Study of the Bed Velocity Induced by Twin Propellers

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Abstract: Twin propellers without a rudder were studied using a physical model with a fixed clearance distance and three different rotating velocities. Experimental results were compared with results from theoretical expressions developed over the past 50 years for the efflux velocity, axial velocity, and maximum bed velocity. It was found that the efflux velocity equations overestimated the experimental results, whereas the computed axial velocities matched the experimental data reasonably well. However, when maximum bed velocity expressions were compared with experimental results, only one method was found to behave better; overestimation resulted if a quadratic superposition of single jets was used. **DOI: 10.1061/(ASCE)WW.1943-5460.0000382.**

Introduction

The marine transportation industry and regular shipping lines have been experiencing significant growth over the last 20 years. The increments in the ships drafts and the power of engines during the docking and undocking maneuvers can generate serious problems in harbors. Currently, the present propulsion systems have powerful engines that are closer to the toes of the docks, causing severe problems for the stability of the docking platforms. At the same time, the eroded sediment is deposited along the inner harbor, thus reducing the water level and operative zones for other maneuvering vessels.

The equations currently used to compute future erosion are based both on theoretical equations with unrealistic hypotheses far from reality and on experimental studies that use one propeller as the propulsion system (Mujal-Colilles et al. 2016). The maximum bed velocity is always expressed as a function of the efflux velocity, which is defined in the literature as the velocity in the downstream propeller plane; this velocity is used as the first parameter to analyze the seabed erosion. However, the expressions for the efflux velocity are based either on the mass continuity equation or on the momentum balance equation, using experimental coefficients [e.g., Broglia et al. (2013); Hamill (1988); Lam et al. (2012); Stewart (1992)]. Both expressions of the efflux velocity are only valid for single propellers, and the effects of two propellers are not included in them.

This paper deals with experimental values obtained for twin-propeller vessels without rudders as the main propulsion system. Experimental results are compared with results from theoretical equations developed over the past 50 years.

Methodology

Experimental Setup

Physical experiments were performed in the Laboratory of Marine Engineering (LIM) at UPC-BarcelonaTech. LaBassA (Fig. 1) is a rectangular concrete tank measuring 12.5 × 4.6 × 2.5 m with three lateral windows to monitor the phenomenon currently running inside the tank. Experiments were performed using two propellers with four blades, each with a diameter D_p of 25.4 cm, a pitch ratio p' of 0.94, and expanded area ratio β of 0.75. Propellers were located at one end of LaBassA with a clearance distance from the bottom of $h_p = 26$ cm (Fig. 2) and a water depth of 70 cm using bollard pull conditions.

Three different rotating velocities (n = 300, 350, and 400 rpm) were measured using five acoustic Doppler velocimeters (ADVs) hanging from an electronic moving reference system and located at several positions to record the magnitude of the velocity decay along the three axes. Table 1 shows the measuring points in the three coordinates assuming that the origin of the reference axis is located at the axis of symmetry at the bottom of LaBassA, as shown in Figs. 1 and 2. A total of 220 points were measured along the three dimensions of the flume.

Initial tests using ADVs located in particular areas of the tank were performed to verify that the thrusters were far from the opposite wall. Lateral walls were also considered to be far enough after the results obtained with the initial tests (Fig. 2) showed that the propeller jet was able to develop freely and resulted in minimum influence on the convective cells created in the tank.

Fig. 2 plots the thruster system with the main distances used during the setup of the experiments and the position of the origin of the coordinate system. The rotating system was symmetric with the right propeller, P1, rotating in a counterclockwise direction and the left propeller, P2, rotating in a clockwise direction at the same speed. Error in the rotational speed was on the order of 10%, with a 3% difference from one propeller to the other; the right thruster rotated slightly faster than the left thruster. Table 2 shows the relative errors found between the theoretical input value of the rotational speed and the real rotation of each propeller. These differences between the theoretical and real rotational speeds are important in terms of the thrust force (a 10% error in rotational speed means

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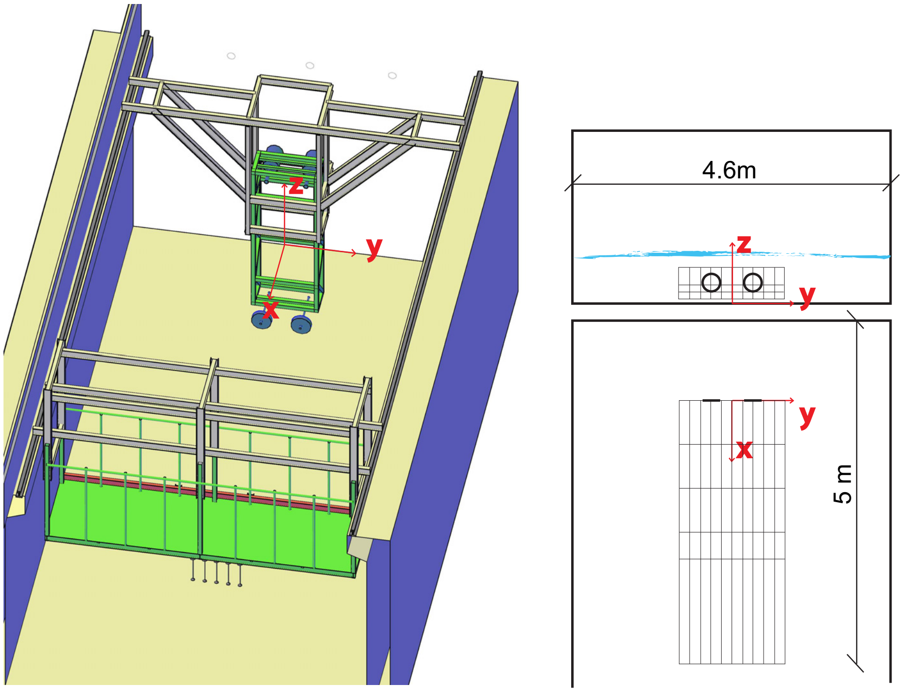

