

An optimization model for the long-term planning of energy distribution networks

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Abstract—Although design is a problem that has been addressed in the literature, maintaining, upgrading and expanding energy distribution networks along the entire life-cycle is a topic that has received scarce attention. The problem includes considering the long-term dependence of the efficiency of the investments along their life span. This work presents a novel model for the optimization of energy distribution networks considering the decaying efficiency caused by equipment aging. An exponential function is used to model the decaying efficiency, thus giving rise to an MINLP formulation. A simplified case study based on a real electricity distribution network has been used as a test bed for the proposed approach. For the sake of comparison, simplified MILP model overlooking the efficiency decrease has also been applied. Results show that infeasible sizing and planning solutions are obtained when the decaying performance is not considered and demonstrate that the proposed MINLP overcomes such limitations.

Index Terms--Optimization, energy distribution networks, decaying performance.

I. INTRODUCTION

In recent times, great effort has been devoted to the study of transmission networks [1], [2] and distributed energy systems [3]. Previous research addresses the issues of generation and consumption (sources and sinks nodes). However, little attention has been paid to distribution and its associated needs and investments. Equipment aging can significantly affect the efficiency of a distribution network. Moreover, further legal, administrative or financial issues could also alter the investment performance along its life span. Thus, a decreasing efficiency should therefore be considered during the investment planning. Otherwise, the designed network may be actually undersized and may significantly underestimate the cost of a network with the capacity of actually satisfying the demand. Despite this, to the best of our knowledge, there are no previous studies which consider the depreciation of energy distribution networks.

This work aims to solve the problems that distribution companies face when managing facilities (put into service, maintain and dismantle) by providing a general tool for the design and long-term planning of distribution networks subject to decaying efficiency. The proposed strategy consists of a mixed-integer non-linear programming (MINLP) model, which is built on a general supply chain model [4].

The model includes energy balances in each node and requires the individual treatment of each facility to accurately assess the decaying performance of the power installed. The model includes energy balances in each node and requires the individual treatment of each facility to accurately assess its regression in terms of both the power supplied and the economic return. Hence, the investment plan (decision variables) consists not only on the start-up and shutdown of alternative distribution facilities, but also on the sizing (i.e. capacity, reliability and cost) of the lines satisfying the energy flows. The general MINLP model developed is aimed at efficiently manage the substitution of obsolete distribution facilities due to their inherent decaying performance, and it has been applied to the particular case of an electricity distribution company in Spain.

The model has been implemented in GAMS. Results considering a variety of scenarios have been discussed and they have proved the value of the proposed model as a practical tool to support the decision-making process in the distribution sector.

II. PROBLEM STATEMENT

A 3 echelon electricity distribution network is considered (Fig. (1)). The set of distribution substations are considered as electricity sources p , with a maximum rated capacity that cannot be exceeded. Transformation centers s enable connection to the distribution network for final consumers c , whose demand must be satisfied. Echelons in the network are connected through lines (i.e., lines p - s and s - c).

The existing configuration of the network, the potential location for transformation centers, distances between the nodes of the network, the time horizon, parameters describing the decaying performance and complete technical and economic data are given parameters.

Hence, the optimal distribution network (lines and transformation centers), taking into account the evolution of the equipment performance over time, must be determined.

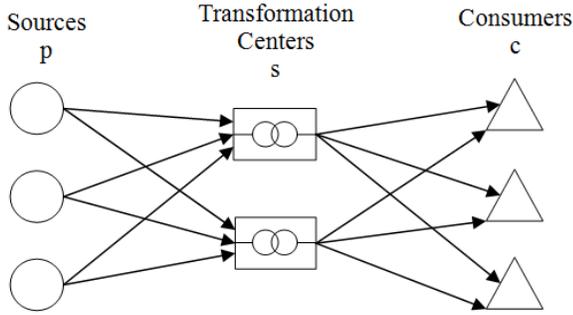


Figure 1. Scheme of the electrical distribution network.

III. MATHEMATICAL FORMULATION

The problem is addressed through the formulation of a Mixed Integer Non-Linear Programming model in which continuous variables model rated capacities and binary variables the structural decisions for the network and its parts (i.e., facilities).

