# Diversity MDIR Receiver for Space-Time Dispersive Channels

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***Abstract*[1]*: A particular property of the cellebrated MDIR receiver is introduced in this communication, namely, the fact that full exploitation of the diversity is obtained with multiple beamformers when the channel is spatially and timely dispersive. Therefore a new structure is developped which provides better performance. The hardware need for this new receiver may be obtained through reconfigurability of the RAKE architectures available at the base station. It will be tested in the FDD mode of UTRA.*

## I. Introduction

The advent of the 3rd generation of mobile communications systems has been accompanied by the recognition of the large increase in system capacity that can be obtained from the use of adaptive antenna arrays. Care has been taken in the definition of the standard to include capabilities for space-time processing of the signals incoming and radiated from the base stations. Therefore, a panoplia of receivers have arosen which improve the performance of the single sensor receiver without conveying the complexity of the optimum multiuser receiver [Jung][Vidal][Rüpf]. One of the most succesful receivers in this trading is the MDIR [Lagunas] whose capabilities for reconfigurability and use in both TDD and FDD modes have been demonstrated [Mestre]. Some unpublished properties of this receiver are reported here, which allow further exploitation of the diversity present in the dispersive mobile channel. Section II shows two possiblities for spatial front-ends: a reduced complexity weighted V-RAKE receiver and the proposed Extended MDIR (E-MDIR). Similarities and differences between are shown and performances compared in terms of BER equations in section III. Simulations in section IV will demonstrate the superiority of the E-MDIR receiver in realistic conditions. Results show a significant improvement in the probability of error with respect to conventional approaches, that is, only spatial beamforming or only VRAKE combining

## II. Spatial front-ends

### *A. Noise-plus-interference matrix inversion (NIMI) receiver*

This receiver is a simplified version of the Weighted V-RAKE, that is, a Vector-RAKE with spatial pre-whitening matrix. The pre-whitening operation can be seen as a bank of beamformers $\mathbf{b}_j$ which are given by the eigenvectors of the spatial correlation matrix (see figure 1)[Vidal]. The optimum combining structure comes from the fact that the errors between branches are uncorrelated. The nature of this receiver is easily seen from a simple case: assume the case of $P<M$ point interferers. If vectors $\mathbf{b}_i$ are taken as the noise eigenvectors of $\mathbf{R}_{w,s}$ each one acts as a spatial interference canceller.

### *B. Extended Matched Desired Impulse Response (EMDIR) receiver*

A different way to build a spatial front-end is to obtain a spatial combiner **b** that maximizes the SINR at its output. Let us first define the signal model of equation as:

$$\mathbf{Y} = \mathbf{DH} + \mathbf{W} \tag{1}$$

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where matrix **D** is a Toeplitz matrix built at chip time from QPSK complex spreaded and scrambled symbols:

$$\mathbf{D}=\begin{bmatrix} d(L-1) & d(L-2) & \cdots & d(0) \\ d(L) & d(L-1) & \cdots & d(1) \\ \vdots & \vdots & \ddots & \vdots \\ d(N-1) & d(N-2) & \cdots & d(N-L-1) \end{bmatrix} \in \mathbb{C}^{(N-L+1)\times L} \tag{2}$$

where $N$ stands for the number of chips in the pilot, $L$ is the length of the estimated physical channel, and all terms $d(n)$ belong to the set {-1-$j$, -1+$j$, 1+$j$, 1-$j$}. It is usually the case that the symbols in matrix **D** are uncorrelated, so $\mathbf{D}^H\mathbf{D} \cong N\mathbf{I}$. **H** contains (column-wise) the $L$-length response of the physical propagation channel seen at each sensor:

$$\mathbf{H}=[\mathbf{h}_1 \quad \mathbf{h}_2 \quad \cdots \quad \mathbf{h}_M] \in \mathbb{C}^{(L+1)\times M} \qquad \mathbf{h}_i^T=[h_i(0) \quad h_i(1) \quad \cdots \quad h_i(L-1)] \tag{3}$$

