occurs (ms<sup>-1</sup>)
- 320  $u_l$  = liquid velocity in the  $x$ -direction (ms<sup>-1</sup>)
- $u_m$  = velocity of the two-phase mixture at the tube centreline (ms<sup>-1</sup>)
- $R$  = tube radius (m)
- $p$  = final pressure (Pa)
- $p_c$  = critical pressure at which cavitation occurs (Pa)
- 325  $p_o$  = initial pressure (Pa)

$p_s$  = saturation pressure (Pa)  
 $p_v$  = vapor pressure (Pa)  
 $\Delta p_c$  = tensile strength of the liquid (Pa)  
 $\Delta P$  = pressure drop (Pa)  
330  $t$  = pipe radius for the volatile removal or time (m)  
 $x$  = volatile concentration ( $\text{m}^{-3}$ )  
 $Z_g$  = total amount of gas  
 $W_T$  = thermal energy for volatilisation (J)  
 $W_W$  = mechanical energy for volatilisation (J)  
335  $T$  = initial temperature of the liquid (K)  
 $T_c$  = critical temperature at which vaporisation occurs (K)  
 $T_s$  = saturation temperature (K)  
 $\Delta T$  = temperature difference (K)

#### 340 **Greek symbols**

$\kappa$  = universal constant  
 $\rho_l$  = liquid density ( $\text{kg m}^{-3}$ )  
 $\rho_g$  = gas density ( $\text{kg m}^{-3}$ )  
 $\rho_v$  = vapor density ( $\text{kg m}^{-3}$ )  
345  $\eta$  = efficiency  
 $\nu_l$  = kinematic viscosity of liquid  
 $\Psi$  = parameter defined by Eq.(10)  
 $\Phi$  = parameter defined by Eq.(11)  
 $\gamma$  = fraction of the volatile compound in the liquid  
350  $\Gamma$  = energy cavitation-to-thermal energy ratio  
 $\Sigma$  = thermal parameter

#### **Subscripts**

$c$  = critical  
355  $l$  = liquid  
 $g$  = gas  
 $v$  = vapor

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