(a) (b)

Fig. 1. (a) Experimental setup in LaBassA (LIM/UPC-BarcelonaTech), with the center of reference located at the symmetry axis in the bottom of the tank; (b) measuring grid used in the experiments

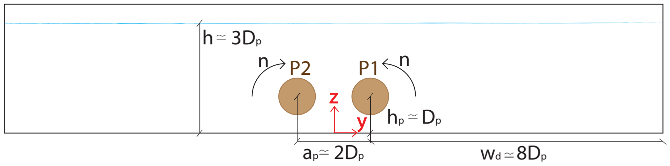

Fig. 2. Thruster system

Table 1. Scenarios and Measuring Points

n (rpm)	x/D_p	y/D_p	z/D_p
300	2.5	0	0.2
350	5	±0.6	0.6
400	7.5	±1.2	1
	10	±1.8	1.8
	15	±2.4	
		±3.0	

Table 2. Relative Errors of the Rotational Speed

	Relative error (%)	
n (rpm)	Left propeller	Right propeller
100	15.0	18.5
200	9.5	12.0
300	7.5	9.0
400	7.5	8.0

more than a 20% change in thrust force, and a 3% difference in rotational speed between propellers is translated as a 6% of difference in the thrust force), but they have a low influence on the results presented herein, as will be further discussed. However, errors have been corrected for future experiments. Rotational speed values were chosen using the upper limit of the structure with a security factor as a reference value (≈500 rpm). In an attempt to reproduce real velocities downstream of the ship's propellers, lower values of the rotational speed were avoided.

Theoretical Aspects

The PIANC (2015) guidelines were used as the reference document to compare the results of the physical model with those of the theoretical formulas given by several authors over the past 50 years [i.e., Bergh and Magnusson (1987); Fuehrer et al. (1987); Hamill and Johnston (1993); Hamill et al. (2004); Hamill (1988); Johnston et al. (2013); Stewart (1992)].

Efflux velocity is defined as the mean axial velocity at the outlet of propeller systems without influence from the rudder, keel, and wall. This velocity is defined for the initial zone of flow establishment, normally delimited for $x \approx 2.5D_p$ (Hamill 1987). Although the influence of a second propeller may change this reference distance or increment the mean velocity at a parallel plane located around 2.5 times the propeller diameter, the authors have considered this distance to allow for comparison of the experimental results of twin-propeller models with those of the theoretical formulas.

Efflux velocity for single propellers based on the momentum equation, Eq. (1), is a function with a dependent parameter called the thrust coefficient, K_T, which is usually given by the propeller manufacturer.

$$V_0 = CnD_p\sqrt{K_T} \qquad (1)$$

The constant parameter, C, varies from 1.33 [e.g., Hamill and Johnston (1993)] to 1.59 from the theoretical assumptions in the momentum equation. Some other authors have developed experimental studies to find empirical relations between the constant parameter and the pitch ratio, the projected area ratio, or the hub diameter (Hamill and McGarvey 1996; Stewart 1992).

However, the thrust coefficient is not always available, and a second expression for the efflux velocity for single propellers can also be used, as follows:

$$V_0 = 1.48\left(\frac{f_p P_D}{\rho_w D_p^2}\right)^{\frac{1}{3}} \tag{2}$$

where f_p = percentage of installed engine power, f_p = 5–15%; P_D = maximum installed engine power (W); and ρ_w = density of the water.

As outlined previously, the expressions for the efflux velocity are based on single propellers. In terms of the maximum bed velocity, which is a function of the efflux velocity, there is only one empirical equation, proposed by Fuehrer et al. (1981), for seaborne vessels with a twin propeller and a central rudder, as follows:

$$V_{b,\max} = 0.52V_0\left(\frac{D_p}{h_p}\right)^{0.275} \quad 0.9 < \frac{h_p}{D_p} < 3 \tag{3}$$

Again, Eq. (3) is based on the computation of the efflux velocity, which is limited to a single propeller. Therefore, the effects of the twin-propeller configuration are included in the constant coefficient.

