RELIABILITY–BASED ECONOMIC MODEL PREDICTIVE CONTROL FOR GENERALISED FLOW–BASED NETWORKS INCLUDING ACTUATORS' HEALTH–AWARE CAPABILITIES

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This paper proposes a reliability-based economic model predictive control (MPC) strategy for the management of generalised flow-based networks, integrating some ideas on network service reliability, dynamic safety stock planning, and degradation of equipment health. The proposed strategy is based on a single-layer economic optimisation problem with dynamic constraints, which includes two enhancements with respect to existing approaches. The first enhancement considers chance-constraint programming to compute an optimal inventory replenishment policy based on a desired risk acceptability level, leading to dynamical allocation of safety stocks in flow-based networks to satisfy non-stationary flow demands. The second enhancement computes a smart distribution of the control effort and maximises actuators' availability by estimating their degradation and reliability. The proposed approach is illustrated with an application of water transport networks using the Barcelona network as the case study considered.

Keywords: model predictive control, flow-based networks, dynamic safety stocks, actuator health, service reliability, chance constraints, economic optimisation.

1. Introduction

The normal functioning of modern society strongly relies on many instances of networks, e.g., communication networks, electrical power networks, public transport networks, road-traffic networks, water networks, oil and gas networks, or supply chains, among other things. Consequently, such networks are critical infrastructures (Negenborn and Hellendoorn, 2010), and maintaining an efficient, reliable and sustainable operation is a must for all network managers (Kyriakides and Polycarpou, 2015).

Although critical infrastructures are conceived and designed to supply different specific services, many of the problems that trigger their operation (e.g., minimisation of displacement times, maximisation of plants throughput, minimisation of energy consumption, maximisation of demand satisfaction, etc.) share a common feature: some commodity (or many at the same time), e.g., water, oil, energy, products, among any other real or abstract entities, needs to be transported through the network infrastructure. Such similarity gave raise to the concept of generalised flow-based networks and to classical network flow problems (cf. Ford and Fulkerson, 1962; Papageorgiou, 1984; Ahuja *et al.*, 1993) that aim to specify some control inputs influencing the flow process in the network so as to optimise a given performance criterion subject to constraints and to continuously varying conditions of both deterministic and probabilistic nature.

The management of generalised flow-based networks is a complex task and has become a research subject worldwide. Strategical and tactical decisions in physical network operation can be addressed by different methods proposed within the supply-chain theory (Papageorgiou, 2009), but the mathematical tools available in control systems theory have shown to be more suitable to handle the problem consisting of time variance, uncertainties, delays, dimensionality and lack of system information (see, e.g., Ortega and Lin, 2004; Sarimveis *et al.*, 2008; Schwartz and Rivera, 2010; Subramanian *et al.*, 2013). Most of the approaches developed in the aforementioned references for the control of dynamic networks are mainly focused on performance and robustness, and the control

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strategy is often implemented in a multi-layer control architecture. However, due to the stochastic nature of customer demands and the ageing behaviour of the infrastructure components, there is an important issue that needs to be considered in the design of the control strategy, which is *system/service reliability*. This latter aspect is the main focus of this paper.

Generally, *reliability* can be defined as the probability that units, components, equipments and systems will accomplish their intended function for a specified period of time under some operating conditions and specific environments (Gertsbakh, 2010). Thus, from the perspective of supply-chain engineering, reliability analysis of a generalised flow-based network may be associated with the α-*service level* (type I) (Goetschalckx, 2011), which is an event-oriented performance criterion that measures the probability that all customer demands will be completely served within a given time interval from the stock on hand without delay, under normal and emergency conditions.

Service reliability and economic optimisation in flow-based networks have been important research topics in the field of inventory management for planning against uncertainty in demand and/or supply. The main strategy reported in the literature to assure a service level in flow-based networks consists in performing demand forecasting to guarantee a safety stock in storage units (if existing) as a countermeasure to secure network performance against forecast inaccuracy. Obtaining and using advanced demand information enable network operators to be more responsive to customer needs and to improve inventory management (Özer, 2003). The interaction between forecasting and stock control is well reviewed by Betts (2011), Guide and Srivastava (2000), Kanet *et al*. (2010), Osman and Demirli (2012), Schoenmeyr and Graves (2009), Strijbosch *et al*. (2011) and the references therein. Nevertheless, to guarantee a service level in flow-based networks, the control strategies should consider not only demand uncertainty but also network topological reliability, which refers to the probability that a network is connected given its components' probability to remain operative at any time.

To the best of the authors' knowledge, reliability and degradation models of system and components have not been addressed simultaneously with dynamic safety stock planning in the framework of generalised flow-based networks control. Reliability in flow-based networks is commonly analysed off-line, i.e., *a posteriori* of the operation cycle, but without a measure of capacity degradation that may exist in the actuators of the network. Relevant attempts to compute the required safety stocks considering the network's health were presented by Blanchini *et al*. (1997; 2000) for the control of production-distribution systems with uncertain demands and system failures. In these works, necessary and sufficient conditions to drive and keep the state within the least storage level are obtained, but under the requirement that the controller must be aware of the demand uncertainty bounds and the actuator failure configuration, which are not always possible to identify and isolate. Most of other approaches that study component-health management and system reliability lie within the framework of fault-tolerant control or in the field of maintenance scheduling (see, e.g., the works of Guida and Giorgio (1995), Martorell *et al*. (1999), Gallestey *et al*. (2002), Khelassi *et al*. (2011), Pereira *et al*. (2010), Chamseddine *et al*. (2014) and the references therein), but they do not consider demand uncertainty.

Several economic-oriented controllers have been recently proposed within the MPC framework (Ellis *et al*., 2014), but without considering reliability issues. Both safety stock and actuator lifetime share the fact that they are conflicting with the economic performance of the system. Therefore, it is desired to have a flexible control strategy that allows a trade-off between the economic optimisation and the reliability of the system. To achieve this aim, only a two-layer hierarchical control strategy has been proposed by Grosso *et al*. (2012) for network flow optimisation considering both economic and reliability criteria. In such a work, first an upper layer performs a local steady-state economic optimisation to set up a uniform back-off of a demand satisfaction constraint due to an assumption of stationary demand uncertainty. At the same stage, a deterministic model of actuator degradation is used to monitor the system health and to set up the maximum allowable degradation of the actuators at each time step to distribute the overall control effort. Later, in a lower layer, an economic MPC algorithm is implemented to compute optimal control set-points that minimise a multi-objective cost function.

The main contribution of this paper consists in an improved reliability-based economic MPC strategy that is aware of the actuator health and allows dynamic management of risk for non-stationary demand uncertainty, extending the results presented by Grosso *et al.* (2012; 2014). Specifically, the two-layer control architecture proposed by Grosso *et al*. (2012) is here simplified and reduced to a less conservative single-layer stochastic approach following the chance-constrained MPC approach presented by Grosso *et al*. (2014). The actuator-health management policy used in this paper follows the one introduced by Pereira *et al*. (2010), but considers stochastic actuator-degradation models and probabilistic actuator lifetime constraints rather than deterministic ones. The customer service level is guaranteed here by means of probabilistic demand satisfaction constraints. The proposed MPC controller optimises directly the economic (possibly multi-objective) performance of the network operation instead of the commonly used tracking cost function.

The reliability-based tuning strategy proposed by Khelassi *et al.* (2011) is here used as part of the constrained optimisation problem to contribute in the optimal allocation of the control effort. The Barcelona water network is used to illustrate and assess the proposed approach.

