

# Field concentration by local PT-symmetry and global P-symmetry

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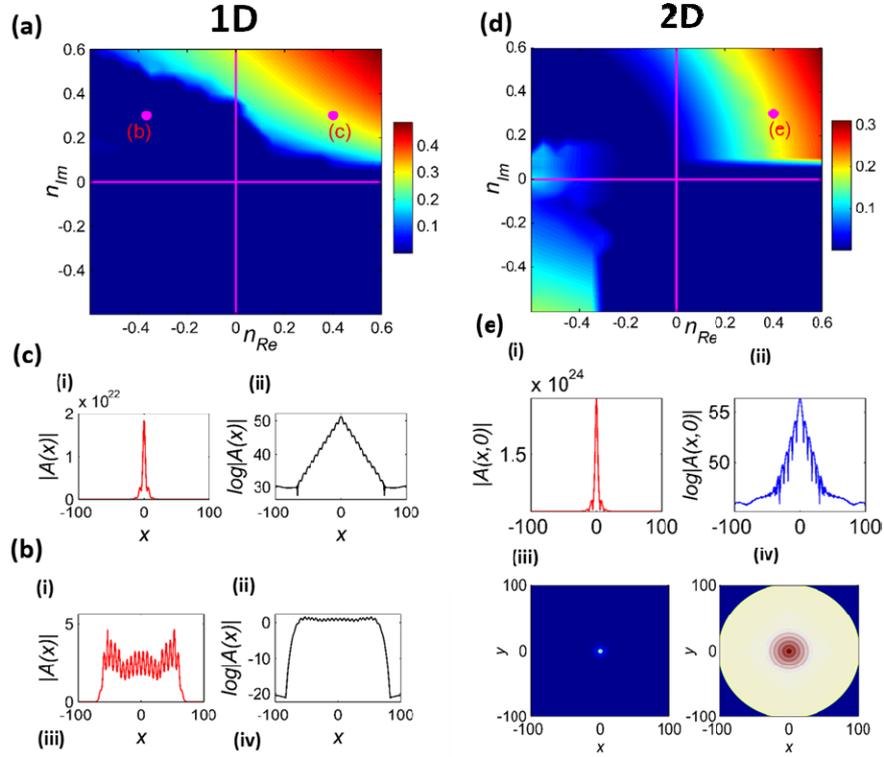
**Abstract-** We propose a new class of systems holding  $PT$ -symmetry only locally; the potential fulfills:  $U(\mathbf{r}-\mathbf{r}_0)=U^*(\mathbf{r}+\mathbf{r}_0)$  in a small neighborhood of any point  $\mathbf{r}_0$  being  $\mathbf{r}_0 \neq \mathbf{0}$ , whereas on a global scale the system is only Parity symmetric:  $U(\mathbf{r})=U(-\mathbf{r})$  for all  $\mathbf{r}$ . We show that such systems lead to a strong field localization at  $\mathbf{r}=\mathbf{0}$ , arising from the merge of the two different symmetries. We explore these new potentials in one and two dimensions for optical systems, combining gain/loss and index modulated in complex nanophotonic structures. We expect the effect to have direct applications as turning broad aperture lasers into bright and narrow output beam sources.

Initially introduced as a curiosity in quantum mechanics [1], Parity-Time ( $PT$ -) symmetric systems have recently found realizations in optics such as asymmetric reflections, transmissions, asymmetric chirality [2-5]. The general requirement for an optic  $PT$ -symmetric system is that the optical potential (complex refractive index including index and gain/loss modulations) fulfills  $n(\mathbf{r}) = n^*(-\mathbf{r})$ . In the simplest one-dimension (1D) realization:  $n(x) = n_0 \exp(iqx)$ , which at resonance,  $k=q/2$  ( $PT$ -transition) asymmetrically couples the left-propagating mode,  $\exp(-ikx)$ , to the right-propagating mode  $\exp(ikx)$ , but not vice versa. However, what happens if such  $PT$ -symmetry condition is not imposed globally, over the whole space, but only locally? Let's consider the situation depicted on Fig.1.a, a complex symmetric potentials, locally  $PT$ -symmetric except for  $x=0$ . The optical complex potential imposes unidirectional coupling "to the right" on the left half-plane, while the coupling is "to the left" on the right half-plane. As a consequence, a strong field localization may be intuitively expected around  $x=0$ . The same idea applies to 2D (or 3D), where radial (or coaxial) dephased modulations of the index and gain/loss may lead to asymmetric coupling between incoming and outgoing waves, see Figs.1.b and 1.c



**Figure 1.** Local unidirectional coupling: (a) 1D complex optical potential, where  $n_{Re}$  is the real index modulation, and  $n_{Im}$  the gain/loss modulation. (b) 2D analogous axisymmetric configuration, (c) 3D coaxial index and gain/loss modulations, analogous for fields propagating along  $z$ . The arrows indicate the direction of the asymmetric mode coupling, at resonance.

Indeed, we explore the field dynamics in spatial and temporal domains within a 1D optical potential in the form:  $n(x) = n_{Re} \cos(|x|) - i n_{Im} \sin(|x|)$  as well the 2D axisymmetric case:  $n(r) = n_{Re} \cos(|r|) - i n_{Im} \sin(|r|)$ . Both systems show equivalent localization regimes, depending upon the real and imaginary modulation amplitudes. Strong field localizations always occur close to the  $PT$ -transition point, see Fig.2 which summarizes the effect.



**Figure 2 1D:** (a) Map of the spatial exponential growth exponent calculated after sufficient long time ( $t > 300$  units) in the parameter space ( $n_{Im}$ ,  $n_{Re}$ ). Spatial field profiles for representative cases (b,c) in linear and logarithmic scales, (i) (ii). **2D:** (d) Axial field localization exponent in parameter space ( $n_{Im}$ ,  $n_{Re}$ ). (e) Axial/total cross-sectional field distribution in real (i)/(iii) and logarithmic scales (ii)/(iv).

We show that the proposed systems simultaneously localize and enhance the field due to the asymmetric radial coupling of inward and outward waves. Since the physical realizations of the above discussed arrangements in 1D and 2D are nowadays available, we expect this light-matter interaction phenomena to find remarkable applications in many linear and nonlinear devices where high degree of localization is desirable such as: beam focusing, narrow beam enhancement, laser emission, switching in nanostructures,...

## REFERENCES

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