To provide other expressions to compute the maximum bed velocity, the PIANC (2015) guidelines propose two methods to account for the effects of a single propeller without a rudder: linear superposition or quadratic superposition of the maximum bed velocity induced by a single propeller. Linear superposition is more realistic in terms of the induced maximum bed velocity, but because the total impulse of the ship is not doubled, the linear superposition is not physically correct.

Blokland and Smedes (1996) provided an equation for single propellers that uses linear superposition to compute the maximum bed velocity for twin propellers. This equation is restricted to a certain clearance-to-propeller separation ratio, as follows (Blokland and Smedes 1996):

$$V_{b,\max}^{L,BS} = 2\frac{h_p}{r_p}V_{b,\max,\text{single}}^{L,BS} \quad 0.3 < \frac{h_p}{a_p} < 0.5 \tag{4}$$

where $r_p = \sqrt{h_p^2 + (a_p/2)^2}$.

The quadratic superposition proposed by PIANC (2015) is

$$V_{b,\max} = V_{b,\max,\text{single}}\sqrt{2} \tag{5}$$

Eq. (5) is valid only when $h_p/a_p < 0.5$ and can be used for any expression of the maximum bed velocity induced by a single propeller.

The experimental expression for the maximum bed velocity caused by a single propeller of Blaaw and van de Kaa (1978) [Eq. (6)] and the expression proposed by Fuehrer et al. (1981) [Eq. (7)] are computed for twin propellers without a rudder using the quadratic superposition previously described in Eq. (5).

$$V_{b,\max,\text{single}}^{BK} = 0.216V_0\left(\frac{D_p}{h_p}\right) \tag{6}$$

$$V_{b,\max,\text{single}}^{F} = 0.42V_0\left(\frac{D_p}{h_p}\right) \tag{7}$$

At the same time, to validate both of the previous equations, the expressions for the axial velocity along the centerline of the propeller used the former empirical relations for the maximum bed velocities; the expressions for the axial velocity were developed by the same authors.

Albertson et al. (1950) were the first authors to propose a function for the axial velocity along the propeller, which is based on experimental data as well, as follows:

$$V_{\text{axis}}(x) = AV_0\left(\frac{D_p}{x}\right)^a \tag{8}$$

where A = 6.17; and a = 1. Blaaw and van de Kaa (1978) and Fuehrer et al. (1981) used the same expression as Eq. (8) with different coefficients, which are detailed in Table 3. Both the Albertson et al. (1950) and Blaaw and van de Kaa (1978) methods are used for single propellers without a rudder. However, the expression proposed by Fuehrer et al. (1981) can also be used for twin configurations, according to PIANC (2015).

Experimental Results

Mean velocity distributions were analyzed for all three components along the zone of established flow ($x > 2.5D_p$). To guarantee statistically stationary data, previous experiments were run with ADVs located at different points in the tank. Results yielded a total run time of 1 h to reach the stationary state at each point of the propeller flow (Mujal-Colilles et al. 2015). Therefore, to simplify the text throughout the document, mean velocities will be referred to directly as velocity, regardless of the component.

The first evolution of the propeller jet is described using planes parallel to the plane containing the propellers, as shown in Fig. 3, where the background variable is the axial component of the measured velocity. In the efflux plane ($x = 2.5D_p$), the two jets were clearly visible, with no difference between the left and right propellers, regardless of the 3% difference in rotational speed between propellers described in the previous section; raw data are available in Table S1. However, in the next plane ($x = 5D_p$), shown in Fig. 3(b), the two jets disappeared and merged into a single jet. It can also be observed that the jet was directed toward the bottom of the tank at some point between $2.5D_p$ and $5D_p$, as shown in Figs. 3(a and b). The same evolution of the jet along the tank is shown in Fig. 3, which confirms the small influence of the opposite wall in the evolution of the jet because the jet was no longer present at a distance $15D_p$ [Fig. 3(d)], which is almost one-third of the total length of the tank. Therefore, the propeller jet was considered to be negligible at a distance larger than $15D_p$ (∼4 m). However, the convective cells generated at the flume could have affected the lateral evolution of the jet. The black dashed line in Fig. 3(a) is the boundary between the positive and negative axial velocity, thus plotting the returning flow. The progression of the contour plots in Fig. 3 shows that the presence of the lateral walls did not affect the growth in the transverse direction of the flow because the jet increased in width, and the returning flow was located beyond $|y/D_p| > 3$. Thus, the experimental values demonstrate the low influence of the lateral walls on the evolution of the jet.