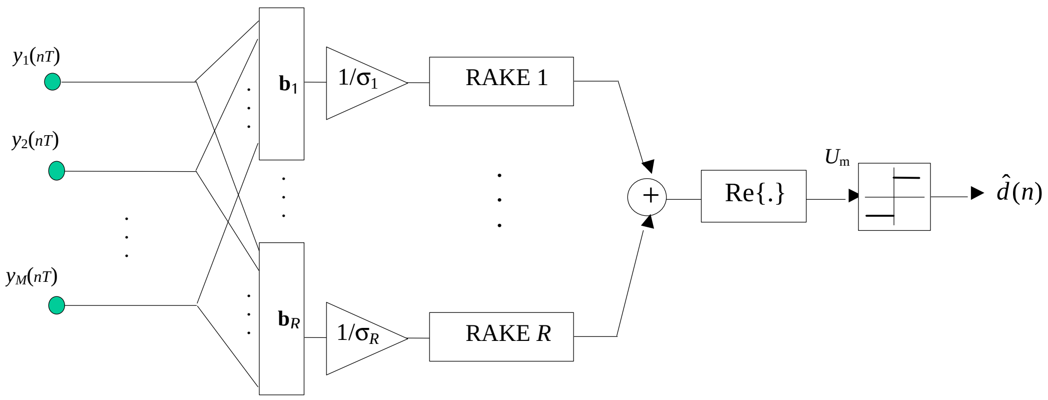


**Figure 1. Optimum combining receiver with reduced rank approximation (M>R). The gains at each branch are given by the eigenvalues of the noise-plus-interference spatial correlation matrix for the NIMI receiver.**

The MDIR receiver developed in [Lagunas] is then given by:

$$\min_{\mathbf{b}} \mathbf{b}^H \mathbf{R}_{w,s}\mathbf{b} \qquad subject\ \ to \quad \mathbf{b}^H\mathbf{H}^H\mathbf{D}^H\mathbf{D}\mathbf{H}\mathbf{b} \ = N \tag{4}$$

It may be shown [Lagunas] that the choice of the beamformer is obtained as the generalised eigenvector **b** of the equation:

$$\mathbf{R}_{w,s}\mathbf{b} = l\mathbf{H}^H\mathbf{D}^H\mathbf{D}\mathbf{H}\mathbf{b} \tag{5}$$

associated to the minimum eigenvalue λ, and the impulse response seen at the output of the beamformer $\mathbf{b}_j$ is $\tilde{\mathbf{h}}_j = \mathbf{H}\mathbf{b}_j$. Being fixed to 1 the signal power at the output of the beamformer $\mathbf{b}_j$ through the restriction in equation 4, $\lambda_j$ takes the value of the inverse of the signal-plus-noise-plus-interference power at the output of the beamformer $\mathbf{b}_j$. At this point, a straightforward question arises: could the different beamformer outputs be efficiently combined as in figure 1? Or, in other words, is there any diversity inherent to the multiple beamformers obtained from the MDIR solution? The answer to this question is given by the following property:

**Property 1**. The noises at the output of the M beamformers obtained from equation 5 are uncorrelated, unless the eigenvalues associated are equal.

**Proof**. Let us extend equation 5 with all the eigenvectors as:

$$\mathbf{R}_{w,s}\mathbf{B} = \mathbf{H}^H\mathbf{D}^H\mathbf{DHBS} = \mathbf{R}_{d,s}\mathbf{BS} \quad (6)$$

By left-multiplying with the conjugate transpose of **B** we obtain on the left hand side of the equation, the correlation matrix of the noises at the output of the different beamformers:

$$\mathbf{B}^H\mathbf{R}_{w,s}\mathbf{B} = \mathbf{B}^H\mathbf{R}_{d,s}\mathbf{BS} = \mathbf{SB}^H\mathbf{R}_{d,s}\mathbf{B} \quad (7)$$

We have used in the last equality the fact that the left-hand side of the equation is an hermitian matrix and the eigenvalues are real. With no loss of generality, let us assume that S has one multiple eigenvalue σ, so since the product above is commutative it can be written as:

$$\mathbf{SB}^H\mathbf{R}_{d,s}\mathbf{B} = \begin{bmatrix} s\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}' \end{bmatrix}\begin{bmatrix} \mathbf{C} & \mathbf{D}^H \\ \mathbf{D} & \mathbf{E} \end{bmatrix} = \begin{bmatrix} \mathbf{C} & \mathbf{D} \\ \mathbf{D}^H & \mathbf{E} \end{bmatrix}\begin{bmatrix} s\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}' \end{bmatrix} \quad (8)$$

By operating it is easy to see that **D** = **0** and that **E** has to be diagonal. Therefore we can conclude that:

$$\mathbf{B}^H\mathbf{R}_{w,s}\mathbf{B} = \begin{bmatrix} \sigma\mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}'' \end{bmatrix} \text{ where } \mathbf{S}'' \text{ is diagonal} \quad (9)$$

