Sensitivity Equations and Calibration Requirements on Airborne Interferometry

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INTRODUCTION

The usage of an airborne interferometric SAR to mass map production is strongly dependent on the capability of performing the data processing in a near automatic mode. One of the main limitations is related with both the accurate calibration of the system parameters, its stability from flight to flight and the correct in-flight recording of aircraft position and attitude. The unstable movements of an airborne SAR platform, if recorded in an accurate manner, can be corrected during the processing step. Measurement errors and time drifts lead to location errors in the final DEM [1]. The system requirements in both accuracy and stability to fulfill a quality-mapping requirement can be resolved from the sensitive equations. The same equations can be also used to calibrate the system parameters from the location errors on the imaging of well-known targets, usually corner reflectors.

THE LOCATION EQUATIONS

Interferometric SAR coherently combines the radar echoes received by the two antennas to get a phase difference for each imaged point. This phase difference can be related to the geometric path length difference to the image point, which depends on the topography. Combining the interferometric phase with the known interferometer geometry, a three-dimensional location of each imaged point can be achieved [2].

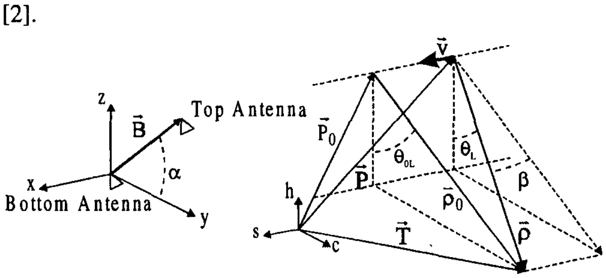

Fig. 1 Interferometer and location geometry.

Two different coordinate systems are used. The first one, $\hat{x}\hat{y}\hat{z}$, is fixed to the aircraft and considers the baseline vector, $\vec{B}$, and the maximum of the antennas radiation patterns contained in the $\hat{y}\hat{z}$ plane. The second one, $\hat{s}\hat{c}\hat{h}$, locally approximates the Earth's ellipsoid, with $\hat{s}$ parallel to the nominal track. Due to the aircraft rotations, described by the yaw, pitch and roll angles, the antenna is pointed forward or backward from the original perpendicular position by a squint angle. Under the geometry presented in Fig. 1, the target position, $\vec{T}$, can be derived from the master antenna position, $\vec{P}$, the attitude angles measured by the inertial unit (yaw, roll and pitch), the unwrapped interferometric phase, Φ, and the baseline length, B, and inclination, α.

$$\vec{T} = \vec{P} + \rho \begin{bmatrix} \sin\beta \\ \sqrt{\sin^2\theta_L - \sin^2\beta} \\ -\cos\theta_L \end{bmatrix} = \vec{P} + \rho \begin{bmatrix} \sin\beta \\ \mu \sin\theta_L \\ -\cos\theta_L \end{bmatrix} \quad (1)$$

Where θ_L is the so-called look angle, the angle the line-of-sight vector, $\vec{\rho} = (\rho_s, \rho_c, \rho_h)$, makes with nadir, and β is the squint angle. Both angles can be derived from the unwrapped interferometric phase and the interferometer geometry.

$$\theta_L = \arccos\left(\cos\theta_p \cos\left(\alpha - \arcsin\left(\frac{\lambda\Phi}{2p\pi B} + \frac{\left(\frac{\lambda\Phi}{2p\pi}\right)^2 - B^2}{2\rho B} \right) + \theta_r \right) \right) \quad (2)$$

The parameter p is set to one in the single baseline case, or single transmitter, and to two in the double baseline case, or double transmitter. The squint angle can be calculated from

$$\sin\beta = \sqrt{\cos^2\theta_p - \cos^2\theta_L} \cdot \frac{\sin\theta_y}{\cos\theta_p} + \cos\theta_L \cdot \cos\theta_y \cdot \tan\theta_p \quad (3)$$

The Doppler centroid depends on the squint angle and the components of the antenna-target relative velocity vector. A simplified expression of the Doppler, assuming the velocity vector has only the along-track component, is

$$f_D = \frac{2v}{\lambda} < \hat{v}, \hat{\rho} > = \frac{2v}{\lambda} \sin\beta \quad (4)$$

The equations are valid for the general squinted geometry, assuming the SAR processor generates *beam-centered* images, defined as the geometry where the focused target is located at a range equal to ρ (and the corresponding azimuth time for the aperture center). However, if compression is done to *zero-Doppler*, which is the perpendicular position with respect to the track, the location equations must be used with zero yaw and pitch and $\vec{P}_0$ instead of $\vec{P}$. This does not apply to the (3) and (4) when calculating the Doppler.