Table 3. Coefficients for Eq. (8) According to Several Methods

Method	A	a
Albertson et al. (1950)	6.17	1
Blaaw and Van de Kaa (1978)	1.95	1
Fuehrer et al. (1981)	0.9	0.25

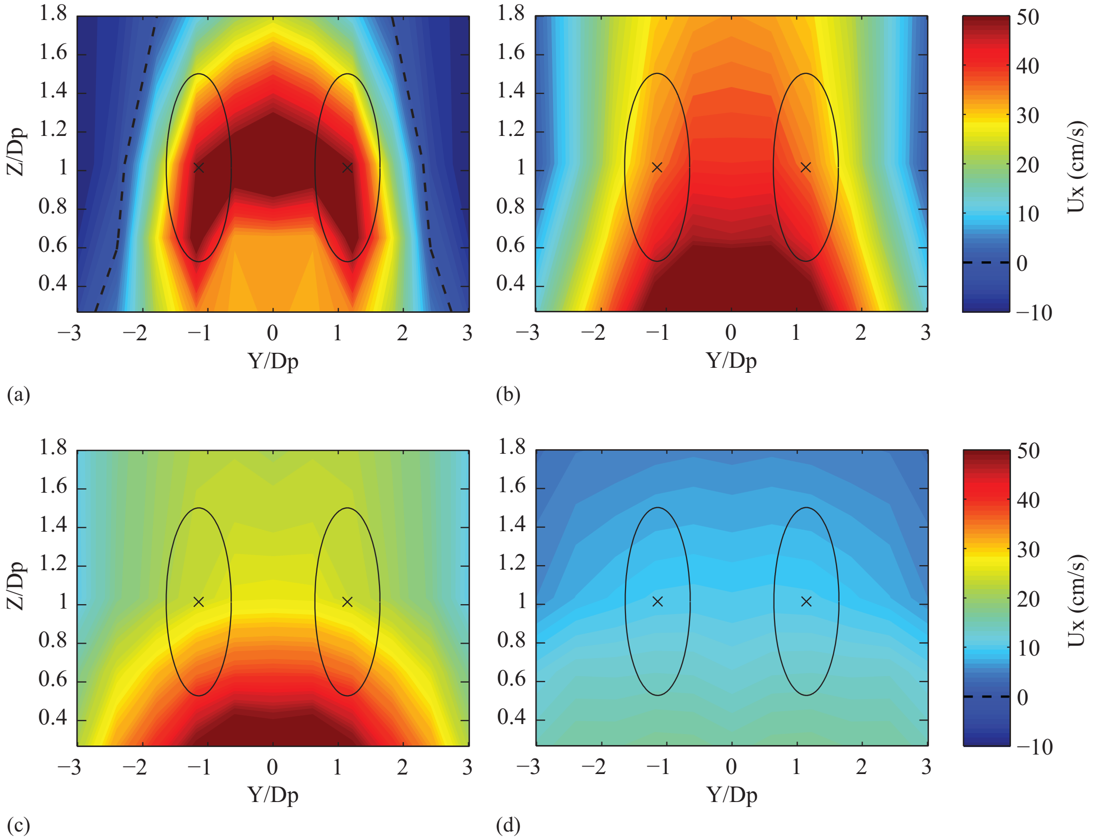

Fig. 3. Axial velocities, U_x, for $n = 400$ rpm in planes parallel to the propellers' plane: (a) $x = 2.5D_p$; (b) $x = 5D_p$; (c) $x = 7.5D_p$; (d) $x = 15D_p$ (Note: Ellipses indicate the helices in real sizes; black dashed line in (a) delimitates the boundary where axial velocity changes direction; data are available in Table S1)

The vertical velocities plotted in Fig. 4 are one order of magnitude lower than the axial velocities, but these contour plots give important information; see the point data in Table S1. As expected, absolute values of the vertical velocities close to the bottom decreased as the distance from the helices increased. That is, at $2.5D_p$, velocities close to the bottom were lower than those at −5 cm/s, whereas at $7.5D_p$, velocities were approximately −1 cm/s; they were almost zero at $15D_p$. The influence of the jet was still visible for vertical velocities at $15D_p$ because the location of the two helices coincided with the zone of higher vertical velocities ($\sim$1 cm/s), but with very low absolute values of velocity. The results in Fig. 4(a) are consistent with the expected values because the rotation of the propellers was still present in this plane. However, further from the propellers' plane, rotational effects dissipated, and when the jet axis reached the bed of the tank [Fig. 4(b)], maximum bed velocities were lower than those in the previous plane. However, the presence of the twin propellers was still visible with the vertical velocities [Figs. 4(b–d)], although the axial velocities showed the opposite. This indicates that the two separate jets were directed toward the bed of the tank at the same angle, maintaining the twin-jet composition in the lower-magnitude components.

