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ON THE TESTING OF THE MAGNETIC FIELD INTEGRAL EQUATION WITH LINEAR TRIANGLE BASIS FUNCTIONS IN METHOD OF MOMENTS

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Abstract. *A formulation of the Method of Moments is presented for analyzing scattering problems involving three dimensional conductors. It is given an insight into the Magnetic Field Integral equation for which a triangle facet is used. It is shown the drawback of the MoM-MFIE when compared with the MoM-EFIE, particularly evident in sharp-cornered bodies of small electrical dimensions. It is introduced an alternative way to compute the solid angle that decreases the MoM-MFIE RCS error. Some MoM-MFIE RCS results on perfectly conducting electrically small cones, cubes and other polyhedrons are presented to confirm our solid angle choice.*

1 INTRODUCTION

The S. M. Rao, D. R. Wilton and A. W. Glisson presented a Galerkin formulation for the Electric Field Integral Equation (EFIE) [3]. An analogous formulation for the Magnetic Field Integral Equation (MFIE) over conductors has also been developed [4]. According to the flat characteristic of the expanding triangles, this raw version of the MoM-MFIE assumes $\Omega_0 = 2\pi$, which is straightforward for smooth geometries. This assumption is unappropriate, though, for those basis functions that border on the geometry edges, where the solid angle value cannot be 2π. For sharp-cornered objects of electrically small dimensions, the contribution of these basis functions is particularly important. When comparing the MoM-MFIE RCS with the MoM-EFIE RCS, that according to its robustness is taken as reference, a significant error appears since a high amount of basis functions present an ill-defined solid angle value. For electrically larger sharp-cornered objects, the electromagnetic diffraction due to the edges is less important. Because of the less number of basis functions attached at the edges, the RCS error due to the wrong solid angle choice is less remarkable, although it still remains. Therefore, we have only taken into consideration sharp-cornered objects of small electrical dimensions. This article provides a correction on this first raw version of the MoM-MFIE by modifying the value of the solid angle so as to approach the MoM-MFIE results to the MoM-EFIE ones. Although the improvement is shown for objects of small electrical dimensions, its validity is extendable to all kind of meshed bodies, such as electrically large and sharp-cornered or even smooth-varying but coarsely meshed.

2 FORMULATION: EFIE, MFIE

$\vec{J}$ is expanded all over the surface of the scatterer by means of the *RWG* basis functions $\vec{f}_n$, a well-known set of linearly interpolating basis functions over triangles defined by *Rao et al.* [3]

$$\vec{J} = \sum_{n=1}^{N} \vec{f}_n(\vec{r}\,')I_n \tag{1}$$

being I_n the unknown current coefficients.

The EFIE in a PeC geometry forces the tangential component of $\vec{E}$ to be nul.

$$\vec{E}^S\Big|_{tan} = -\vec{E}^i\Big|_{tan} \tag{2}$$

where the electrical scatterer field $\vec{E}^S$ is defined as

$$\vec{E}^{S} = -jk\boldsymbol{h}\int_{S'} \vec{J}(\vec{r}\,')\, G(\vec{r}-\vec{r}\,')\, dS' + j\frac{\boldsymbol{h}}{\boldsymbol{k}}\int_{S'} \nabla'\,\vec{J}(\vec{r}\,')\,\nabla' G(\vec{r}-\vec{r}\,')\, dS' \tag{3}$$

In matrix form, as shown in [3], we have

$$\left[Z^{E}_{mn}\right]\left[I_{n}\right] = -\left[E^{i}_{m}\right] \qquad n,m = 1..N \tag{4}$$

where $\left\langle \vec{f}_m\,,\,\vec{E}^{\,S}\left(\vec{f}_n\,(\vec{r}\,')\right)\right\rangle$, $\left[E^{i}_{m}\right]=\left\langle \vec{f}_m,\vec{E}^{i}\right\rangle$ and $\left\langle \vec{f},\vec{g}\right\rangle$ denotes the Hilbert inner product $\int \vec{f}(\vec{r}\,')^{*}.\vec{g}(\vec{r}\,')dS'$.

Similarly, up from the harmonic expression of the scattered magnetic field $\vec{H}^{S}$ we have

$$\vec{H}^{S} = \int_{S'} \vec{J}(\vec{r}\,')\times\nabla' G(\vec{r}-\vec{r}\,')\, dS' \tag{5}$$

Appealing to the expression of the Magnetic Field Integral Equation (MFIE),

$$\hat{n}\times\vec{H}^{S} - \vec{J} = -\hat{n}\times\vec{H}^{i} \tag{6}$$

where $\hat{n}$ stands for the unit normal vector to surface.