THE SENSITIVITY EQUATIONS

Sensitivity equations are derived by differentiating the basic location equation (1) with respect to the different interferometer parameters. There are two different error sources related with the parameters to be taken into account. One source corresponds to the static parameters that are not expected to change with time, like baseline length and inclination. The other is related with the different data recorded in-flight, like attitude angles, DGPS aircraft position, etc. In both cases, errors on its estimation or measurement bias lead to errors on the final DEM. Although the equations have been derived for the three location vector components, only the elevation ones are presented in this paper.

The different expressions have been particularized to the JPL TOPSAR system characteristics in C band. The chirp center frequency is 5287.5 MHz, the aircraft speed is 450 kts, the altitude 8100 m, the baseline length 5 m and inclination 65°.

Absolute Time Delay

The Absolute Time Delay is the time delay for the interferometric channel identified as the reference. Errors on its calibration lead to range errors. Its effects are smaller on the far range, which corresponds to larger look angles.

$$\rho = \frac{1}{2} c\tau \longrightarrow \frac{\partial \rho_h}{\partial \tau} = -\frac{1}{2} c \cos\theta_L \qquad (5)$$

Differential Time Delay

This is the difference between the time delays for the two interferometric channels. Both interferometric channels are co-registered to reduce the error.

The Lever-Arms

The lever-arm, $\vec{\ell}$, is the vector from the DGPS to the master antenna used to calculate the true antenna position.

$$\vec{P}_{master} = \vec{P}_{DGPS} + [Y\,P\,R]\,\vec{\ell}_{master} \qquad (6)$$

Being [YPR] the Euler rotation matrix. The position of the slave antenna is calculated with the baseline vector.

Baseline Length and Inclination/INU Roll bias

One of the most critical parameters of the interferometer. Small errors on its estimation lead to large errors on the target location.

$$\frac{\partial \rho_h}{\partial B} = \frac{-\rho}{B}\sqrt{\cos^2\theta_p - \cos^2\theta_L}\cdot \frac{\sqrt{\cos^2\theta_p - \cos^2\theta_L}\cdot\cos(\alpha+\theta_r) - \cos\theta_L\cdot\sin(\alpha+\theta_r)}{\cos\theta_L\cdot\cos(\alpha+\theta_r) + \sqrt{\cos^2\theta_p - \cos^2\theta_L}\cdot\sin(\alpha+\theta_r)} \qquad (7)$$

Equation (2) shows as the roll angle is directly added to the baseline inclination one and, consequently, determines the baseline orientation. This makes it the most critical attitude angle to be measured by the INU.

$$\frac{\partial \rho_h}{\partial \alpha} = \rho\sqrt{\cos^2\theta_p - \cos^2\theta_L} \qquad (8)$$

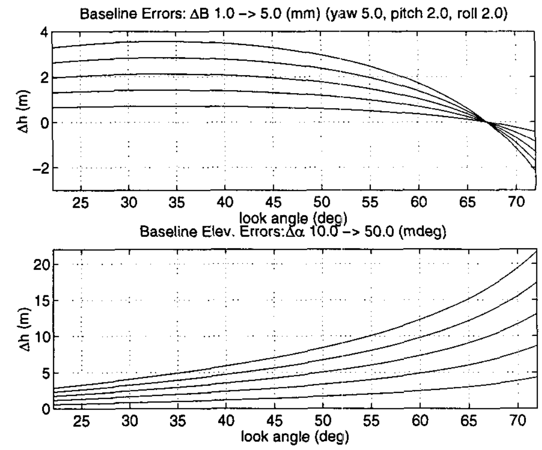

Fig. 2 Sensitivity of elevation with baseline length and inclination.

Phase Offset

The number of integer cycles on the unwrapped phase can be easily derived from the a-priori information of the terrain elevation and the aircraft position. But there still is a remaining phase offset, caused by the different elements of the radar circuitry, to be estimated. The sensitivity of elevation with the unwrapped phase is

$$\frac{\partial \rho_h}{\partial \Phi} = -\rho\frac{\lambda}{2\pi\, pB}\cdot \frac{\cos\theta_p\sqrt{\cos^2\theta_p - \cos^2\theta_L}}{\cos\theta_L\cdot\cos(\alpha+\theta_r) + \sqrt{\cos^2\theta_p - \cos^2\theta_L}\cdot\sin(\alpha+\theta_r)} \qquad (9)$$

