TIME-DELAY ESTIMATION OF THE LINE-OF-SIGHT SIGNAL IN A MULTIPATH ENVIRONMENT*

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ABSTRACT

The problem of estimating the time-delay of the line-of-sight (LOSS) or direct signal received by an antenna array in a multipath scenario is addressed. This problem is essential in many communication, radar, sonar and navigation systems, since the signal that propagates through the direct path may be the only one that bears useful information for the operation of the receiver. In many of these systems, it is possible to have an approximate a priori knowledge about the direction-of-arrival (DOA) of the direct signal. We analyze several time-delay estimators that exploit this information. In order that they can operate in the presence of co-channel interference (CCI), it is assumed that the noise field has an arbitrary and unknown spatial correlation. We show that the maximum likelihood (ML) estimator is the only one that presents a sufficient robustness against calibration or modeling errors. Starting from the ML estimator, we derive a new method that takes the uncertainty about the steering vector of the LOSS into account.

1 INTRODUCTION

Time-delay estimation or timing synchronization is a key task in diverse areas, such as radar, sonar and communications [1]. Some systems that arouse great interest at present and wherein the time-delay estimation is fundamental are the Global Navigation Satellite Systems (GNSS). These systems are based on the measurement of the distance between the transmitting satellite and the receiver. The distance is obtained by means of the propagation-delay of the signal that propagates through the direct path. The reflections received in a multipath scenario together with the interference constitute the main sources of error in the time-delay measurement [2]. It is important to note that the delays of the reflections are not relevant parameters for the receiver because they are not related to the geometric distance of the direct path. A particular characteristic of the GNSS systems is that, using a calibrated antenna array, the steering vector of the direct signal can be *a priori* known. This information is possible because the receiver knows its position and that of the satellite (transmitted by the satellite itself) with high accuracy [3]. This situation is not exclusive of the GNSS. For instance, in a radar system, one of the parameters of interest is the distance to the target, whose angular location is approximately given by the transmit direction of the radar. In the downlink of a satellite communication system, there may also be a direct propagation path, and the synchronization of the ray that arrives from this path is essential because it does not suffer from severe fading.

Most of the work presented up to date tackles the problem of estimating the time-delays of all the received reflections in spatially white noise [4]. Besides presenting a high computational load, these methods are not suited for situations involving strong CCI and do not exploit the knowledge of the spatial signature of the direct signal. In this paper, we analyze a ML time-delay estimator that allows us to estimate only the relevant parameter (*i.e.* the delay of the LOSS) in the presence of strong CCI. Moreover, we also propose a method that makes the original ML estimator more robust against calibration or pointing errors in the nominal steering vector of the LOSS.

2 SIGNAL MODEL

We assume that an arbitrary m element array receives d scaled and delayed replicas of a known signal $s(t)$. The baseband array output can be expressed by the following $m \times 1$ vector:

$$\mathbf{y}[n] = \boldsymbol{\alpha}_0 \, s(n\,T_s - \tau_0) + \sum_{k=1}^{d-1} \boldsymbol{\alpha}_k \, s(n\,T_s - \tau_k) + \mathbf{v}[n] \ , \quad (1)$$

where T_s is the sampling period, $\boldsymbol{\alpha}_k$ and τ_k are the spatial signature and time delay of the k^{th} arrival, and $\mathbf{v}[n]$ represents additive noise and interference. If N samples are collected from the array, they can be grouped together into the following $m \times N$ matrix:

$$\mathbf{Y} = \Big[\mathbf{y}[1] \cdots \mathbf{y}[N]\Big] = \boldsymbol{\alpha}_0 \, \mathbf{s}^T(\tau_0) + \sum_{k=1}^{d-1} \boldsymbol{\alpha}_k \, \mathbf{s}^T(\tau_k) + \mathbf{V} \ , \quad (2)$$

where $\mathbf{s}(\tau) = [s(T_s - \tau) \cdots s(N\,T_s - \tau)]^T$ and $\mathbf{V}$ is formed identically to $\mathbf{Y}$.

It is clear that the standard "narrowband assumption" used in many array signal processing problems is made here. The first term in equations (1) and (2), namely the one with the subscript 0, represents the line-of-sight signal. The terms inside the summations correspond to the reflections of the direct signal. From an obvious physical reasoning, it is clear that the delays of the reflections are always greater than that of the direct signal, *i.e.*, $\tau_k > \tau_0 \quad k = 1..d-1$, since they travel a longer distance from the transmitter to the receiver.