The remainder of this paper is organised as follows. Section 2 briefly describes a control-oriented model of generalised flow-based networks, and states the safety stock allocation policy and the actuator-health management policy. Section 3 is devoted to the formulation of the proposed reliability-based economic MPC strategy. Section 4 describes the case study where the effectiveness of the proposed approach is analysed via simulations. Finally, Section 5 highlights the concluding remarks that can be drawn from the results presented in this paper, as well as some ideas for future research.

Notation. Throughout this paper, $\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$ and $\mathbb{R}_+$ denote the field of real numbers, the set of column real vectors of length n, the set of m by n real matrices and the set of non-negative real numbers, respectively, while $\mathbb{Z}_+$ denotes the set of non-negative integer numbers including zero. Define $\mathbb{Z}_{[a,b]} := \{x \in \mathbb{Z}_+ \mid a \leq x \leq b\}$ for some $a, b \in \mathbb{Z}_+$ and $\mathbb{Z}_{\geq c} := \{x \in \mathbb{Z}_+ \mid x \geq c\}$ for some $c \in \mathbb{Z}_+$. For a vector $x \in \mathbb{R}^n$, $x_{(i)}$ denotes the i-th element of x. Similarly, $X_{(i)}$ denotes the i-th row of a matrix $X \in \mathbb{R}^{n\times m}$. Additionally, $\|\cdot\|_Z$ denotes the weighted 2-norm of a vector, i.e., $\|x\|_Z = (x^\top Zx)^{1/2}$. If not otherwise noted, all vectors are column vectors. Transposition is denoted by the superscript $^\top$ and the operators $<, \leq, =, >, \geq$ denote element-wise relations of vectors. Moreover, 0 denotes a zero column vector and I the identity matrix, both of appropriate dimensions. For a given vector $x \in \mathbb{R}^n$, let $\mathrm{diag}(x)$ denote a diagonal matrix in $\mathbb{R}^{n\times n}$ whose main diagonal contains the elements of x. For a symmetric matrix $Z \in \mathbb{R}^{n\times n}$, let $Z \succ 0$ ($\succeq 0$) denote that Z is positive definite (semi-definite).

2. Problem statement

Consider a *generalised flow-based network* being denoted as $\mathcal{N} = (\mathcal{G}, p, \mathcal{S})$, which consists of a *directed graph* $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ formed by a finite set of *nodes* $\mathcal{V} \subseteq \mathbb{Z}_{\geq 1}$, and a finite set of *arcs* $\mathcal{A} \subseteq \mathcal{V} \times \mathcal{V}$, with an arc $a \in \mathcal{A}$ being an ordered link between a pair of nodes (i, j) with $i, j \in \mathcal{V}$, whose order indicates the direction of the flow between the two nodes. The network has a special subset of nodes $\mathcal{S} \subset \mathcal{V}$ called *terminals*. A terminal is either a *source* or a *sink*. The set of source nodes is denoted as $\mathcal{S}^+$ and the set of sink nodes is denoted as $\mathcal{S}^-$, and it follows that $\mathcal{S} = \mathcal{S}^+ \cup \mathcal{S}^-$. The rest of nodes $i \in \mathcal{V} \setminus \mathcal{S}$, are called *intermediate* nodes. These latter nodes can be further classified according to their flow storage capacity into dynamic nodes and static nodes. The dynamic nodes have non-zero storage capacity, while in the static ones the transshipment of the commodity is immediate. The functioning of the network is driven by a vector function p containing the functions that define the dynamic attributes of the graph, i.e., capacities, transit times, gains, supplies, demands. It is supposed here that only the attributes conforming p are time varying, while the structure of the network (defined by $\mathcal{G}$ and $\mathcal{S}$) remains unchanged.

In this paper, the following initial assumptions are considered regarding the network operation.

Assumption 1. The network operates in a push-flow regime with zero transit time for all $a \in \mathcal{A}$.

Assumption 2. The flow through each arc $a \in \mathcal{A}$ is controlled by an actuator for all $a = (i, j)$ with $i, j \in \{\mathcal{V} \setminus \mathcal{S}^-\}$. The flow does not experience any gain or loss while traversing an arc.

In order to derive a control-oriented model, define the state vector $x \in \mathbb{R}^n$ to represent the storage at the dynamic nodes. Similarly, define the vector $u \in \mathbb{R}^m$ of controlled inputs as the collection of the flow rate through the arcs $(i, j) \in \mathcal{A}_u := \{(i, j) \in \mathcal{A}$ such that $i, j \in \mathcal{V} \setminus \mathcal{S}^-\}$, and the vector $d \in \mathbb{R}^p$ of uncontrolled inputs (demands) as the collection of flow rate through the arcs $(i, j) \in \mathcal{A}_d := \{(i, j) \in \mathcal{A}$ such that $i \in \mathcal{V} \setminus \mathcal{S}^-$ and $j \in \mathcal{S}^-\}$. Following flow/mass balance principles as well as Assumptions 1 and 2, a discrete-time model based on linear difference-algebraic equations can be formulated for the network $\mathcal{N}$ as follows:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + B_d d_k, & \text{(1a)} \\ 0 = E_u u_k + E_d d_k, & \text{(1b)} \end{cases}$$

where $k \in \mathbb{Z}_+$ is the current time step while A, B, B_d, E_u and E_d are matrices of compatible dimensions dictated by the network topology. Specifically, (1a) represents the mass balance at dynamic nodes while (1b) represents the mass balance at static nodes. The system is subject to state and input constraints considered here in the form of convex polyhedra defined as

$$x_k \in \mathbb{X} := \{x \in \mathbb{R}^n \mid Gx \leq g\}, \tag{2a}$$

$$u_k \in \mathbb{U} := \{u \in \mathbb{R}^m \mid Hu \leq h\}, \tag{2b}$$

for all k, where $G \in \mathbb{R}^{r_x\times n}$, $g \in \mathbb{R}^{r_x}$, $H \in \mathbb{R}^{r_u\times m}$, $h \in \mathbb{R}^{r_u}$, being $r_x \in \mathbb{Z}_+$ and $r_u \in \mathbb{Z}_+$ the number of state and input constraints, respectively.

Assumption 3. The states in x and the demands in d are measured at any time step $k \in \mathbb{Z}_+$.

Assumption 4. The realisation of demands at any time step $k \in \mathbb{Z}_+$ can be decomposed as

$$d_k = \hat{d}_k + e_k, \tag{3}$$

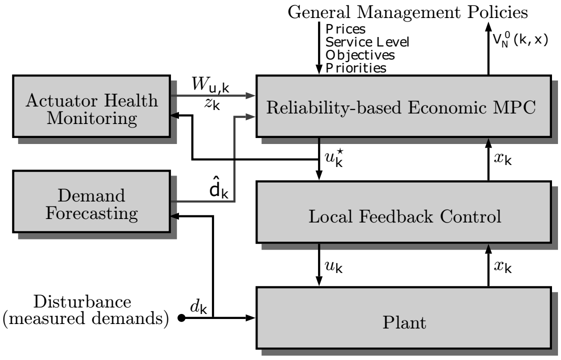


Fig. 1. Reliability-based MPC structure.

where $\hat{d}_k \in \mathbb{R}^p$ is the vector of expected disturbances and $e_k \in \mathbb{R}^p$ is the vector of forecasting errors with non-stationary uncertainty and a known (or approximated) quasi-concave probability distribution $\mathcal{D}(0, \Sigma(e_{(j),k}))$. The stochastic nature of each j-th row of d_k is described then by $d_{(j),k} \sim \mathcal{D}_i(\hat{d}_{(j),k}, \Sigma(e_{(j),k}))$, where $\hat{d}_{(j),k}$ denotes its mean and $\Sigma(e_{(j),k})$ its variance.