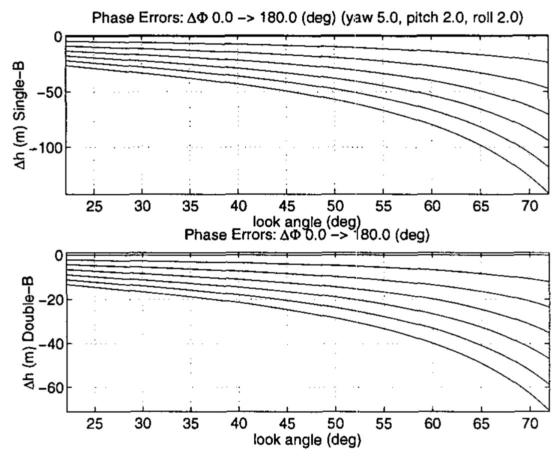

Fig. 3 Sensitivity of elevation with the unwrapped phase for the single and double baseline cases.

INU Yaw and Pitch bias

Those two angles only affect the location if the image is beam-centered, but not in the zero-Doppler case. The yaw bias has no effect on the elevation, but causes errors on the along and cross-track locations in beam-centered interferometers.

$$\frac{\partial \rho_h}{\partial \theta_y} = 0 \tag{10}$$

$$\frac{\partial \rho_h}{\partial \theta_p} = \frac{\rho}{\mu} cos\theta_L \cdot tan\theta_p \tag{11}$$

Aircraft/Antenna position

The errors on the aircraft position directly translate the reconstructed location in the same direction.

Doppler Centroid

The Doppler centroid estimation strongly depends on the yaw and pitch. Deriving equation (4):

$$\frac{\partial f_D}{\partial \theta_y} = \frac{2v}{\lambda}\left[\sqrt{cos^2\theta_p - cos^2\theta_L} \cdot \frac{cos\theta_y}{cos\theta_p} - cos\theta_L \cdot sin\theta_y \cdot tan\theta_p\right] \tag{12}$$

$$\frac{\partial f_D}{\partial \theta_p} = \frac{2v}{\lambda}\left[\frac{-cos^2\theta_L \cdot sin\theta_p}{\sqrt{cos^2\theta_p - cos^2\theta_L}} + cos\theta_L \cdot cos\theta_y\right] \cdot \frac{1}{cos^2\theta_p} \tag{13}$$

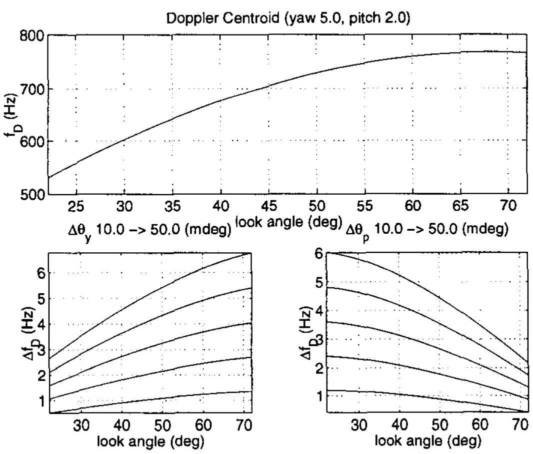

Fig. 4 Doppler centroid sensitivity to yaw and pitch estimation.

CALIBRATION

The sensitivity equations can be used to calibrate the interferometer parameters from the errors on the location of several well-known corner reflectors on a calibration site. From the raw data the Doppler centroid for each corner reflector could be estimated, assuming its signal is powerful enough to perform a good estimation. Once compared with the value predicted by the INU measurements, we may solve the yaw and pitch bias with the following equations system in an iterative process.

$$[\Delta f_{D\_CR}] = [Sens\_f_D] \cdot \begin{bmatrix} \Delta\theta_y \\ \Delta\theta_p \end{bmatrix} \tag{14}$$

In a similar way the rest of parameters can be solved from the errors on the location of the different corner reflectors. For instance

$$[\Delta \vec{T}_{CR}] = [Sens] \cdot \begin{bmatrix} \Delta B \\ \Delta\alpha \\ \Delta\Phi \end{bmatrix} \tag{15}$$

Preliminary results have shown the large condition numbers of some of the matrices involved in the calibration procedure. This makes the calibration strongly sensitive to random errors due to the DGPS and the INU, which can lead to large errors on the parameter estimation. The condition numbers can be slightly reduced by spreading the corner reflectors along the whole range and its effects smoothed by increasing the number of corner reflectors on the calibration site and averaging several calibration flights.

CONCLUSION

The sensitive equations for the different system parameters in a squinted geometry have been derived in compact form. The demand of precise estimation and measurement of the different interferometer parameters has been shown. The equations can be used not only to determine the system requisites for a desired mapping quality, but also to perform the calibration iterative procedure to get the system parameters error estimation.

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