As stated in the introduction, we are only interested in estimating the delay of the direct signal τ_0. Therefore, all disturbing terms are lumped together in an "equivalent noise"

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term $\mathbf{e}[n]$, which is expressed as:

$$\mathbf{e}[n] = \mathbf{v}[n] + \sum_{k=1}^{d-1} \boldsymbol{\alpha}_k \, s(n T_s - \tau_k) \; . \qquad (3)$$

Thus, the received signal in (2) can be written as:

$$\mathbf{Y} = \boldsymbol{\alpha}_0 \, \mathbf{s}^T(\tau_0) + \mathbf{E} \; , \qquad (4)$$

where $\mathbf{E}$ is formed identically to $\mathbf{Y}$. We model $\mathbf{e}[n]$ as a zero-mean, circularly complex Gaussian vector, with an arbitrary and unknown covariance matrix, and white in the temporal domain:

$$\mathrm{E}\{\mathbf{e}[n]\} = \mathbf{0} \qquad \mathrm{E}\left\{\mathbf{e}[n]\,\mathbf{e}[m]^H\right\} = \mathbf{Q}\,\delta_{n,m} \; , \qquad (5)$$

where $(\cdot)^H$ denotes the transpose conjugate operation. This model is obviously an approximation, but it will allow us to develop estimators that take the spatial covariance of the "noise" into account and thus, the antenna array will carry out a spatial filtering of the disturbing signals. On the other hand, when the noise is assumed spatially white, as in [4], the antenna array does not cancel the interferences. Although that approximation comes at the price of a certain performance degradation, the increase of the complexity due to the use of estimators that are derived from the exact model in (1) may not be justified in an application where only τ_0 is of interest. These estimators present a superior performance, but also much more complexity, because they explicitly estimate the parameters of the reflections [1]. The approximate model that we consider captures the most significant effects of the noise, interference and multipath, and leads to tractable algorithms. For simplicity the process $\mathbf{e}[n]$ has been taken as temporally white. This assumption does not entail any loss of generality, since the approaches in [3] and [5] can be used to include the temporal correlation in to the model.

3 ANALYSIS OF DIFFERENT TIME-DELAY ESTIMATORS

In this section, we present and analyze several time-delay estimators that can be applied to the problem at hand. We consider: *i)* the effect that the errors in the assumed spatial signature have on the variance of the delay estimate; *ii)* the bias of the delay estimate produced by reflections that are not exactly modeled by (4).

3.1 Maximum Likelihood Estimator Using Only Temporal Information

In this case, we assume that the knowledge about the steering vector of the direct signal, $\boldsymbol{\alpha}_0$, is not available or is not to be exploited. It is easy to show that under the model (4) the ML time-delay estimate, $\hat{\tau}_t$, is given by the argument maximizing:

$$\Lambda_t(\tau) = \frac{1}{\hat{P}_s(\tau)} \, \hat{\mathbf{r}}_{ys}^H(\tau) \, \hat{\mathbf{R}}_{yy}^{-1} \, \hat{\mathbf{r}}_{ys}(\tau) \; , \qquad (6)$$

where

$$\hat{P}_s(\tau) = \frac{\|\mathbf{s}(\tau)\|^2}{N} \qquad \hat{\mathbf{r}}_{ys}(\tau) = \frac{1}{N}\,\mathbf{Y}\,\mathbf{s}^*(\tau) \qquad (7)$$

$$\hat{\mathbf{R}}_{yy} = \frac{1}{N}\,\mathbf{Y}\,\mathbf{Y}^H \; . \qquad (8)$$

Using a standard first order analysis, the asymptotic (large N, hereafter) variance of the estimate can be computed as:

$$\sigma_t^2 = \mathrm{E}\left\{(\hat{\tau}_t - \tau_0)^2\right\} = \frac{1}{N} \frac{\lim_{N\to\infty} N \mathrm{E}\left\{\left(\Lambda_t'(\tau_0)\right)^2\right\}}{\left(\lim_{N\to\infty} \Lambda_t''(\tau_0)\right)^2} \; , \qquad (9)$$

where $(\cdot)'$ and $(\cdot)''$ denote the first and second derivatives, respectively. Equation (9) is completely general and will also be employed to obtain the variance of the estimators below. If the model in (4) holds exactly, then using (9) the inverse of the variance of $\hat{\tau}_t$ is shown to be:

$$\sigma_t^{-2} = 2\left(\mathbf{t}^H(\tau_0)\,\mathbf{P}_{\mathbf{s}(\tau_0)}^{\perp}\,\mathbf{t}(\tau_0)\right)\left(\boldsymbol{\alpha}_0^H \mathbf{Q}^{-1} \boldsymbol{\alpha}_0\right) \; , \qquad (10)$$

where

$$\mathbf{P}_{\mathbf{s}(\tau)}^{\perp} = \mathbf{I} - \mathbf{s}(\tau)\left(\mathbf{s}^H(\tau)\,\mathbf{s}(\tau)\right)^{-1}\mathbf{s}^H(\tau) \qquad (11)$$

$$\mathbf{t}(\tau) = \frac{\mathrm{d}\,\mathbf{s}(\tau)}{\mathrm{d}\,\tau} \; . \qquad (12)$$

Although the variance coincides with the CRB, this estimator is inadequate for the estimation of the delay of the LOSS in a multipath environment. It has been shown in [6] that the reflections produce large biases in the estimates. This result is completely logical, since we are not explicitly modeling the reflections and we are not taking into account any reference to the LOSS. Therefore, it is clear that, in order to obtain an acceptable performance without estimating the delays of the reflections, a clear reference to the direct signal has to be introduced. Estimators that assume the knowledge of the steering vector of the LOSS are analyzed in the two sections below.

3.2 Minimum Variance Beamformer

A common approach would be to spatially filter (*i.e.* beamforming) the received signals and then estimate the time-delay from the beamformer-output signal. If we assume that the steering vector of the LOSS is $\mathbf{a}_0$ (ideally it only differs from $\boldsymbol{\alpha}_0$ in a scaling by a complex amplitude), a natural choice may be the minimum variance (MV) beamformer: $\mathbf{w}_{mv} = \hat{\mathbf{R}}_{yy}^{-1}\,\mathbf{a}_0$. Using this beamformer, the time-delay estimate $\hat{\tau}_{mv}$ is the argument maximizing

$$\Lambda_{mv}(\tau) = \frac{1}{\hat{P}_s(\tau)} \left|\mathbf{a}_0^H \, \hat{\mathbf{R}}_{yy}^{-1} \, \hat{\mathbf{r}}_{ys}(\tau)\right|^2 . \qquad (13)$$

Using equation (9) it can be shown that, under the model in (4), the variance of this estimator is

$$\sigma_{mv}^2 = \frac{\mathbf{a}_0^H \, \mathbf{R}_{yy}^{-1} \, \mathbf{Q} \, \mathbf{R}_{yy}^{-1} \, \mathbf{a}_0}{2\left(\mathbf{t}^H(\tau_0)\,\mathbf{P}_{\mathbf{s}(\tau_0)}^{\perp}\,\mathbf{t}(\tau_0)\right)\left|\mathbf{a}_0^H \mathbf{R}_{yy}^{-1} \boldsymbol{\alpha}_0\right|^2} \; , \qquad (14)$$

where $\mathbf{R}_{yy}$ is the asymptotic limit of $\hat{\mathbf{R}}_{yy}$. The variance coincides with the CRB only if $\mathbf{a}_0$ is parallel to $\boldsymbol{\alpha}_0$. Again, the MV method has proved to be unsuitable for the problem at hand. The reason is twofold. First, as it is well-known and is corroborated by (14), the variance is extremely sensitive to errors in the vector $\mathbf{a}_0$. The cause is that when this vector has a component in the noise subspace of $\mathbf{R}_{yy}$, the numerator of (14) rapidly increases. Second, in the presence of reflections, the estimator suffers from the desired-signal cancellation phenomenon, which causes an increased bias and variance.

3.3 Maximum Likelihood Estimator

From the analysis above, it is evident that the use of an estimator that employs the available spatial information in a efficient way becomes necessary. To this end, assuming that the spatial signature of the LOSS is $\mathbf{a}_0$ up to a multiplicative constant, the ML estimator for the model in (4) was derived in [3]. The ML time-delay estimate is the parameter that maximizes:

$$\Lambda_{ml}(\tau) = \frac{|\mathbf{a}_0^H \hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{r}}_{ys}(\tau)|^2}{\left(\hat{P}_s(\tau) - \hat{\mathbf{r}}_{ys}^H(\tau) \hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{r}}_{ys}(\tau)\right)\left(\mathbf{a}_0^H \hat{\mathbf{R}}_{yy}^{-1} \mathbf{a}_0\right)} . \tag{15}$$