The control goal is to minimise a convex (possibly multi-objective and time-varying) stage cost function $\ell : \mathbb{Z}_+ \times \mathbb{X} \times \mathbb{U} \to \mathbb{R}_+$, which might bear any functional relationship to the economics of the system operation. To do so, the control strategy addressed in this paper is based on the control system structure shown in Fig. 1, where the information given by forecasting demand and actuator health estimation modules is used within an economic MPC controller to ensure a given service level in the network. In such a structure, z_k is the state of the cumulative degradation of actuators and $W_{u,k}$ is a reliability-dependant matrix that adjusts the tuning of the MPC controller (see Section 2.2 for details). Moreover, $u_k^\star$ and V_N^0 denote respectively the optimal control action computed in the predictive control block and the optimal value of the cost function that is embedded and minimised within the proposed MPC controller (see Section 3).

2.1. Safety stock allocation policy. There is often the need for guaranteeing a safety stock at each storage node of a generalised flow-based network in order to decrease the probability of stock-outs (when a node has insufficient resources to satisfy either external demands or the flow requested by other intermediate nodes) due to possible uncertainties in the network. As discussed in Section 1, stock allocation problems have been addressed before in the literature of supply chain management, where solutions are mainly based on inventory planning strategies that incorporate, within deterministic formulations, safety mechanisms to cope with randomness and risks associated with network operation (Christopher, 2005). Most techniques from inventory management suppose a hierarchical and descendant flow of products, even in multi-stage multi-echelon schemes, in a way that predicted safety stock changes are easily communicated backwards in order to support availability of quantities when they are needed (Kanet *et al.*, 2010). Nevertheless, this behaviour is not true in real large-scale generalised flow-based networks since a meshed topology with multi-directional flows between nodes prevails instead of spread tree configurations.

To circumvent the aforementioned limitation and determine optimal dynamic safety stocks, the chance-constrained MPC strategy described by Grosso *et al.* (2014) is used here. Such a strategy relaxes the original state constraint (2a) by using probabilistic statements, leading to the form of the so-called (probabilistic) chance constraint, i.e.,

$$x_k \in \{x \in \mathbb{R}^n \mid \mathbb{P}[G_{(j)}x \leq g_{(j)}\ ,\ \forall j \in \mathbb{Z}_{[1,r_x]}] \geq 1-\delta_x\}, \tag{4}$$

for all k, where $\mathbb{P}$ denotes the probability operator, $\delta_x \in (0,1)$ is the *risk acceptability level* of constraint violation for the states, and $G_{(j)}$ and $g_{(j)}$ denote the j-th row of G and g, respectively. This requires that all rows j have to be jointly fulfilled with the probability $1 - \delta_x$. A lower δ_x implies a harder constraint. As discussed by Grosso *et al.* (2014), the constraint (4) is difficult to be addressed since it lacks analytic expressions due to the multivariate probability distributions involved. Nevertheless, there are tractable approximations that can be derived if each element of the demand vector follows a log-concave univariate distribution with a known stochastic description; see the work of Grosso *et al.* (2014, Section 3) for details. Specifically, (4) can be enforced by the following constraints:

$$G_{(j)}(Ax_k + Bu_k) \leq g_{(j)} - F^{-1}_{G_{(j)}B_d d_k}(1 - \delta_{x,j}), \tag{5}$$

$$\sum_{j=1}^{r_x} \delta_{x,j} \leq \delta_x, \tag{6}$$

$$0 \leq \delta_{x,j} \leq 1, \tag{7}$$

for all $j \in \mathbb{Z}_{[1,r_x]}$, where $F_{G_{(j)}B_d d_k}(\cdot)$ and $F^{-1}_{G_{(j)}B_d d_k}(\cdot)$ are the cumulative distribution and the left-quantile function of $G_{(j)}B_d d_k$, respectively. The constraints (5) are the deterministic equivalent of the set of r_x resultant individual chance constraints. Moreover, (6) and (7) are conditions imposed to bound the new single risks in such a way that the joint risk bound is not violated. Any solution that satisfies the above constraints is guaranteed to satisfy (4). As suggested by Nemirovski and Shapiro (2006, Remark 2.1), assigning a fixed and equal value of risk to each individual constraint, i.e., $\delta_{x,j} = \delta_x / r_x$ for all $j \in \mathbb{Z}_{[1,r_x]}$, satisfies (6) and (7).

In this way, the safety stocks are optimally allocated and represented by the constraint back-off effect caused by the term $F_{G_{(j)}B_d d_k}(1-\delta_{x,j})$ in (4). Hence, the original state constraint set $\mathbb{X}$ is contracted by the effect of the r_x deterministic equivalents in (5) and replaced with the stochastic feasibility set given by

$$\begin{aligned}
&\mathbb{X}_{s,k} \\
&:= \{x_k \in \mathbb{R}^n \mid \exists u_k \in \mathbb{U}, \text{ such that} \\
&\qquad G_{(j)}(Ax_k + Bu_k) \leq g_{(j)} - F^{-1}_{G_{(j)}\tilde{B}_d d_k}(1-\delta_{x,j}), \\
&\qquad \forall j \in \mathbb{Z}_{[1,r_x]} \text{ and } E_u u_k + E_d \hat{d}_k = 0\},
\end{aligned}$$

for all k, where $\hat{d} = \mathbb{E}[d]$ is the first moment of d. From convexity of $G_{(j)}x_{k+1} \leq g_{(j)}$ and the log-concavity assumption of the distribution, it follows that the set $\mathbb{X}_{s,k}$ is convex when non-empty for all $\delta_{x,j} \in (0,1)$ in most distribution functions (Kall and Mayer, 2005). For some particular distributions, e.g., Gaussian, convexity is retained for $\delta_{x,j} \in (0, 0.5]$.

Remark 1. This strategy deals specifically with storage node reliability (assuming their faulty behaviour as the inability to satisfy their own demands), which is affected by both the capacity and reliability of the elements supplying flow to them. If the flow capacity is less than the average demand, no storage unit will probably be large enough to provide a sustained service.

2.2. Actuator-health management policy.

Unless some damage mitigating policy is adopted to ensure the availability of actuators for a given maintenance horizon, their inherent degradation could compromise the overall service reliability of the network. Therefore, system safety can be enhanced by taking into account the health of the components explicitly in controller design. Several models have been proposed in the literature to describe reliability and ageing of actuators under nominal operation; see the works of Gorjian *et al*. (2009), Guida and Giorgio (1995), and Letot and Dehombreux (2012) for a review. Nevertheless, as pointed out by Khelassi *et al*. (2011) and Martorell *et al*. (1999), a realistic health measurement should also include the trend of actuator ageing according to the variation of the operating conditions. Rates of degradation can be assumed constant for some equipment, but others present a highly variable and non-linear rate depending on the degradation mechanism and the local conditions. For the sake of simplicity, the linear proportional degradation model presented by Pereira *et al*. (2010) and its uniform rationing heuristic are adopted in this paper, but with the inclusion of an additive uncertainty. The approach considers the health condition of each actuator being described by a wear process with the rate associated with the exerted control effort as follows:

$$z_{k+1} = z_k + \varphi|u_k| + \eta_k, \tag{8}$$

where $z_k \in \mathbb{R}^m$ denotes the state of cumulative degradation of actuators at time step k and $\varphi := \mathrm{diag}(\psi_1, \ldots, \psi_m)$ is a diagonal matrix of constant degradation coefficients $\psi_i \in \mathbb{R}$, $i \in \mathbb{Z}_{[1,m]}$, associated with the m actuators. Moreover, $\eta \in \mathbb{R}^m$ is a random vector whose components lie in a normal distribution $\mathcal{N}(0, \Sigma_{\eta_{(i)}})$.