A. Energy balances

Energy balances in each transformation center must be satisfied (1-2). These energy balances are posed as inequalities rather than as equalities in order to prevent the model from forcing equipment aging to be the same at each side of the transformation center.

$$\sum_p P_{psn}^{in} \leq P_{sn}^{sub} \quad \forall s, n \quad (1)$$

$$\sum_c P_{scn}^{out} \geq P_{sn}^{sub} \quad \forall s, n \quad (2)$$

In these constraints, P_{sn}^{sub} , P_{psn}^{in} and P_{scn}^{out} denote the power capacity in time period n of transformation center s , line from source p to transformation center s and line from transformation center s to consumer c , respectively. Constraint (3) forces the demand of each customer c , D_{cn} , to be satisfied in each time period n , while (4) indicates that the distribution station rated capacity, \overline{PPUB}_{pn} , should not be exceeded.

$$\sum_s P_{scn}^{out} \geq D_{cn} \quad \forall c, n \quad (3)$$

$$\sum_s P_{psn}^{in} \leq \overline{PPUB}_{pn} \quad \forall p, n \quad (4)$$

B. Capacity constraints of transformation centers

Equation (5) calculates the capacity of transformation center s in a given time period n from $P0_s^{sub}$, which is a parameter accounting for the capacity of the original facilities in node s (if any); $PC0_{sn'}^{sub}$, which is a continuous variable denoting the initial capacity in location s and which is

dismantled in time period n' ; $PA_{sn'}^{sub}$, which is a continuous variable indicating the capacity expansion performed in each of the previous time periods n' in location s ; and $PC_{sn'n''}^{sub}$, which is a continuous variable accounting for the capacity of that expansion that is dismantled in time period n'' . Continuous variables $PR0_{sn}^{sub}$ and $PR_{sn'n}^{sub}$ indicate respectively the performance in time period n of existing or new facilities (i.e., installed in time period n'). These variables can take any value between 1 and a lower bound and are used to penalize individually the performance of each part of the installation (i.e., original vs expanded) thus providing an accurate representation of equipment aging.

$$P_{sn}^{sub} = (P0_s^{sub} - \sum_{n' \leq n} PC0_{sn'}^{sub}) \cdot PR0_{sn}^{sub} + \sum_{n' \leq n} \left((PA_{sn'}^{sub} - \sum_{n'' | n' \leq n'' \leq n} PC_{sn'n''}^{sub}) \cdot PR_{sn'n}^{sub} \right) \forall s, n \quad (5)$$

Since $P0_s^{sub}$ decreases with equipment aging, it cannot be directly used for the calculation of maintenance costs, as otherwise these costs would also decrease with time for a given installed capacity. In order to prevent this from happening, we defined the continuous variable PI_{sn}^{sub} which penalizes the installed capacity so that the maintenance cost finally increases with equipment aging (6).

$$PI_{sn}^{sub} = (P0_s^{sub} - \sum_{n' \leq n} PC0_{sn'}^{sub}) \cdot (2 - PR0_{sn}^{sub}) + \sum_{n' \leq n} \left((PA_{sn'}^{sub} - \sum_{n'' | n' \leq n'' \leq n} PC_{sn'n''}^{sub}) \cdot (2 - PR_{sn'n}^{sub}) \right) \forall s, n \quad (6)$$

Other general constraints for capacities must be defined (7)-(11). These equations make use of binary variables X_{sn}^{pon} which indicates whether the expansion of a facility is performed in node s in time period n ; $X0_{sn}^{poff}$, which denotes whether the original facility of node s is dismantled in time period n ; and $X_{sn'n'}^{poff}$, which takes a value of 1 when the capacity expansion performed in time period n is dismantled in time period n' .

$$\frac{PLB^{sub}}{PUB^{sub}} \cdot (X_{sn}^{pon} - X_{sn'n'}^{poff}) \leq PA_{sn}^{sub} - PC_{sn'n'}^{sub} \leq \overline{PUB}^{sub} \cdot (X_{sn}^{pon} - X_{sn'n'}^{poff}) \quad \forall s, n, n' | n' \geq n \quad (7)$$

$$\frac{PLB^{sub}}{PUB^{sub}} \cdot X_{sn'n'}^{poff} \leq PC_{sn'n'}^{sub} \leq \overline{PUB}^{sub} \cdot X_{sn'n'}^{poff} \quad \forall s, n, n' | n' \geq n \quad (8)$$

$$X_{sn'n'}^{poff} \leq X_{sn}^{pon} \quad \forall s, n, n' | n' > n \quad (9)$$

$$P0_s^{sub} \cdot X0_{sn}^{poff} \leq PC0_{sn}^{sub} \leq P0_s^{sub} \cdot X0_{sn}^{poff} \quad \forall s, n \quad (10)$$

$$\sum_n X0_{sn}^{poff} \leq 1 \quad \forall s \quad (11)$$

S_{sn}^{pon} and $S_{sn'n''}^{poff}$ are binary variables denoting the set-up of a transformation center and its final dismantling, respectively.