Even though this estimator is a combination of the two estimators above (*i.e.*, $\Lambda_t(\tau)$ and $\Lambda_{mv}(\tau)$), the performance analysis shows that it overcomes the drawbacks of the former two. We have shown that the variance of the ML estimate is:

$$\sigma_{ml}^2 = \frac{\mathbf{a}_0^H \mathbf{Q}^{-1} \mathbf{a}_0}{2\left(\mathbf{t}^H(\tau_0)\, \mathbf{P}_{\mathbf{s}(\tau_0)}^{\perp}\, \mathbf{t}(\tau_0)\right) |\mathbf{a}_0^H \mathbf{Q}^{-1} \boldsymbol{\alpha}_0|^2} , \tag{16}$$

which coincides with the CRB in absence of errors in the steering vector. An interesting and new result is that the variance σ_{ml}^2, unlike the variance in (14), is not very sensitive to those errors. Its sensitivity is simply given by the array beampattern in the norm of $\mathbf{Q}^{-1}$. Moreover, the presence of reflections does not entail a significant degradation of the bias and variance.

4 ROBUSTNESS TO POINTING OR CALIBRATION ERRORS

The effect of calibration or pointing errors is a subject of primary interest in any method that relies on the *a priori* knowledge of a steering vector, such as the ML estimator in section 3.3. Although this technique presents an inherent low sensitivity to such errors, it is worth investigating methods to achieve better robustness. That is, we address the problem of designing an estimator that assumes that the direct signal arrives from a subspace close to a given steering vector.

Let consider that the steering vector is parameterized by a nuisance parameter: $\mathbf{a}(\rho)$, in such a way that $\mathbf{a}(0) = \mathbf{a}_0$ is the nominal vector [1]. It is possible to deal with the nuisance parameter ρ following with two approaches. The first, usually referred to as autocalibration, attempts to estimate the value of ρ that optimizes a given criterion [7]. Meanwhile, in the second approach a probability density function (PDF) is assigned to the nuisance parameter, and a certain criterion is averaged over the distribution of the parameter [8].

Since one of our goals in this work has been to avoid the introduction of more unknowns into the problem than those strictly necessary, we choose to apply the second approach. The criterion that we consider is evidently the ML estimator in section 3.3, because it is the one with the best performance. However, the expression of the estimator in (15) does not facilitates the computation of the expectation with respect to ρ. A more convenient expression, which is equivalent to (15), is the following Rayleigh quotient:

$$\Lambda_{ml,2}(\tau|\rho) = \frac{\mathbf{a}^H(\rho)\, \hat{\mathbf{W}}^{-1}(\tau)\, \mathbf{a}(\rho)}{\mathbf{a}^H(\rho)\, \hat{\mathbf{R}}_{yy}^{-1}\, \mathbf{a}(\rho)} , \tag{17}$$

[1]The extension to the multiparameter case is not difficult but obscure the exposition.

where we have defined

$$\hat{\mathbf{W}}(\tau) = \hat{\mathbf{R}}_{yy} - \frac{1}{\hat{P}_s(\tau)} \hat{\mathbf{r}}_{ys}(\tau)\, \hat{\mathbf{r}}_{ys}^H(\tau) . \tag{18}$$

The cost function averaged over the uncertainty in the steering vector is:

$$\Psi(\tau) = \mathrm{E}_\rho\left\{\Lambda_{ml,2}(\tau|\rho)\right\} . \tag{19}$$

This is the criterion to be maximized in order to estimate the time-delay. In general, the expectation in (19) presents impassable obstacles. Since, in the cases of practical interest, ρ has zero mean and small variance σ_ρ^2, it is possible to derive an approximation of (19) regardless of the particular distribution of ρ. This approximation is exact up to order σ_ρ^2, *i.e.* only the terms involving $\sigma_\rho^4, \sigma_\rho^6, \ldots$ are approximated. To this end, the following second-order series expansions are employed:

$$\mathbf{a}(\rho) \simeq \mathbf{a}_0 + \rho\, \mathbf{d}_0 + \tfrac{\rho^2}{2} \mathbf{h}_0 \quad \tfrac{1}{1+x} \simeq 1 - x + x^2 , \tag{20}$$

where

$$\mathbf{d}_0 = \left.\frac{\mathrm{d}\,\mathbf{a}(\rho)}{\mathrm{d}\,\rho}\right|_{\rho=0} \quad \mathbf{h}_0 = \left.\frac{\mathrm{d}^2\,\mathbf{a}(\rho)}{\mathrm{d}\rho^2}\right|_{\rho=0} . \tag{21}$$