The three components of bed velocities recorded with ADVs are shown in Fig. 5, which plots the axial U_x, transverse U_y, and vertical U_z components of the velocity. Unlike Figs. 3 and 4, Fig. 5 plots the planes perpendicular to the propellers' plane (i.e., the planes parallel to the bed of the tank). Axial velocities are the velocity components used to compute the Shields parameter to determine whether there will be erosion or not. In the present case, axial velocities at the bottom were found to increase with the speed of revolution, as expected, but the point with larger maximum axial velocities moved further from the propeller plane [from Figs. 5(a–c)]. This point coincided with the zone of the jet impact on the boundary, which, according to Johnston et al. (2013), occurs at a distance between 5 and $6D_p$; this is confirmed in Fig. 5. At the same time, the transverse velocities, which are shown in Figs. 5(d–f), were of the same order of magnitude as the axial velocities, meaning that the jet spread in the x-direction at the same growth rate as it increased in diameter. This comparison indicates that the opening angle of the jet flow might not be on the order of 10–15° (Hamill 1988) but almost double that amount. Moreover, the transverse velocities [Figs. 5(d–f)] confirm that the center of both jets could merge, but the influence of the twin propellers was maintained along the tank. However, the influence of the vertical velocities [Figs. 5(g–i)] did not increase significantly with the speed of revolution; they decreased abruptly after $7.5D_p$ in all of the cases.

Discussion

To compare the results of the present study with those of formulas from the literature, the efflux velocity was obtained from the maximum velocity in the plane $x = 2.5D_p$ (Fig. 3). As shown in Table 4, the theoretical results obtained with both Eqs. (1) and (2) were approximately 2 times the efflux velocity yielded by the experiments. Eq. (1) was computed with the assumption of a thrust coefficient of 0.35, obtained from Bernitsas et al. (1981), and an experimental coefficient of $C = 1.33$ as proposed by Hamill (1987). Power

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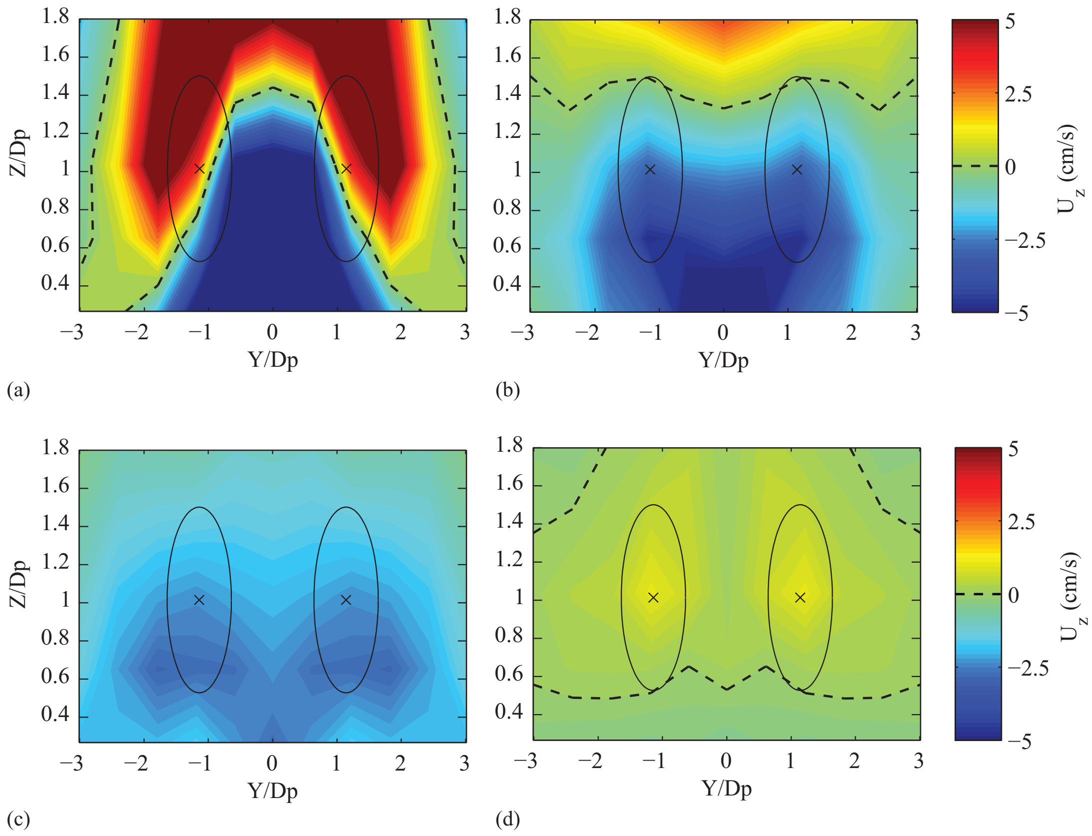


Fig. 4. Vertical velocities, U_z, for $n = 400$ rpm in planes parallel to the propellers' plane: (a) $x = 2.5D_p$; (b) $x = 5D_p$; (c) $x = 7.5D_p$; (d) $x = 15D_p$ (Note: Data are available in Table S1)