Their relation with other the binary variables previously defined is given by (12)-(15).

$$S_{sn}^{pon} \leq X_{sn}^{pon} \quad \forall s, n \quad (12)$$

$$S_{snn'}^{poff} \leq X_{snn'}^{poff} + X0_{snn'}^{poff} \quad \forall s, n, n' \quad (13)$$

$$S0_s^{pon} + \sum_{n' \leq n} S_{sn'}^{pon} - 1 \leq \sum_{n' \leq n} \sum_{n'' | n' \leq n'' \leq n} S_{snn''}^{poff} \leq S0_s^{pon} + \sum_{n' \leq n} S_{sn'}^{pon} \quad \forall s, n \quad (14)$$

$$\begin{aligned} & \underline{PLB}^{sub} \cdot (S0_s^{pon} + \sum_{n' \leq n} (S_{sn'}^{pon} - \sum_{n'' | n' \leq n'' \leq n} S_{snn''}^{poff})) \\ & \leq P_{sn}^{sub} \leq \overline{PUB}^{sub} \cdot (S0_s^{pon} + \sum_{n' \leq n} (S_{sn'}^{pon} - \sum_{n'' | n' \leq n'' \leq n} S_{snn''}^{poff})) \quad \forall s, n \end{aligned} \quad (15)$$

C. Decaying performance of transformation centers

The performance of the facilities in transformation centers is assumed to follow an exponential decay as shown in (16) and (17). Its calculation depends on the performance parameters ($C^{sub}, k^{sub}, PR^{sub\infty}$) and the antiquity of each facility (calculated in (18) and (19)). In these equations, $PR0_{sn}^{sub}$ denotes the performance in period n of the originally existing installation in center s , $PR_{snn'}^{sub}$ indicates the performance in period n' of the capacity expansion performed in period n . $A_{snn'}^{sub}$ is a continuous variable denoting the antiquity in time period n' of the facility expansion performed in time period n in location s , $A0_{sn}^{sub}$ is a continuous variable denoting the antiquity in period n of the facility originally existing in location s , and $A0_s^{pon}$ is a parameter indicating the antiquity at the start of the operation of the existing facility installed in location s .

$$PR_{snn'}^{sub} = C^{sub} \cdot \exp(-k^{sub} \cdot A_{snn'}^{sub}) + PR^{sub\infty} \quad \forall s, n, n' \geq n \quad (16)$$

$$PR0_{sn}^{sub} = (C^{sub} \cdot \exp(-k^{sub} \cdot A0_{sn}^{sub}) + PR^{sub\infty}) + (1 - (C^{sub} \cdot \exp(-k^{sub} \cdot A0_{sn}^{sub}) + PR^{sub\infty})) \quad \forall s, n \quad (17)$$

$$A_{snn'}^{sub} = X_{sn}^{pon} ((n' - n) - \sum_{n'' | n \leq n'' \leq n'} (X_{snn''}^{poff} (n' - n''))) \quad \forall s, n, n' \geq n \quad (18)$$

$$A0_{sn}^{sub} = S0_s^{pon} (A0_s^{pon} + (n) - \sum_{n' | n \leq n'} (X0_{snn'}^{poff} (n - n'))) \quad \forall s, n \quad (19)$$

D. Lines

Equations (5) to (19) are also applied to lines source-transformation station and transformation station-consumer. Due to space limitation, they are reported by the implicit equations (20) to (23).

$$h^{in} \begin{pmatrix} P_{psn}^{in}, PA_{psn}^{in}, PC_{psnn'}^{in}, PI_{psn}^{in}, LPS_{psn}^{pon}, \\ LPS_{psnn'}^{poff}, \gamma_{psn}^{pon}, \gamma_{psnn'}^{poff}, Y0_{psn}^{poff}, PC0_{psn}^{in}, \\ A_{psnn'}^{in}, A0_{psn}^{in}, PR_{psnn'}^{in}, PR0_{psn}^{in} \end{pmatrix} = 0 \quad (20)$$

$$g^{in} \begin{pmatrix} P_{psn}^{in}, PA_{psn}^{in}, PC_{psnn'}^{in}, PI_{psn}^{in}, LPS_{psn}^{pon}, \\ LPS_{psnn'}^{poff}, \gamma_{psn}^{pon}, \gamma_{psnn'}^{poff}, Y0_{psn}^{poff}, PC0_{psn}^{in}, \\ A_{psnn'}^{in}, A0_{psn}^{in}, PR_{psnn'}^{in}, PR0_{psn}^{in} \end{pmatrix} \leq 0 \quad (21)$$