Finally, the cost function $\Psi(\tau)$ can be computed by substituting (20) into (17). A case with special interest is when the array is uniform and linear with antennas spaced δ wavelengths apart, and ρ represents the error with respect to the nominal DOA, θ_0. In this case, the final criterion can be expressed as:

$$\begin{aligned}\Psi(\tau) \simeq\ & \frac{\mathbf{a}_0^H \left(\hat{\mathbf{W}}^{-1}(\tau) \odot \mathbf{B}\right) \mathbf{a}_0}{\mathbf{a}_0^H \left(\hat{\mathbf{R}}_{yy}^{-1} \odot \mathbf{B}\right) \mathbf{a}_0} \cdot \\ & \cdot \left(1 + 4\sigma_\rho^2 \frac{\mathrm{Re}^2\left\{\mathbf{a}_0^H \hat{\mathbf{R}}_{yy}^{-1} \mathbf{d}_0\right\}}{\left(\mathbf{a}_0^H \hat{\mathbf{R}}_{yy}^{-1} \mathbf{a}_0\right)^2} - \right. \\ & \left. - 4\sigma_\rho^2 \frac{\mathrm{Re}\left\{\mathbf{a}_0^H \hat{\mathbf{R}}_{yy}^{-1} \mathbf{d}_0\right\} \mathrm{Re}\left\{\mathbf{a}_0^H \hat{\mathbf{W}}^{-1}(\tau)\, \mathbf{d}_0\right\}}{\left(\mathbf{a}_0^H \hat{\mathbf{R}}_{yy}^{-1} \mathbf{a}_0\right)\left(\mathbf{a}_0^H \hat{\mathbf{W}}^{-1}(\tau)\, \mathbf{a}_0\right)}\right) ,\end{aligned} \tag{22}$$

where $\odot$ denotes the Hadamard (element-wise) product and the p, q^{th} element of $\mathbf{B}$ is

$$[\mathbf{B}]_{p,q} = \mathrm{e}^{-2\pi^2\delta^2\cos^2(\theta_0)\sigma_\rho^2(p-q)^2 + j\pi\delta\sigma_\rho^2\sin(\theta_0)(q-p)} . \tag{23}$$

5 NUMERICAL RESULTS

In all the simulation experiments, an uniform linear array with 6 antennas spaced half wavelength apart is employed. The signal vector $\mathbf{s}(\tau)$ is built as the concatenation of 3 Nyquist square root raised cosine pulses. Each pulse has a bandwidth equal to $(1+\alpha)/2T_c$, is truncated to the interval $[-3T_c, 3T_c]$, and the sampling period is $T_s = T_c/2$. The roll-off factor is set equal to $\alpha = 0.2$. The LOSS arrives from DOA 0^o, and its signal to noise ratio (SNR) is 16dB. The nuisance parameter ρ represents the pointing error, which is the difference between the DOA used to form the vector $\mathbf{a}_0$ and the true DOA of the LOSS. The robust estimator is designed using an *a priori* standard deviation $\sigma_\rho = 8\pi/180$. In the first experiment, a wide-band Gaussian interference at DOA -30° is received with a signal to interference ratio of -3dB. This situation corresponds exactly to the model in (2). The simulated (using 500 Monte Carlo realizations) and theoretical Root Mean Squared Errors (RMSE) of the MV beamformer-based estimator and the ML estimator with spatio-temporal reference are shown in Fig.1. Also the simulated RMSE of the robust estimator $\Psi(\tau)$ is plotted in this figure. The probabilities of outlayers (P_{out}) of the same techniques are displayed in Fig.2. In the ML and robust methods, an estimate is considered to be an outlayer when its distance to the true value is greater than $\min\{0.5T_c, 6\sigma\}$, where σ is the theoretical RMSE of the method under consideration. In the MV method, the threshold for the outlayers is simply $0.5T_c$. It can be observed that the simulated RMSEs agree with the values predicted by (14) and (16). An exception is the MVB-based estimator in absence of pointing errors, which has worse finite-sample performance than the ML approach. Moreover, the RMSE and the P_{out} of the MVB-based estimator undergo a severe degradation for tiny pointing errors, whereas the ML estimator tolerates errors even larger than 10^o. The robust estimator still outperforms the ML method. The improvement is especially important in the probability of outlayers, since the P_{out} of the robust method is smaller than 1.5% in all the $\pm 20^o$ simulated range of pointing errors. In the second experiment (Fig.3), instead of an interference, a specular reflection is received. It arrives from DOA 30^o, is attenuated 3dB and delayed $0.25T_c$ with respect to the LOSS; and both signals are in phase at the first antenna. Unlike the first experiment, all the estimators are in general biased, so the RMSEs do not coincide with the standard deviations (STD). To compute these performance metrics, the outlayers are removed, being the threshold $0.5T_c$. We have omitted the MVB-based estimator because it completely fails due to the coherent multipath propagation, as it is well known. Even in the presence of the reflection, the performance of the ML estimator is virtually insensitive to errors up to 10^o. This *flat region* is further extended about 5^o using the robust estimator.