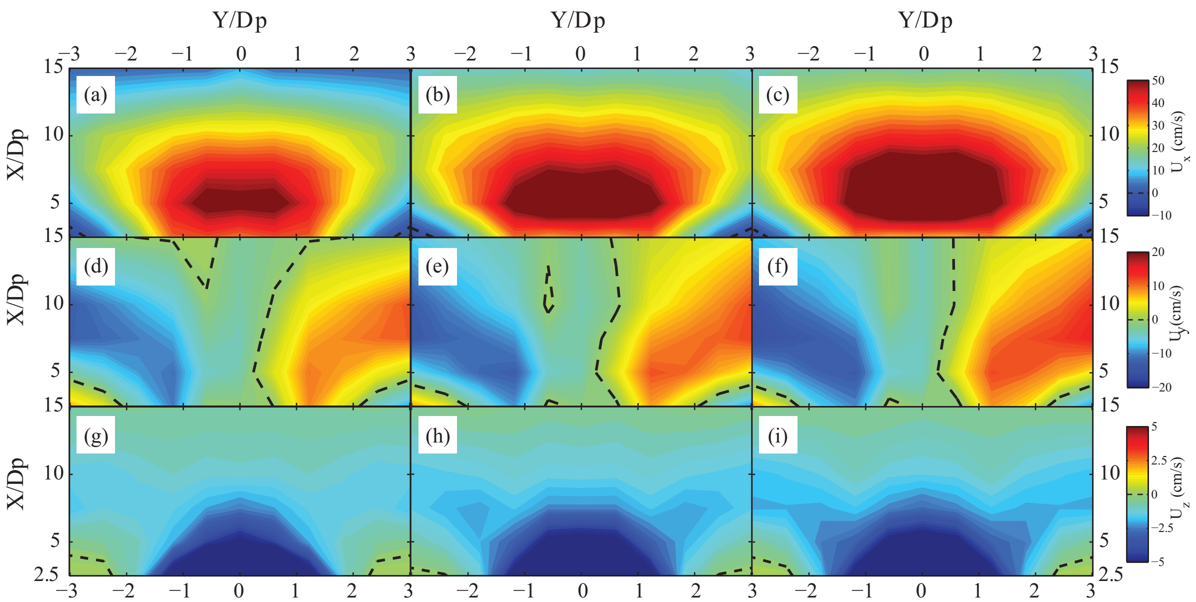


Fig. 5. Axial, transverse, and vertical velocities at $z/D_p = 0.25$ of LaBassA: (a–c) $n = 300$ rpm; (d–f) $n = 350$ rpm; (g–i) $n = 400$ rpm

values of Eq. (2) were measured in situ in the laboratory and yielded final values of Eq. (2) similar to the results from Eq. (1).

Table 4 shows that the experimental results were not substantially different between scenarios. However, the scenario with a speed of 350 rpm had a higher efflux velocity value than the scenario with a speed of 400 rpm. This may be attributable to the location of the efflux velocity plane. The efflux velocity plane is located beyond $2.5D_p$ (Felli et al. 2011), so if the efflux velocity is measured at $3D_p$, this error should disappear. For instance, in the plane $x = 5D_p$, higher values of velocity correspond to higher values of revolution, as shown in Fig. 6; this indicates that for twin propellers, the flow was not totally established at $2.5D_p$.

In Fig. 6, the three scenarios reveal that the axial velocity at the zone between the propellers [Fig. 6(a)] was higher than the axial velocity at the center of the propeller [Fig. 6(b)]. This is attributable to the cumulative effect of each propeller given the small distance between the propellers in the present experiment. It is expected that a configuration with a larger separation between propellers should not produce this effect. In this section, axial velocities at the center of symmetry will be used to compare the experimental values with the theoretical expressions proposed in the previous section.

Fig. 6(a) shows that the axial velocity at the axis of symmetry for the case of $n = 350$ rpm was slightly larger than the axial the velocity for $n = 400$ rpm. Fig. 6(b) shows that larger velocity values at $x = 2.5D_p$ were obtained for the scenarios with low-speed velocity. The first result combined with the second indicates that the small differences in the rotational velocities of the propellers created a nonsymmetric flux for which the axis of symmetry between the propellers might not have been the point with higher values. However, it is important to point out that small errors in velocity will be corrected in the future to avoid these problems.

The axial velocities along the x-axis are plotted in Fig. 7, and a comparison of the experimental results (black line) with the theoretical results detailed in Eq. (8) and Table 3 reveals that all of the theoretical expressions overestimated the axial velocity. The axial velocity shown in Fig. 7 was located at the axis of symmetry of LaBassA and the middle point between both propellers. If axial velocity located along the x-axis at the center of the propellers had been used, this overestimation would be even larger, as shown in Fig. 6. In any case, it seems that the method proposed by Blaaw and van de Kaa (1978) is the only method that was found to fit the experimental data with reasonably accurate results, regardless of the low estimation.

Finally, the maximum bed velocities obtained in the experimental results were also compared with the results of the theoretical expressions. It is important to recall that the theoretical expressions used herein considered the action of twin propellers. Fuehrer et al. (1981) developed an equation for seaborne vessels with twin propellers and twin rudders, although the rudder angle is not an independent variable in the function [see Eq. (3)]. The Blokland and Smedes (1996) method is used for twin propellers considering a linear superposition of two single propellers. According to PIANC (2015) guidelines, with linear superposition, the total impulse increases, which is not consistent with reality; however, if quadratic superposition is used (PIANC 2015), the total impulse of the jet remains constant, but velocities are underestimated when both jets start to merge. Several authors suggest the use of quadratic superposition for single-propeller maximum bed velocity (Blaaw and van de Kaa 1978; Fuehrer et al. 1981) because the maximum velocity at the bed for a single propeller is computed using each method in Eqs. (6) and (7), respectively.