Therefore, we can use the multiple beamformers provided by the MDIR receiver and coherently combine their outputs according to figure 1. Note however that the number of eigenvectors given by equation (5) depend on the rank of the signal matrix $\mathbf{H}^H\mathbf{D}^H\mathbf{DH}$ (assuming full rank of the noise-plus-interference matrix), each yielding a different beamformer output with different noise level, given by the eigenvalue. A rank higher than one, precludes the existence of both spatial and temporal dispersion in the desired user. Another property of interest for the analysis of the performance of the MDIR receiver is the following:

**Property 2**. The channels associated to the beamformers having different eigenvalues $\tilde{\mathbf{h}}_i = \mathbf{Hb}_i$ are orthonormal, that is, $\tilde{\mathbf{h}}_i^H\tilde{\mathbf{h}}_j = d_{i-j}$ .

**Proof**. The diagonality of matrix $\mathbf{B}^H\mathbf{H}^H\mathbf{D}^H\mathbf{DHB}$ stated in property 1 (equation 8), along with the restriction in equation 4 completes the proof.

## III. BER equations

BER equations are derived here for the two receivers, assuming Gaussian temporally white interference. This assumption is quite restrictive and conclusions should be drawn carefully.

### *A. NIMI Receiver*

For the M sensor NIMI receiver, the multiple beamformer operation in figure 1 can also be expressed as a spatial whitening of the received signal [Vidal]. The selected R branch outputs are maximum-ratio combined with orthogonal interference plus noise terms $n_r$. The resulting decision variable $U_m$ (see figure 1) is given by:

$$U_m = b_m\sum_{r=1}^{R}\sum_{l=0}^{L}\left|h_{rl}^w\right|^2 SF.E_{Pc} + \sum_{r=1}^{R} n_r = b_m \, trace\left(\mathbf{HR}_w^{-1}\mathbf{H}^H\right) SF \, E_{Pc} + \sum_{r=1}^{R} n_r \quad (10)$$

The variance is given by (since the noise is spatially whitened after the spatial processor):

$$s_n^2 = \sum_{r=1}^{R} s_r^2 = \frac{E_{Pc}\ SF}{2} trace(\mathbf{H}\mathbf{R}_w^{-1}\mathbf{H}^H) \tag{11}$$

and the BER yields ($R$ is the number of branches in figure 1):

$$BER_{NIMI} = Q\left(\sqrt{2E_b trace(\mathbf{H}\mathbf{R}_w^{-1}\mathbf{H}^H)}\right) = Q\left(\sqrt{2E_b trace(\sum_{i=1}^{R} l_i \mathbf{v}_i \mathbf{v}_i^H)}\right) = Q\left(\sqrt{2E_b \sum_{i=1}^{R} l_i}\right) \tag{12}$$

where $\lambda_i$ are the P non-zero eigenvalues of the Hermitian matrix $\mathbf{H}\mathbf{R}_w^{-1}\mathbf{H}^H$.

***B. EMDIR Receiver***

For the M sensor EMDIR, one has to analyze the number P of significant branches (the number of different non-zero eigenvalues). The P outputs are combined with orthogonal interference plus noise terms: $n_r$ are incorrelated as it has been previously demonstrated. If the resultant decision variable is considered gaussian distributed:

$$U_m = b_m \sum_{i=1}^{R} \tilde{\mathbf{h}}_i^H \tilde{\mathbf{h}}_i\ SF\ E_{Pc} + \sum_{i=1}^{R} n_i = \begin{Bmatrix} \tilde{\mathbf{h}}_i = \mathbf{H}\mathbf{b}_i \\ \tilde{\mathbf{h}}_i^H \tilde{\mathbf{h}}_i = 1 \end{Bmatrix} = b_m R\, SF\ E_{Pc} + \sum_{i=1}^{R} n_i \tag{13}$$