Degradation of each actuator will accumulate until the element reaches a state in which it will not perform its function at an acceptable level. At such a point, it can be considered that the actuator operation may be compromising the network supply service unless demands result reachable from other redundant flow paths or a fault-tolerant mechanism is activated. Therefore, instead of incurring into a failure that requires corrective control actions, a preventive strategy can be implemented to improve the overall system reliability by guaranteeing that each actuator remains available until the instant of a programmed maintenance intervention.

To circumvent the system availability problem, an obvious approach is to constrain the accumulated degradation of actuators at each time instant to remain below a safe threshold until a predefined maintenance horizon is reached. Here, the health management is considered to be ruled by the probabilistic version of the constraints proposed by Pereira *et al*. (2010), that is,

$$\mathbb{P}[z_{k+N|k} \leq z_{\max,k}] \geq (1-\delta_z), \tag{9}$$

$$z_{\max,k} := z_k + N\frac{z_{\text{thresh}} - z_k}{M + N - k}, \tag{10}$$

where $N \in \mathbb{Z}_+$ is a prediction horizon used for prognosis, $\delta_z \in (0,1)$ is a risk acceptability level, $z_{\max,k} \in \mathbb{R}^m$ is the vector of maximum accumulated degradation of actuators' allowed for the time step k, and $z_{\text{thresh}} \in \mathbb{R}^m$ is the vector of thresholds for the terminal degradation at a maintenance horizon $M \in \mathbb{Z}_+$. Notice that (9) restricts the predicted accumulated degradation of actuators health at N-steps ahead from the current time step k and its deterministic equivalent can be obtained similarly to Section 2.1. The right-hand side of (10) is a uniform rationing of the remaining allowed degradation ($z_{\text{thresh}} - z_k$) that is updated at each time step according to the control actions applied and ensures that $z_k \leq z_{\text{thresh}}$ for $k = M$.

Remark 2. Despite the inherent relation, a degraded state is not the same as a faulty state (see Hsu *et al*., 1991). In fact, under nominal conditions of operation, degradation always precedes failure. When a component is degraded, maintenance actions should be executed to improve its performance to acceptable levels, but when the component is faulty, repairing actions are needed to restore its functionality.

Keeping in mind the difference between degraded and faulty states, it can be noticed that the strategy for uniform rationing of degradation should be complemented with an another other safety mechanism to incorporate the remaining useful life of the actuators on the basis of their reliability and keep them available as long as possible. Accordingly, here the improvement of the safety and reliability of a generalised flow-based network is proposed using a smarter control allocation policy following the results of Khelassi *et al*. (2011) and the proportional hazard model reported by Weber *et al*. (2012). The main idea is to add to the process cost function a penalisation on control actions, which is weighted with a matrix $W_u \in \mathbb{R}_+^{m\times m}$ that depends directly on actuators' reliability. This strategy leads to a smart use of actuators minimising the frequency of unscheduled downtimes and related costs.

Consider that actuators' reliability can be estimated for the variable operating conditions with the following modified exponential distribution:

$$R_{i,k} = \exp\big(-\lambda_i^0 \exp\big(\beta_i\|\tilde{u}_i\|^2\big)k\Delta t\big), \qquad (11)$$

with $i \in \mathbb{Z}_{[1,m]}$, where $\lambda_i^0 \in \mathbb{R}_+$ is the nominal failure rate of the i-th actuator, $\beta_i \in \mathbb{R}_+$ is a shape parameter of the actuator failure for an expected life $t_M \in \mathbb{Z}_+$, and $\exp(\beta_i\|\tilde{u}_i\|^2) \in \mathbb{R}_+$ is the load function that modifies the failure rate according to the root-mean-square (denoted by $\tilde{u}_i$) of the control actions applied from the initial time until the time step k. From (11), it follows that the cumulative probability of the failure rate can be written as $F_{i,k} = 1 - R_{i,k}$. Hence, the optimal control actions can be distributed among actuators so that components with larger accumulated damage are relieved. This can be achieved by adding to the original economic cost function a weighed term for the suppression of control moves, i.e., $\|\Delta u_k\|^2_{W_{u,k}}$, in which the weighing matrix is given by

$$W_{u,k} \coloneqq \operatorname{diag}(w_1, w_2, \ldots, w_m), \qquad (12)$$

where $w_{i,k} = F_{i,k} = 1 - R_{i,k}$ for $i \in \mathbb{Z}_{[1,m]}$. Notice that the weighing matrix is re-computed on-line at each time step k to take into account the variation of the control actions and actuators' reliability. Hence, this weighing strategy allows us to improve system availability, i.e., to retain the operability of the network elements for longer times.

3. Reliability-based economic MPC problem

After discussing reliability aspects of storage and supply infrastructure, next the setting of the proposed reliability-based economic MPC controller is shown, which incorporates into its optimisation problem both the dynamic safety stock policy and the actuator-health management policy, in order to improve the flow supply service level in a given network, facing demand uncertainty and equipment wear. The design of the controller is based on Interpretation 1.

Interpretation 1. (Sup-Inf type information) *At any time step k, when computing the corresponding controlled flow u_k, both the state x_k and the demand (uncontrolled flow) d_k are known. Future demands d_{k+i} are unknown for all $i \in \mathbb{Z}_+$, but forecast information of their first two moments (i.e., the expected value and the variance) is available for a given prediction horizon $N \in \mathbb{Z}_+$. The controller has also knowledge of the current estimated accumulated degradation z_k of the network actuators.*

Therefore, for a given demand sequence $\hat{\mathbf{d}}_k = \{\hat{d}_{k+i|k}\}_{i\in\mathbb{Z}_{[0,N-1]}}$, estimated actuator degradation z_k, acceptable risk levels δ_x and δ_z, and reliability-based weight $W_{u,k}$, the proposed approach relies on solving the following optimisation problem at each time step k:

$$\begin{aligned}\min_{\mathbf{u}_k,\boldsymbol{\xi}_k^x,\boldsymbol{\xi}_k^z} \sum_{i=0}^{N-1} [\ell(k+i, x_{k+i|k}, W_{u,k}\, u_{k+i|k}) \\ + \|\Delta u_{k+i|k}\|^2_{W_{u,k}} + \|\xi^x_{k+i|k}\|^2_{W_x} \\ + \|\xi^z_{k+i|k}\|^2_{W_z}],\end{aligned} \qquad (13a)$$

subject to (10), (12),

$$x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k} + B_d\hat{d}_{k+i|k}, \qquad (13b)$$

$$z_{k+i+1|k} = z_{k+i|k} + \varphi|u_{k+i|k}|, \qquad (13c)$$

$$E_u u_{k+i|k} + E_d\hat{d}_{k+i|k} = 0, \qquad (13d)$$

$$G_{(j)}(Ax_{k+i|k} + Bu_{k+i|k}) \le g_{(j)} - \phi^x_{k,j}(\delta_x) + \xi^x_{k+i|k}, \qquad (13e)$$

$$\Delta u_{k+i|k} = u_{k+i|k} - u_{k+i-1|k}, \qquad (13f)$$

$$z_{(l),k+N|k} \le z_{\max(l),k} - \phi^z_{k,l}(\delta_z) + \xi^z_{k+i|k}, \qquad (13g)$$

$$u_{k+i|k} \in \mathbb{U}, \qquad (13h)$$

$$\xi^x_{k+i|k} \ge 0, \quad \xi^z_{k+i|k} \ge 0, \qquad (13i)$$

$$(x_{k|k}, z_{k|k}, u_{k-1|k}, \hat{d}_{k|k}) = (x_k, z_k, u_{k-1}, d_k), \qquad (13j)$$