Table 4. Measurements at $x = 2.5D_p$

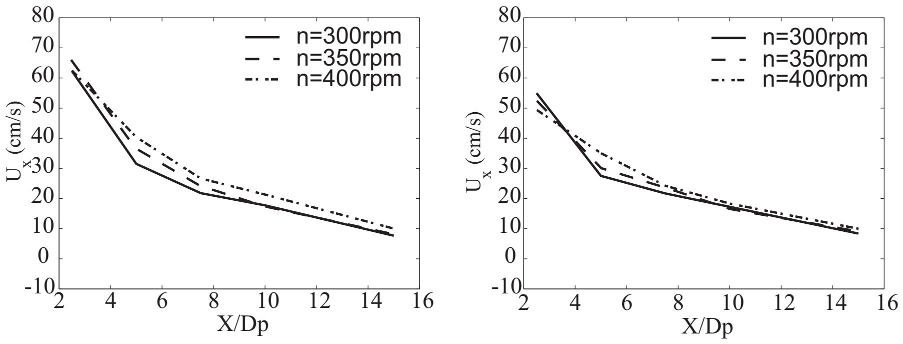
	Efflux velocity [V_0 (m/s)]		
Value type	300 rpm	350 rpm	400 rpm
Experimental	0.62	0.66	0.62
Theoretical [Eq. (1)]	1.00	1.17	1.33
Theoretical [Eq. (2)]	1.04	1.20	1.34

Maximum bed velocities are shown in Fig. 8; in this case, the Fuehrer et al. (1981) method was found to more accurately predict the maximum bed velocities. In contrast to the theory described in the PIANC (2015) guidelines, the quadratic approximation using the Fuehrer et al. (1981) method overestimated the maximum bed velocity. Both superpositions of the jets using the formulas of Blokland and Smedes (1996) and Blaaw and van de Kaa (1978) clearly underpredicted the experimental results.

The results shown in Fig. 8 are consistent with the results obtained for the axial velocities because the Fuehrer et al. (1981) method always gives larger values than the Blaaw and van de Kaa (1978) method. However, the overestimation detected for the axial velocities was not as large as the overestimation found for the maximum bed velocities, indicating that the relation between axial and maximum bed velocities should be further investigated for twin propellers. In fact, the best method to approximate maximum bed velocities, the Fuehrer et al. (1981) method, clearly overpredicted the axial velocity, whereas the maximum bed velocity formulas proposed by Blaaw and van de Kaa (1978) were found to be best at fitting the axial velocity but underestimated the maximum bed velocity.

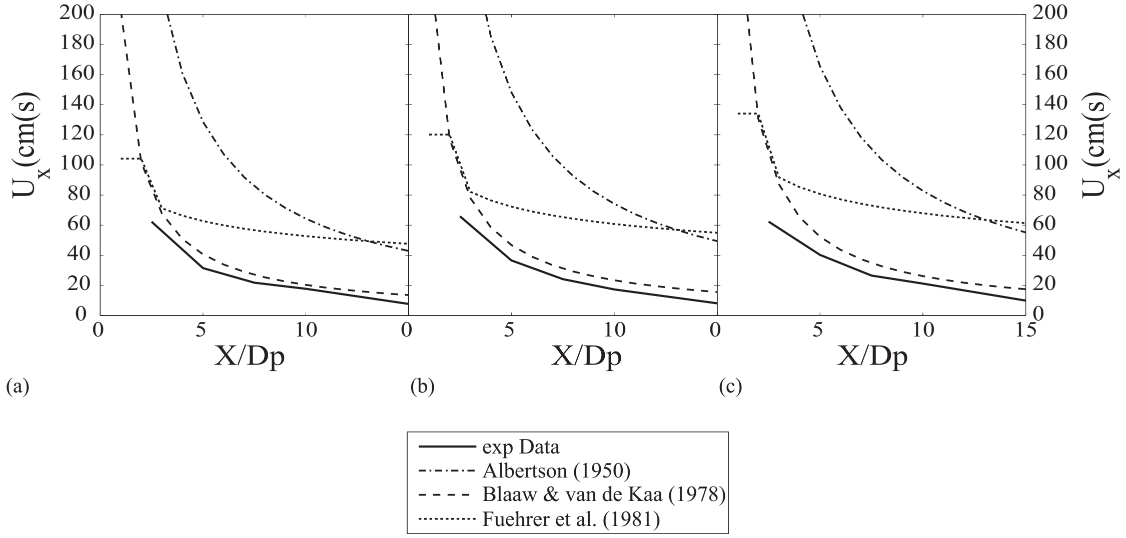
Conclusions

An accurate description of jet hydrodynamics is needed to estimate the forces acting on quay structures and harbor basin beds. Likewise, an understanding of the relation between these forces and variables as controlled by the maneuvering of vessels (e.g., power propeller, clearance distance, rudder angle, pitch ratio) can help with the maintenance of harbor structures and their management.