Since, as previously proved, the noises at each branch are uncorrelated (and assumed all temporally white), the variance is now computed as:

$$s_n^2 = \sum_{i=1}^{R} s_i^2 = \frac{SF\ E_{Pc}}{2} \sum_{i=1}^{R} l_i \tag{14}$$

where $\lambda_j$ is the noise power associated to the $j$-th branch:

$$\begin{aligned} \mathbf{R}_w \mathbf{b}_j = l_j \frac{1}{N} \mathbf{H}^H \mathbf{D}^H \mathbf{D}\mathbf{H}\mathbf{b}_j \quad &\Rightarrow \quad \frac{1}{l_j}\mathbf{b}_j = \mathbf{R}_w^{-1}\mathbf{H}^H\mathbf{H}\mathbf{b}_j \\ \frac{1}{l_j}\tilde{\mathbf{h}}_j = \mathbf{H}\mathbf{R}_w^{-1}\mathbf{H}^H \tilde{\mathbf{h}}_j \quad &\Rightarrow \quad \tilde{\mathbf{H}} \mathrm{L}^{-1} = \mathbf{H}\mathbf{R}_w^{-1}\mathbf{H}^H \tilde{\mathbf{H}} \end{aligned} \tag{15}$$

Using property 2 ($\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H = \mathbf{I}$) then it turns out that:

$$s_n^2 = \sum_{i=1}^{R} s_i^2 = \frac{SF\ E_{Pc}}{2} \sum_{i=1}^{R} l_i = \frac{SF\ E_{Pc}}{2} trace\left(\mathbf{H}\mathbf{R}_w^{-1}\mathbf{H}^H\right)^{\#} \tag{16}$$

where $(.)^{\#}$ is the pseudoinverse operator. Note that the $\lambda$ are the same as in equation 12. The BER yields:

$$BER_{EMDIR} = Q\left(\sqrt{\frac{2E_b R^2}{trace\left(\mathbf{H}\mathbf{R}_w^{-1}\mathbf{H}^H\right)^{\#}}}\right) = Q\left(\sqrt{\frac{2E_b R^2}{trace\left(\sum_{i=1}^{R} \frac{1}{l_i} \mathbf{v}_i \mathbf{v}_i^H\right)}}\right) = Q\left(\sqrt{\frac{2E_b R^2}{\sum_{i=1}^{R} \frac{1}{l_i}}}\right) \tag{17}$$

It can be concluded that:

- The E-MDIR and W-VRAKE are not equivalent, but in two cases both receivers achieve the same BER (in Gaussian scenario): when $\mathbf{R}_w$ is diagonal and, when $R$=1 is chosen. Otherwise, according to the equations 12 and 17, NIMI outperforms E-MDIR, which is not an unexpected result since we have derived the equations considering Gaussian noise-plus-interference.

- For the non-Gaussian case, simulations in section IV indicate that E-MDIR is better. Its performance depends on the noise-plus-interference spatial distribution as well as the desired signal spatial-temporal distribution: highly dispersive channels yield a matrix $\mathbf{H}^H\mathbf{H}$ with low dispersion of eigenvalues, so the use of multiple branches becomes interesting. Equations 12 and 17 can be used to have an estimate of the gain obtained in using diversity.
- The NIMI receiver requires the computation of the noise-plus-interference matrix, which is usually difficult to compute and may lead to numerical errors. On the contrary, the EMDIR receiver may be easily reformulated using $\mathbf{R}_y$ instead of $\mathbf{R}_w$ [Lagunas].
- The architecture in figure 1 is dynamically re-configurable by nature. Assume that the number of users at a certain moment is low. Certainly, some of the RAKE combiners at the base station will be unused and therefore (hardware permitting) might be incorporated to additional branches.

# IV. Experimental performance evaluation

### *A. Propagation channel model*

In order to evaluate the receiver in a realistic mobile scenario, we have carried out simulations based on a Gaussian stationary uncorrelated hypothesis for the channel, assuming independence between angular and Doppler spread, as it has been experienced from measurements taken in downtown Stockholm in the 1,8 GHz band [Pedersen]. There, it is empirically shown that azimuth spectrum follows a Laplacian law, along with Gaussian distribution for the directions of arrival ($f$) around the mean angular position of the user. The angular spread (that is the standard deviation of the Gaussian, $s_f$) is taken 8º. The number of rays impinging the array is fitted as a Poisson random variable of mean 25. The power delay spread will be based on the pedestrian and vehicular models for temporal spreading recommended in the SMG2 documents for UTRA [ETSI-UTRA]. The amplitude associated with each propagation path ($a$) is a complex Gaussian random variable whose power decreases as the time delay and the angular direction of arrival with respect to the mobile position increase. A classical Clarke's bath-shaped Doppler spectrum is obtained by assuming multiple reflections close around the mobile. The carrier frequency is 2,0 GHz. All sensors have flat spatial response in a sectored area of 120º, and are linearly and uniformly spaced at $d/\lambda$=0,5.