$$h^{out} \begin{pmatrix} P_{scn}^{out}, PA_{scn}^{out}, PC_{scnn'}^{out}, PI_{scn}^{out}, LSC_{scn}^{pon}, \\ LSC_{scnn'}^{poff}, Z_{scn}^{pon}, Z_{scnn'}^{poff}, Z0_{scn}^{poff}, PC0_{scn}^{out}, \\ A_{scnn'}^{out}, A0_{scn}^{out}, PR_{scnn'}^{out}, PR0_{scn}^{out} \end{pmatrix} = 0 \quad (22)$$

$$g^{in} \begin{pmatrix} P_{psn}^{in}, PA_{psn}^{in}, PC_{psnn'}^{in}, PI_{psn}^{in}, LPS_{psn}^{pon}, \\ LPS_{psnn'}^{poff}, \gamma_{psn}^{pon}, \gamma_{psnn'}^{poff}, Y0_{psn}^{poff}, PC0_{psn}^{in}, \\ A_{psnn'}^{in}, A0_{psn}^{in}, PR_{psnn'}^{in}, PR0_{psn}^{in} \end{pmatrix} \leq 0 \quad (23)$$

Note that the variables in these equations are analogous to those in transformation centers differing only in the superscripts (i.e., superscript sub is replaced by in for lines source-transformation center and by out for lines transformation center-consumer) and the subscripts (i.e., subscript s is replaced by pairs ps and sc for lines upstream and downstream of the transformation centers, respectively). The only exceptions for this are binary variables S_{sn}^{pon} and $S_{snn'}^{poff}$, which are replaced by binary variables LPS_{psn}^{pon} and $LPS_{psnn'}^{poff}$ for input lines to transformation centers, and by binary variables LSC_{psn}^{pon} and $LSC_{psnn'}^{poff}$ for output lines from transformation centers.

E. Objective function

The objective function is the total cost, $TCost$, which includes fixed and variable investment for facilities in lines and transformation centers (the parameters used for this are $\alpha^{in}, \alpha^{out}, \beta^{in}, \beta^{out}, \gamma^{in}, \gamma^{out}, \delta, \epsilon, \zeta$), their maintenance (parameters $\iota^{in}, \iota^{out}, \kappa$) and cost related to dismantling facilities (parameters $\eta^{in}, \eta^{out}, \eta^{sub}$).

$$\begin{aligned} TCost = & \sum_n (\sum_p \sum_s PA_{psn}^{in} \cdot (L_{ps}^{in} \cdot \alpha^{in}) + \sum_s \sum_c PA_{scn}^{out} \\ & \cdot (L_{sc}^{out} \cdot \alpha^{out}) + \sum_p \sum_s (Y_{psn}^{pon} \cdot L_{ps}^{in} \cdot \beta^{in}) \\ & + \sum_s \sum_c (Z_{scn}^{pon} \cdot L_{sc}^{out} \cdot \beta^{out}) + \sum_p \sum_s (LPS_{psn}^{pon} \cdot \gamma^{in}) \\ & + \sum_s \sum_c (LSC_{scn}^{poff} \cdot \gamma^{out}) + \sum_s (PA_{sn}^{sub} \cdot \delta) + \sum_s (X_{sn}^{pon} \cdot \epsilon) \\ & + \sum_s (S_{sn}^{pon} \cdot \zeta) + \sum_p \sum_s ((Y0_{psn}^{poff} + \sum_{n'} Y_{psnn'}^{poff}) \cdot \eta^{in}) \\ & + \sum_s \sum_c ((Z0_{scn}^{poff} + \sum_{n'} Z_{scnn'}^{poff}) \cdot \eta^{out}) \\ & + \sum_s ((X0_{sn}^{poff} + \sum_{n'} X_{snn'}^{poff}) \cdot \eta^{sub})) \\ & + \sum_p \sum_s (PI_{psn}^{in} \cdot \iota^{in}) + \sum_p \sum_s (PI_{scn}^{out} \cdot \iota^{out}) \\ & + \sum_s (PI_{sn}^{sub} \cdot \kappa) \end{aligned} \quad (24)$$

Finally, the MINLP model can be formally posed as follows:

$$DP \quad \min TCost \quad \text{s. t. Eqs. (1 - 24)}$$

IV. CASE STUDY

The proposed methodology has been applied to a case study, consisting of a real problem of a distribution company in Spain. It includes a distribution station as the source that must supply electricity to 10 consumer nodes through 10 transformation centers. No potential locations for transformation centers are available.

In the calculus process, the apparent power has been used as the demand to be supplied to consumers. Its value has been derived from the contracted power and power factor associated to each consumption point.