for all $i \in \mathbb{Z}_{[0,N-1]}$, $j \in \mathbb{Z}_{[1,r_x]}$ and $l \in \mathbb{Z}_{[1,m]}$, where $\mathbf{u}_k = \{u_{k+i|k}\}_{i\in\mathbb{Z}_{[0,N-1]}}$, $\boldsymbol{\xi}_k^x = \{\xi^x_{k+i|k}\}_{i\in\mathbb{Z}_{[0,N-1]}}$ and $\boldsymbol{\xi}_k^z = \{\xi^z_{k+i|k}\}_{i\in\mathbb{Z}_{[0,N-1]}}$ are the decision variables, with $\mathbf{u}_k$ being the sequence of controlled flows while $\boldsymbol{\xi}_k^x$ and $\boldsymbol{\xi}_k^z$ sequences of slack variables introduced to retain feasibility of the optimisation problem. Moreover, $\hat{d}_{k+i|k}$ is the forecasted demand for the i-step ahead of k. Additionally, the terms

$$\phi^x_{k,j}(\delta_x) = F^{-1}_{G_{(j)}B_d d_{k+i}}\Big(1 - \frac{\delta_x}{r_x N}\Big)$$

and

$$\phi^z_{k,l}(\delta_z) = F^{-1}_{\eta_{(l)}}\Big(1 - \frac{\delta_z}{mN}\Big)$$

are the quantile functions involved in the state- and actuator-health deterministic equivalent constraints. Weighing matrices $W_x \in \mathbb{R}_+^{n\times n}$ and $W_z \in \mathbb{R}_+^{m\times m}$ are used to manage the penalisation of the slack variables $\xi_{k+i|k}^x$ and $\xi_{k+i|k}^z$, while $W_{u,k} \in \mathbb{R}_+^{m\times m}$ is the reliability-based weighing matrix introduced to relieve the actuators with larger accumulated degradation. The constraint (13j) represents the measurements available at time step k.

Denote by $(\mathbf{u}_k^\star, \boldsymbol{\xi}_k^{x\star}, \boldsymbol{\xi}_k^{z\star})$ the optimal solution of (13) at time step k. Then, following the MPC philosophy, only the first optimal control action is applied, i.e., $u_k = u_{k|k}^\star$.

Remark 3. The core of the proposed reliability-based economic MPC approach relies on the dynamic handling of constraints that allows a trade-off between reliability and economic optimisation to obtain an enhanced robust performance. Note that the worse the demand forecasting and actuator degradation models, the stricter the constraints and the more conservative control policy. The proposed controller gives just an enhancement of robustness, without guaranteeing robust feasibility and stability. In particular, the authors have addressed the case of economic recursive feasibility for periodic operation in different works by means of periodic terminal equality or inequality constraints; see preliminary results of Grosso (2015) and Limon *et al.* (2014). Such references do not include explicitly the reliability component, but it can be incorporated in the recursively feasible schemes by augmenting the state vector with the degradation state z.

4. Numerical results

In this section, the performance of the proposed reliability-based economic MPC approach is assessed with a case study consisting of a large-scale real system reported by Ocampo-Martinez *et al.* (2009), specifically, the Barcelona drinking water network (DWN). The general role of this system is the spatial and temporal re-allocation of water resources from both superficial (i.e., rivers) and underground water sources (i.e., wells) to distribution nodes located all over the city. The structure of this network (i.e., its directed graph $\mathcal{G}$ and the set $\mathcal{S}$ of source and sink nodes) can be obtained from the layout shown in Fig. 2 and its model in the form of (1) can be derived by setting the state $x_k \in \mathbb{R}^{63}$ as the volume (in m^3) of water stored in tanks at time step k, the control input $u_k \in \mathbb{R}^{114}$ as the flow rate through all network actuators (expressed in m^3/s) and the measured disturbance $d_k \in \mathbb{R}^{88}$ as the flow rate of customer demands (expressed in m^3/s). This network is currently managed by AGBAR[1] and supplies potable water to the Metropolitan Area of Barcelona (Catalunya, Spain). The main control task for managers is to economically optimise the network flows while satisfying customer demands. These demands are characterised by patterns of water usage and can be forecasted by different methods, (see, e.g., Billings and Jones, 2008; Sampathirao *et al.*, 2014).

In this way, the function ℓ in (13a) is defined as $\ell := c_{u,k}^\top W_{u,k} u_k \Delta t$ and represents the economic cost of network operation at each time step k, which depends on the reliability-based weight $W_{u,k}$ defined in (12) and on a time-of-use pricing scheme driven by a time-varying price $c_{u,k} := \big(c_1 + c_{2,k}\big) \in \mathbb{R}_+^{114}$ of the water flow, which in this application takes into account a fixed water production/treatment price $c_1 \in \mathbb{R}_+^{114}$ and a water pumping price $c_{2,k} \in \mathbb{R}_+^{114}$. This latter price is time dependant because it changes according to the electricity tariff, which is assumed to be periodically time varying. All prices are given in economic units per cubic meter (e.u./m^3) due to confidentiality reasons. The state and input constraint sets for this case study are given by $\mathbb{X} = \{x \in \mathbb{R}^{63} \mid x_{s,k} \le x \le x_{\max}\}$ and $\mathbb{U} = \{u \in \mathbb{R}^{114} \mid 0 \le u \le u_{\max}\}$, respectively, where $x_{s,k} \in \mathbb{R}_+^{63}$ is a desired time-varying safety threshold, $x_{\max} \in \mathbb{R}^{63}$ is the vector of maximum storage capacity in tanks (expressed in m^3) and $u_{\max} \in \mathbb{R}^{114}$ is the vector of maximum flow rates of actuators (expressed in m^3/s). The prediction horizon and the sampling time used in the simulations are $N = 24$ hours and $\Delta t = 1$ hour, respectively. The simulation horizon was $n_s = 96$ hours.

To analyse and highlight the benefits of the proposed reliability-based economic MPC approach, a numeric comparison with respect to baseline control strategies that were previously reported for the same case study is shown in Table 1. Specifically, the assessed approaches are the following.

Certainty-equivalent economic MPC (CE-MPC). This approach was proposed by Ocampo-Martinez *et al.* (2009). It does not consider uncertainty explicitly in the controller design and might require on-line tuning to ensure an appropriate robust performance. In fact, the common action to deal with demand uncertainty for such an approach is to heuristically define a conservative constant safety threshold $x_{s,k} = \beta x_{\max}$ for all k, with $\beta \in (0, 1)$, and incorporate a constraint of the form $x_k \ge x_{s,k}$ (or a softened version of it).

Chance-constrained economic MPC (CC-MPC). This approach was proposed by Grosso *et al.* (2014). It incorporates robustness only for demand uncertainty by replacing the state deterministic constraints with chance constraints. In this approach, every constraint that involves random variables is dynamically managed by the CC-MPC controller causing a back-off with respect to the original hard constraints. The level of back-off is variable and depends on the volatility of the forecasted demand at

[1] Aguas de Barcelona S.A. Company, which manages the drinking water transport and distribution in Barcelona (Spain).