### *B. Simulations*

A set of simulations has been performed using up to 9 users of spreading factor 16 in the FDD mode of UTRA [ETSI-UTRA]. Equal power for the traffic and the pilot bits has been assumed. The channel has been estimated using all the chips of the pilot, so the autointerference of the traffic is very low. All users are assumed to have controlled transmitted power with no errors (1 dB) errors showed no difference in performance. $E_b/N_o$ is 15 dB. The speeds of users are 3 km/h, for the pedestrian channel and 50 Km/h for the vehicular channel. The E-MDIR, with different number of eigenvectors, the conventional V-RAKE and NIMI have been tested and its performance plotted in figure 2.

### *C. Evaluation of results*

In all cases, the performance was superior to the conventional VRAKE receiver, so we can conclude that substantial gain from the use of spatial beamforming is achieved. E-MDIR also performs better than the NIMI receiver in all cases. Not surprisingly, a significant reduction in BER is obtained for the E-MDIR when using 2 beamformers, in particular for the vehicular channel, whose delay spread is larger than for the pedestrian. This is verified in figure 3, where the cumulative function of the ratio between the increasing eigenvalues and the maximum eigenvalue of the MDIR receiver is depicted, for different number of active users. It is clear that the second eigenvalue (that is, the SNIR associated to the output of the second beamformer) is always significant, although it decreases slightly as the number of active users increase. Note that the third eivenvalue is only significant for a low number of users, so it can be discarded.

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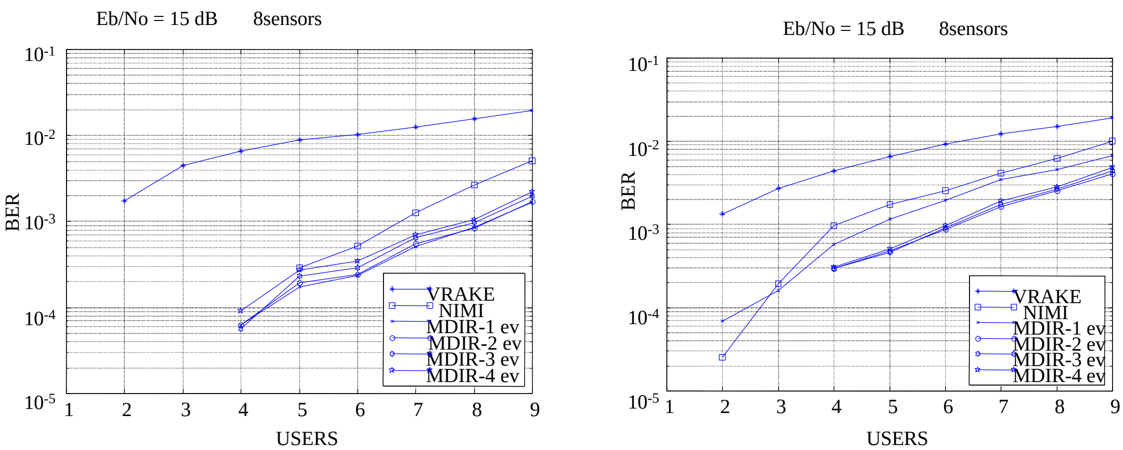
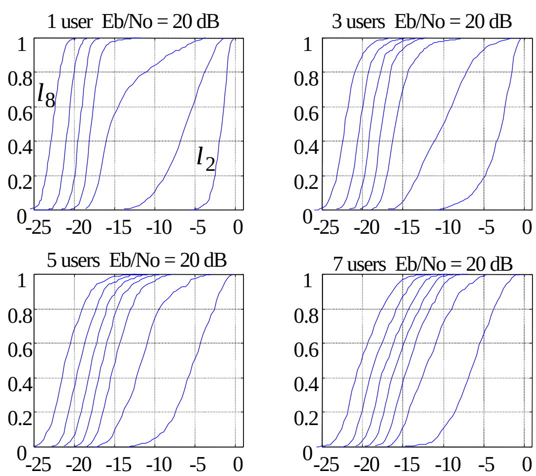



**Figure 2. Probability of error for a different number of active users, all transmitting controlled power, for the different receivers. Pedestrian channel (left) and vehicular channel (right).**




$P(l_i / l_{max}) < abs$ is plotted vs $l_i / l_{max} (dB)$

**Figure 3. Cumulative functions of the ratio of the MDIR eigenvalues to the maximum eigenvalue. Vehicular channel.**