The decaying performance applied to the different facilities is estimated to tend asymptotically to a 70% of the initial value (i.e., $PR^{sub\infty} = 0.7$).

The time horizon analyzed comprises 10 years with a 5% annual increase in the demand. The investment costs have been calculated based on the recommendations by the National Commission of Markets and Competence [5].

TABLE I. COST SUMMARY.

Type	Cost (€)	
	NDP	DP
Line equipment	484212	571758
Line installation	406245	666465
Line set-up	0	0
T.C. equipment	52771	70557
T.C. installation	30426	91278
T.C. set-up	0	0
Dismantling	60000	0
Maintenance	1171166	1380514
Total	2204821	2780573

V. RESULTS AND DISCUSSION

As a preliminary test, we first solve the problem with the traditional approach, that is, we build a simplified version of our model which does not include the decaying performance (NDP). This MILP model is solved by CPLEX 12.6.2.0 providing an optimal solution entailing a $TCost$ of 2.20 million €. Figure 2.a depicts the evolution of $\sum_c P_{scn}^{out}$ for each transformation center s in the optimal solution of NDP. This solution seems feasible since it satisfies the demand in all the time periods, yet if the decaying performance is then applied

to the solution in order to assess the effect of equipment aging (Figure 2.b), it becomes evident that the network is indeed undersized and henceforth unable of meeting the aggregated demand. This demonstrates that the traditional approach must be avoided when the effect of equipment aging may be significant.

Next, we addressed the same problem by means of the proposed model with decaying performance. Model DP, featuring 59375 equations, 45136 continuous variables and 21879 binary variables, was coded in GAMS 24.4.6 and solved with DICOPT providing an optimal solution with a $TCost$ of 2.78 million €. The solution of DP (Figure 2.c), allows overcoming aforementioned limitation, thus satisfying the aggregated demand in all the time periods even when the performance of the facilities decays as a result of their aging. Note that solution to model NDP, which proved to be infeasible, entails a lower cost than that of model DP (i.e., 2.78 vs 2.20 million €). This brings to light that the traditional approach (i.e., model NDP) underestimates the real cost of the required network (in this case, in a 26%).

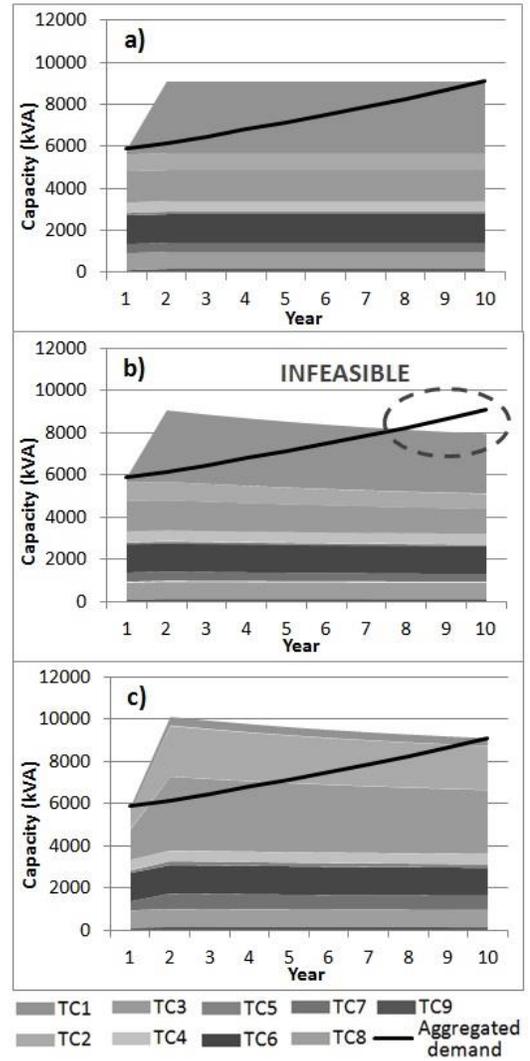


Figure 2. $\sum_c P_{scn}^{out}$ in the optimal solutions a) NDP; b) NDP with decaying performance; c) DP.

VI. CONCLUSIONS

This work presents a novel model for the long-term planning and sizing of electricity distribution networks under decaying performance. The model is built on that of a general supply chain and therefore it can be readily applied to similar problems (i.e., from chemical supply chains, to distribution networks).

Results have demonstrated the importance of accurately modeling the decaying performance of the system as otherwise, equipment sizing will likely become insufficient when facing real operation.

Based on the promising findings, future work could address the acceleration of the search algorithm by using global optimization methods and the consideration of uncertainty. In addition, the model could be expanded to include reliability and supply quality considerations.

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