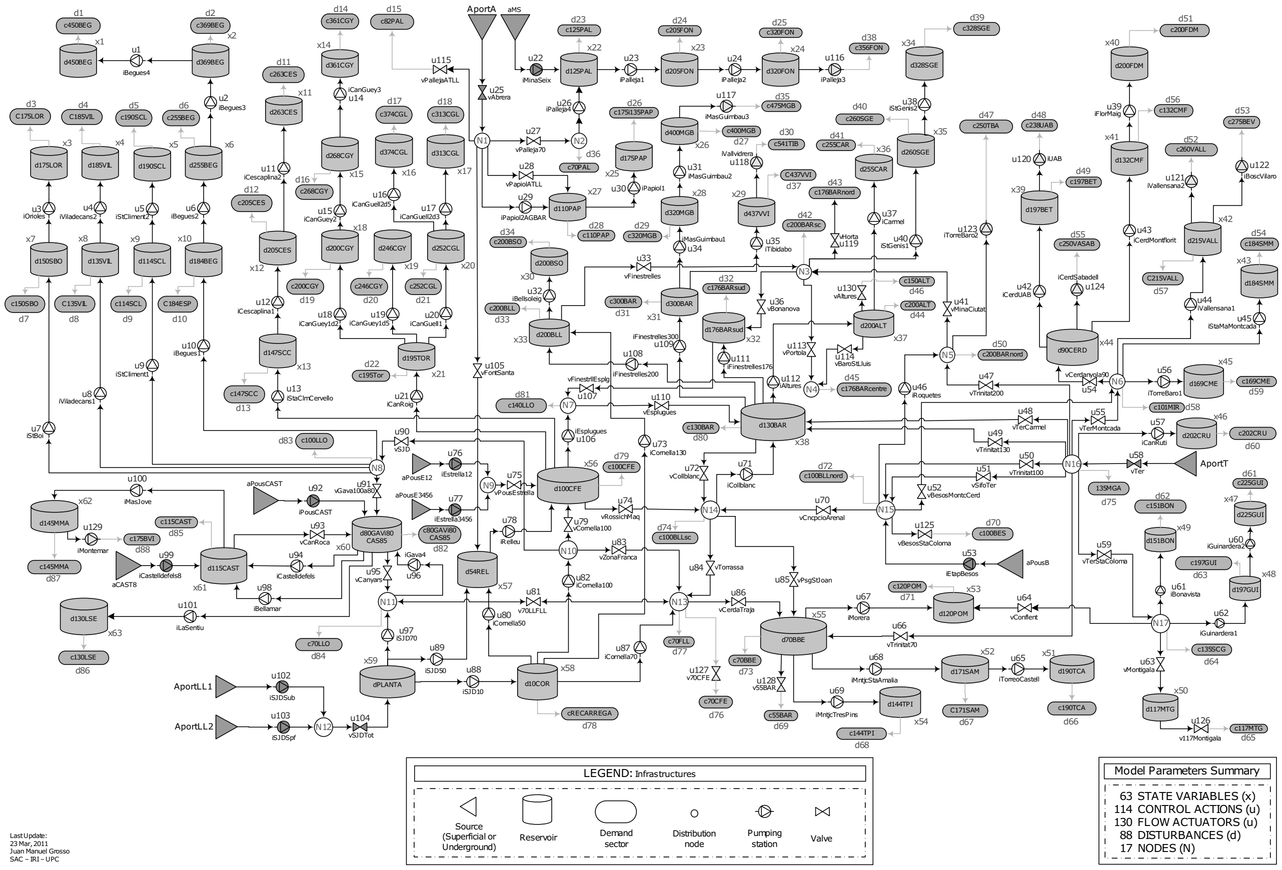


Fig. 2. Barcelona DWN layout.

each prediction step. The approach relies on a prediction model of the stochastic properties of disturbances, which should be running in parallel with the MPC model. The CC-MPC controller is not aware of the health of the network actuators.

Reliability-based economic MPC with stationary uncertainty (RB-MPC). This approach was proposed by Grosso *et al*. (2012). It uses the original output bounds, but incorporates a dynamic state soft constraint to guarantee a desired service level under demand uncertainty. In this approach, the stochastic description of demands, used to define the soft constraint, is computed *a posteriori* before each MPC execution, based on the sample mean and sample deviation of water demands. Uncertainty is considered stationary within the MPC algorithm and, as a consequence, the controller keeps a uniform back-off of demand, whose amount represents the safety stock along the prediction horizon. Additionally, this approach incorporates also the actuator-health management policy of Section 2.2, but using a deterministic actuator degradation model.

Reliability-based economic MPC using chance-constraints (RB-SMPC). This is the approach proposed by this paper, which relies on solving the problem (13). It considers non-stationary stochastic demand uncertainty and stochastic actuator degradation. Hence, the base stock constraint, the hard bounds of the states and the terminal constraint of actuator degradation are in the form of chance constraints (see Section 2).

The numeric assessment of the aforementioned approaches is carried out through different key performance indicators (KPIs), which are defined as follows:

$$\mathrm{KPI}_E := \frac{1}{n_s+1}\sum_{k=0}^{n_s} c_{u,k}^{\top} u_k \Delta t, \tag{14a}$$

$$\mathrm{KPI}_{\Delta U} := \frac{1}{n_s+1}\sum_{i=1}^{m}\sum_{k=0}^{n_s} \left(\Delta u_{(i),k}\right)^2, \tag{14b}$$

$$\mathrm{KPI}_S := \sum_{i=1}^{n}\sum_{k=0}^{n_s} \max\left\{0, x_{s(i),k} - x_{(i),k}\right\}, \tag{14c}$$

$$\mathrm{KPI}_Z := \frac{1}{n_s+1}\sum_{i=1}^{m}\sum_{k=0}^{n_s} z_{(i),k}, \tag{14d}$$

$$\mathrm{KPI}_V := \sum_{k=1}^{n_s} v_k, \tag{14e}$$

$$\mathrm{KPI}_O := t_{\mathrm{opt},k}, \tag{14f}$$

where KPI_E is the average economic performance of the DWN operation, $\mathrm{KPI}_{\Delta U}$ measures the smoothness of the control actions, KPI_S is the amount of water used from safety stocks, KPI_Z accounts for the average degradation of actuators, KPI_V measures the number of safety constraint violations that have occurred during the simulation, with v_k being the number of tanks that required the use of their safety stock at time step k, and KPI_O determines the difficulty to solve the optimisation tasks involved in each strategy accounting $t_{\mathrm{opt},k}$ as the average time that takes to solve the corresponding MPC optimisation problem. A lower KPI value represents a better performance result. Simulations have been carried out using $\gamma = \{80, 95\}\%$ for RB-MPC and $\delta = \{5, 20\}\%$ for both the CC-MPC and RB-SMPC (where $\delta_x = \delta$ and $\delta_z = \delta$). In addition, Table 3 discloses details of the production and operational costs related to each strategy, which are the primary objectives for DWN managers. Furthermore, Table 2 summarises the capabilities handled by each controller. This qualitative information complements the quantitative evaluation of the assessed strategies in order to highlight the benefits of the proposed RB-SMPC design.

An important aspect in any MPC controller is the handling of constraints. In the Barcelona DWN, manipulated variables can always be kept within bounds by the controller, but output constraints, which are subject to measured and/or unmeasured uncertainties, must be properly handled. Since the baseline CE-MPC approach relies on proper tuning of heuristic safety stocks, its robustness and economic performance might be compromised. Contrarily, the RB-MPC, CC-MPC and RB-SMPC approaches focus on economic robust performance of the DWN. They enhance the robustness of the baseline CE-MPC by performing a dynamic handling of constraints while keeping tractability of the optimisation problems even for the large-scale model of the case study. In particular, the RB-SMPC approach proposed in this paper integrates the health-aware capabilities of the RB-MPC approach with the stochastic technique of the CC-MPC approach. Figure 3 shows the mechanism that both RB-MPC and RB-SMPC approaches use to guarantee a service level in the DWN and to avoid the violation of real output constraints due to uncertainty. The plot shows the response of both controllers for a forecasted demand with confidence levels of 80% and 95%. Notice that both the approaches dynamically generate a back-off of the original constraints. An important observation regarding the handling of constraints by both RB-MPC and RB-SMPC controllers is the inherent relation between