6 CONCLUSIONS

We have analyzed the effect of pointing/calibration errors on several estimators of the time-delay of the LOSS. The signals are received by an antenna array in a noise field with unknown spatial correlation. In particular, we have considered the ML estimator using only temporal information, an estimator based on the MV beamformer, and the ML estimator using both temporal and spatial information. The expressions of the variances of these methods have been derived in the absence of reflections of the direct signal. The MV beamformer may seem a natural alternative to introduce a spatial reference to the LOSS, and hence to reduce the degradation suffered by the ML method with temporal reference when reflections are received. However, the MV beamformer is extremely sensitive to pointing errors, it has poor finite-sample performance and also fails in the presence of reflections. The ML estimator that employs the space-time information proves to be an excellent choice to overcome the drawbacks of the aforementioned methods. It is inherently robust to pointing errors and to the reception of multipath components. Finally, we have derived a modification of this ML estimator that further extends the range of tolerable pointing errors.

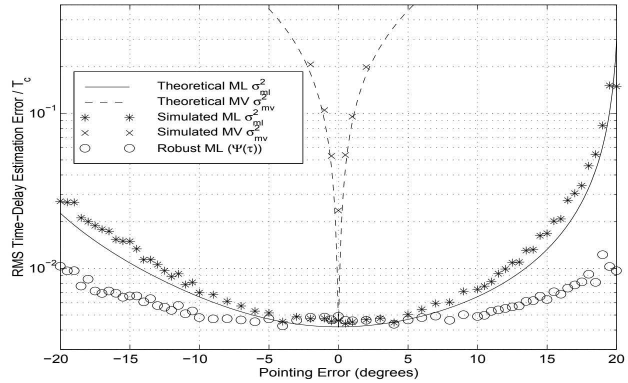

Figure 1: RMSE in the presence of one interference.

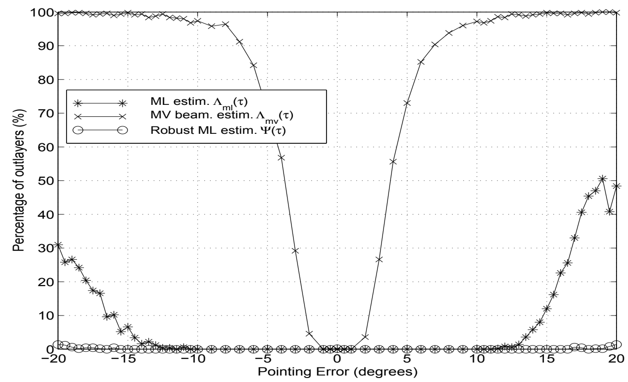

Figure 2: Prob. of outlayers in the presence of one interference.

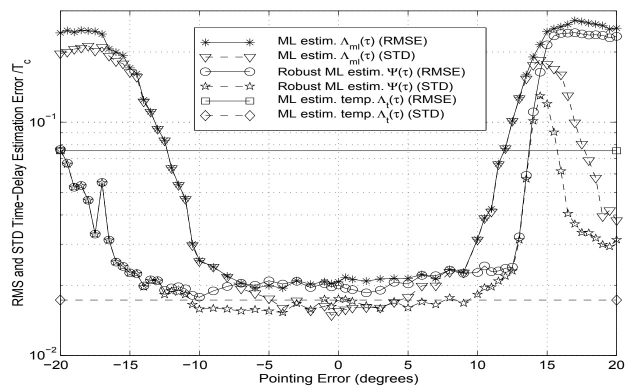

Figure 3: RMSE and STD in the presence of one reflection.

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