(a) (b)

Fig. 6. Axial velocities: (a) axis located at the center of symmetry, $y = 0$; (b) axis located at the center of the right propeller, $y = a_p/2$



(a) (b) (c)

Fig. 7. Theoretical and experimental values of axial velocity along the axis of symmetry: (a) 300 rpm; (b) 350 rpm; (c) 400 rpm

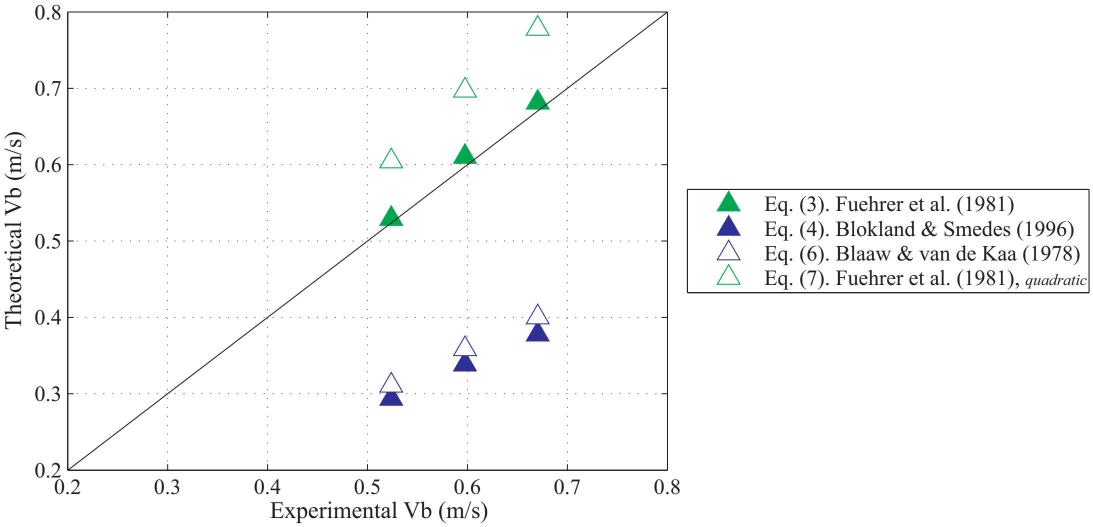


Fig. 8. Theoretical and experimental values of the maximum bed velocity, $V_{b,\max}$

The contribution of this research consists of the application of formulas available in the literature to an experimental study using twin propellers without a rudder, with a fixed clearance distance and separation between propellers. Most of the expressions published in the literature are used by structural and design engineers; they have been developed for single propellers and are extrapolated to the case of twin propellers without the support of experimental research.

Experimental results compared with the theoretical work published so far indicate that the efflux velocities obtained during experiments were clearly lower than those predicted by the theoretical results. At the same time, the formulas proposed in the literature to obtain axial velocities were found to overpredict the experimental results. The best equation to predict axial velocities for twin propellers was determined to be the method proposed by Blaaw and van de Kaa (1978), although this method clearly underpredicted the maximum bed velocity. However, the formula of Fuehrer et al. (1981), particularly developed for twin propellers, was able to match the bed velocity experimental results described in this research reasonably well.

In the present experiments, viscosity did not influence the difference between the theoretical and experimental values because all scenarios took place in a highly turbulent regime.

Supplementary data are provided in Table S1 to facilitate comparison with the real results and to provide information to be used for future benchmarks; these data are available to any interested researcher.

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Notation

The following symbols are used in this paper:

A = constant coefficient in Eq. (8);
a = exponential coefficient in Eq. (8);
a_p = distance between the propellers;
C = constant coefficient in Eq. (1);
D_p = propeller diameter;
f_p = percentage of installed engine power;
h_p = clearance distance;
K_T = thrust coefficient;
P_D = maximum installed engine power;
P' = pitch-to-diameter ratio;
V_{axis} = axial velocity;
$V_{b,\max}$ = maximum bed velocity;
$V_{b,\max}^{L,BS}$ = maximum bed velocity proposed by Blokland and Smedes (1996);
$V_{b,\max,\text{single}}$ = maximum bed velocity for a single propeller;
$V_{b,\max,\text{single}}^{BS}$ = maximum bed velocity for a single propeller proposed by Blokland and Smedes (1996);
$V_{b,\max,\text{single}}^{BK}$ = maximum bed velocity for a single propeller proposed by Blaaw and van de Kaa (1978);
$V_{b,\max,\text{single}}^{F}$ = maximum bed velocity for a single propeller proposed by Fuehrer et al. (1981);
V_0 = efflux velocity;
W_d = wall distance;
β = expanded area ratio; and
ρ_w = water density.

Supplemental Data

Table S1 is available online in the ASCE Library (www.ascelibrary.org).

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