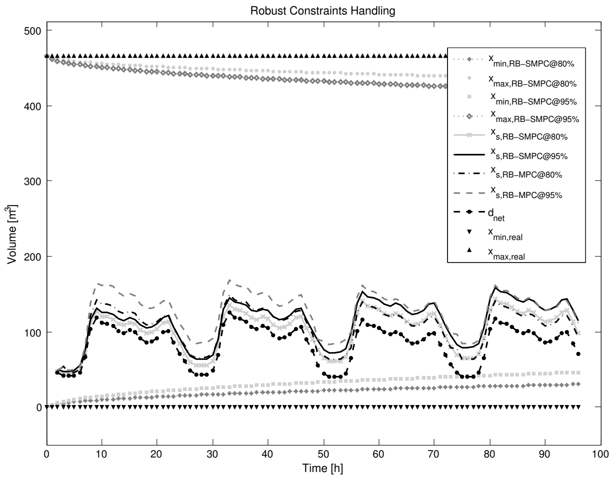


Fig. 3. Risk averse mechanism using the RB-MPC and RB-SMPC approaches.

the service level in RB-MPC and the joint risk level in RB-SMPC. Despite being defined under different philosophies, both parameters represent a measure of reliability for the DWN function. Nevertheless, Fig. 3 shows that the dynamic safety stock computed by the RB-MPC controller is uniform and more conservative than the one computed by RB-SMPC, which increases according to the forecast error along the prediction horizon. This fact highlights the importance of a suitable forecasting model and the effect of the explicit propagation of uncertainty within the MPC model. In general, decreasing the value of the service level, e.g., from 95% to 80% (equivalent to increasing the value of the risk level from 5% to 20%), causes a reduction in the safety stock and leads the base stock closer to the demand pattern, which means that the probability of not achieving the customer requirements increases due to the demand uncertainty.

After reviewing the results in Tables 1 and 2, it can be said that the robustness enhancements of the MPC strategy proposed in this paper outperform the CE-MPC controller in terms of reliability, i.e., CE-MPC may have low values in most of KPIs but without any guarantee of reliability and robust or probabilistic feasibility. Despite having the lowest KPI_S, the baseline CE-MPC approach is the one that presents the highest number of soft constraint violations, which means that the safety thresholds might be overestimated (as observed in several tanks in the DWN), causing more oscillations in the excursion of water, or keeping states near the threshold with easiness of activating the constraints in the controller. Therefore, the baseline CE-MPC approach, with fixed and empirical safety stocks, limits the economic optimisation. Instead, the CC-MPC approach reached the lowest KPI_E (in both 80% and 95% risk levels) by incorporating robust and

Table 1. Comparison of the performance of controllers.

Controller	KPI_E	KPI_S	$\text{KPI}_{\Delta U}$	KPI_Z	KPI_V	KPI_O	Simulation Time
CE-MPC	2442.97	0.18011	0.8419	0.1374	2245	1.83	202.37
$\text{CC-MPC}_{@5\%}$	2390.57	9421.46	1.0223	0.1373	1822	2.65	624.36
$\text{CC-MPC}_{@20\%}$	2362.64	710.22	1.1556	0.1374	1960	2.41	603.61
$\text{RB-MPC}_{@95\%}$	2569.59	3029.94	2.1023	0.1098	1699	9.18	892.34
$\text{RB-MPC}_{@80\%}$	2560.72	1625.29	2.0665	0.1187	1761	9.17	891.38
$\text{RB-SMPC}_{@5\%}$	2761.48	3364.82	2.8664	0.1270	1710	2.50	603.91
$\text{RB-SMPC}_{@20\%}$	2560.36	4946.13	2.2038	0.1076	1715	2.67	629.35

Table 2. Comparison of capabilities handled by each controller.

Controller	Dynamic safety stocks	Dynamic output bounds	Actuator health	Smart tuning
CE-MPC				
CC-MPC	$\surd$	$\surd$		
RB-MPC	$\surd$		$\surd$	$\surd$
RB-SMPC	$\surd$	$\surd$	$\surd$	$\surd$

$\surd$: handled.

Table 3. Comparison of daily average costs of MPC strategies.

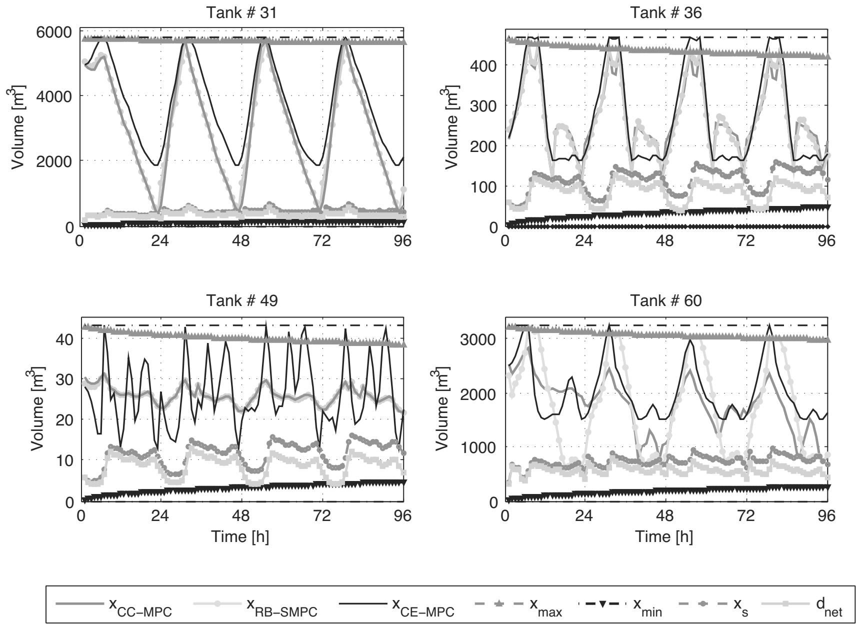
MPC approach	Water average cost (e.u./day)	Electric average cost (e.u./day)	Daily average cost (e.u./day)
CE-MPC	29037.21	29594.14	58631.35
CC-MPC	27706.72	29666.85	57373.58
RB-MPC	42072.97	19597.27	61670.25
RB-SMPC	53179.29	13096.23	66275.53

e.u.: economic units.

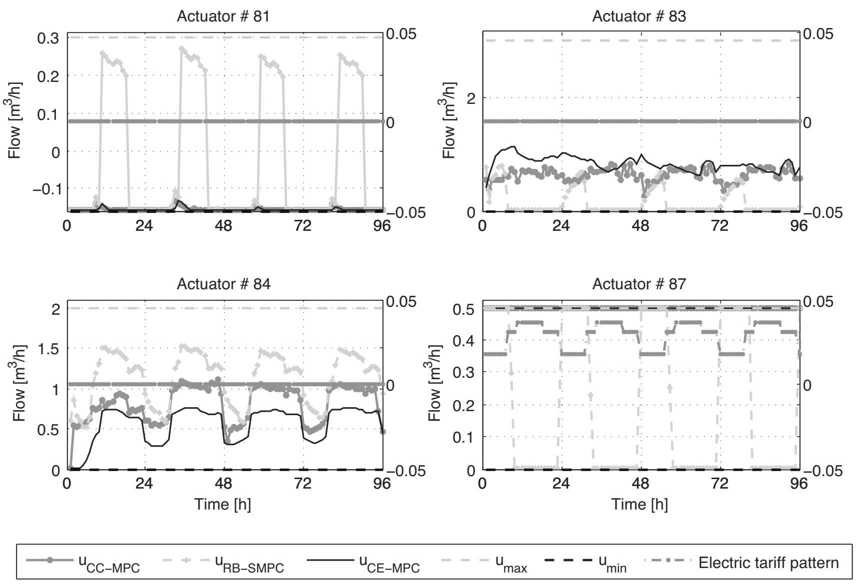
optimal safety stocks to face demand uncertainty with minimum storage of water. This stochastic approach has lower KPI_V, i.e., it reduces the number of violations of the base stocks but increases the amount of safety stocks used to meet demands (higher KPI_S). This is expected behaviour due to the policy of minimum storage behind the computation of the base stocks, which prefers using the safety stocks instead of keeping more volume of water than required. The lower cost of water in the Barcelona DWN (see Table 3), comparing both the CE-MPC and CC-MPC approaches, reinforces this observation. The main disadvantage of these cheaper controllers is that control actions are computed based on economic criteria, accounting for tanks reliability but not for actuators' reliability. This fact leads to higher values of the KPI_Z, i.e., the controllers overexploit those actuators that have lower operational costs, accelerating their wear and compromising the service reliability.

In order to manage the overall system reliability, the RB-MPC and RB-SMPC controllers incorporate actuator-health models and restrict their maximal cumulative degradation at each time step to ensure their proper functioning until a maintenance horizon is reached. As seen in Table 3, the ability to compute control actions for efficient management of actuator reliability implies an important reduction in the electric costs. This improvement is achieved at the expenses of (i) an increment in KPI_E due to the higher water cost, (ii) an increment in $\text{KPI}_{\Delta U}$ due to the distribution of control effort that avoids (if possible) constant control actions that could cause an imbalance degradation of actuators, and (iii) an increment in KPI_S due to the narrowing of constraints. It is important to point out that the RB-MPC controller has greater KPI_O than the RB-SMPC one. The reason is that the former has to solve a bi-level optimisation problem on-line, compared with the RB-SMPC controller, which just requires solving a single optimisation problem. Nonetheless, all the compared controllers are suitable for real-time control considering the sampling time in the DWN is one hour.

As can be expected, the RB-MPC and RB-SMPC approaches have higher KPI_E than the CC-MPC approach. The reason is that the inclusion of actuator degradation constraints leads to control actions that sacrifice (if necessary) economic performance in order to guarantee the availability of actuators for a given maintenance horizon. The rationing of actuator degradation also leads to an increase in control action



(a)



(b)

Fig. 4. Operation of the Barcelona DWN with stochastic strategies: management of water storage with stochastic strategies (a), management of actuators with stochastic strategies (b).

smoothness ($\mathrm{KPI}_{\Delta U}$), specially due to the operation of pumps associated with Tank 55 to Tank 63 in the bottom-right part of the DWN diagram (see Fig. 2). With the CC-MPC approach, the volumes of water in the aforementioned tanks are managed near the safety constraints without complete replenishments, while with the RB-SMPC approach the excursion of water is periodic within the full range of operation. The actuator-health management policy forces cycling the operation of several pumps instead of keeping some of them always active, and therefore requires exploiting the full capacity of the related tanks. Furthermore, the safety performance indicator (KPI_S) is drastically higher in the CC-MPC approach; the reason is that the water volume in tanks tends to keep longer time in the limit of constraints, which leads to an increase in the frequency of violation of safety thresholds. Figure 4 illustrates the mentioned behaviour of the system. In general, chance constraints cause an optimal back-off from real constraints as a risk-averse mechanism to face the non-stationary uncertainty involved in the prediction of states.

Table 3 details the water production and electricity costs of each strategy. The CC-MPC approach has quite similar costs to those of the baseline CE-MPC approach, but with the benefit of better handling of constraints, automatic computation of safety stocks and management of risk near to the output bounds. On the other hand, the RB-SMPC approach achieves a constant improvement in electric costs, although at the expense of increasing stored volumes of water (no matter how expensive the source could be) and consequently water costs.

In general, the proposed RB-SMPC approach leads to a higher total closed-loop operational cost if considering only the water and electric costs as indicators for economic performance. This is the price to be paid for enlarging the availability of the actuators by using the proposed health-aware policy. Nevertheless, the economic advantages of the RB-SMPC approach might be seen when considering the long term operation, e.g., $N \ll k < M$, where high corrective maintenance costs (due to the possibly overexploitation and consequent failure of actuators) could appear if the actuator-degradation management policy is not considered. Therefore, the RB-SMPC controller indirectly takes into account the cost maintenance tasks by ensuring that the actuators will be available until a pre-scheduled maintenance horizon M, and consequently it might lead to a better long term closed-loop economic performance.

Figure 5 shows the accumulated degradation of a set of redundant actuators. Notice how the RB-SMPC approach smartly decides to decrease the rate of degradation of Actuator 87 (pump) by distributing the control effort among the other three plotted actuators (which are valves that have smaller coefficients of degradation) according to their flow capacity. This behaviour is equivalent to the one obtained with RB-MPC, the difference being that the chance-constrained approach narrows the maximum level of degradation allowed at each time step according to the uncertainty in the health prediction model of actuators. The wear process with both the CE-MPC and CC-MPC approaches is neglected, compromising the reliability of the supply infrastructure even if safety stocks are optimally computed for a reliable service.

Looking at the results discussed before, the RB-SMPC strategy should be preferred given its tractability for large-scale systems (as shown with the Barcelona DWN case study) and ability to handle probabilistic constraints related to demand and service reliability.

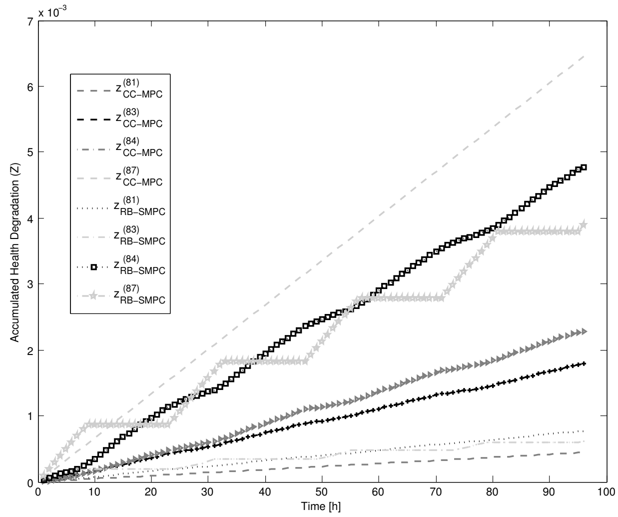


Fig. 5. Degradation of a set of redundant actuators under stochastic approaches.

5. Conclusions

In this paper, a reliability-based economic MPC approach relying on chance-constrained programming has been proposed to deal with the management of generalised flow-based networks, considering both demand uncertainty and actuator-health degradation. The approach avoids relying on heuristic fixed safety volumes such as those used in the CE-MPC or RB-MPC schemes proposed in previous publications, which is traduced in better robust economic performance. This latter is achieved by incorporating dynamic planning of safety stocks and actuators' health monitoring, to assure reliability in the flow supply and to minimise operational costs for a given customer service level.

According to the results obtained with the case study considered, the methodology is applicable to real-size problems. The level of the resultant back-off is variable and depends on the volatility of the forecasted demand and actuator degradation at each prediction step as well as the suitability of the probabilistic distributions used to model uncertainties. The fact of unbounded disturbances in the system precludes the guarantee of robust feasibility with these schemes. Hence, the approach proposed in this paper is based on a service-level guarantee and probabilistic feasibility. Even when RB-SMPC increased the operational costs by around 2.5%, it allowed improving service reliability by more than 90% when compared with a baseline CE-MPC setting.

Future research will be focused on incorporation of parametric uncertainty and unmeasured disturbances in the model, in addition to deriving conditions for robust feasibility and stability. From the economic point of view, considering plant equipment depreciation and actuator ageing models enriched with the effect of maintenance quality and costs could be advantageous for network management. Moreover, it is of interest to extend the results and develop non-centralised stochastic MPC controllers for large-scale complex flow networks.

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