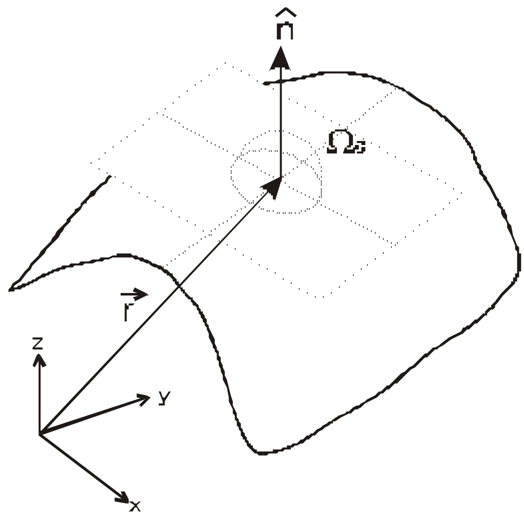

Figure 1: Local solid angle

The MFIE operator (5) requires the integration to be splitted into two definite parts [2]

$$\hat{n} \times \vec{H}^S(\vec{r}) = \int_{PV} \hat{n} \times \left(\vec{J} \times \nabla' G\right) dS' + \frac{\Omega_0(\vec{r})}{4\pi} J(\vec{r}) \tag{7}$$

where PV denotes the Cauchy's principal value integration and $\Omega_0(\vec{r}) = 2\pi$.

After the MoM discretization of (5) and (6), accounting for [4], the matrix results in

$$\begin{aligned} &\left[Z_{mn}^H\right][I_n] - [D_{mn}][I_n] \\ &= \left[Z_{mn}^H\right]_{PV}[I_n] + \frac{1}{2}[D_{mn}][I_n] - [D_{mn}][I_n] \\ &= \left[Z_{mn}^H\right]_{PV}[I_n] - \frac{1}{2}[D_{mn}][I_n] \qquad n, m = 1..N \end{aligned} \tag{8}$$

being $D_{mn} = \left\langle \vec{f}_m, \vec{f}_n \right\rangle$, $\left[Z_{mn}^H\right] = \left\langle \vec{f}_m, \hat{n} \times \vec{H}^S\left(\vec{f}_n\right) \right\rangle$ and $\left[H_m^i\right] = \left\langle \vec{f}_m, \hat{n} \times \vec{H}^i \right\rangle$.

3 COMPUTATION

4.1 Source integration

So as to achieve the maximum accuracy, the singular terms of both operators ($\propto \frac{1}{R}$ for EFIE and $\propto \frac{1}{R}, \frac{1}{R^3}$ for MFIE) are analytically integrated ([4],[5]). We have let the rest of the terms be numerically computed considering a four point quadrature rule. Furthermore, the MoM-MFIE formulation moves the cross-product out of the integral according to [5]. This accurate formulation is rather important to avoid errors when analyzing electrically small geometries, which are preferably chosen, in order to ensure unbiased values for $\vec{H}^S$ and $\vec{E}^S$.

4.2 Field integration

The weighting integrals, inherent to the MoM, have been distinguished by numerically integrating the field contributions coming from either all the source triangles (Rf large) or from only the same field triangle (Rf=0), being Rf the field domain radius where the field integration is precisely carried out. As shown in figures 3, 4, 5, 6 and 7, which refer respectively to a cube with side of 0.1λ and 48 facets, a cube with side of 0.2λ and 48 facets, a rectangular basis pyramid with side 0.07λ and 72 facets, a regular octahedron with side 0.07λ and 72 facets and a cone with basis radius of 0.05λ and height of 0.04λ and 288 facets, all under axial +z incidence and with the electric field x-polarized. One remarks on the unaccordance of the MoM-MFIE results with the MoM-EFIE ones despite the imposed high accuracy for the computed fields. The MoM-EFIE is very robust since its results , according to the fine enough meshing, are very similar when varying the reach of the field integrating area. The MoM-MFIE, however, shows a higher dependance on the number of field integrating points. Indeed, when enlarging the reach of the field integrating area to the whole geometry, the MoM-MFIE approaches slowly the RCS to the EFIE reference but still an error remains.

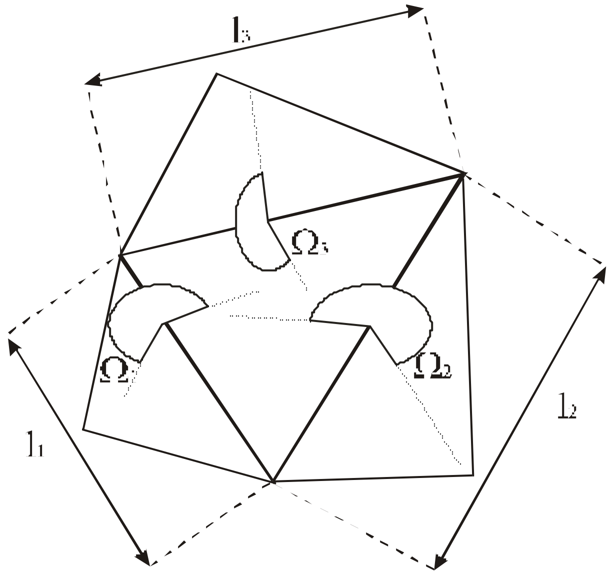

Figure2: Local solid angle estimate

5 MOM-MFIE CORRECTION AND IMPROVEMENT

Since the conditioning of the linear systems (4) and (8) is good, we consider that this biased dissimilarity between both MoM operators may come from the unsatisfactory aproximation $\Omega_0 \approx 2\pi$ on the edges of the geometries.

$$\Omega_{eq} = \frac{(l_1\Omega_1 + l_2\Omega_2 + l_3\Omega_3)}{(l_1 + l_2 + l_3)} \tag{9}$$

We have thus undertaken a direct correction on the MoM-MFIE operator by slightly modifying the solid angle value. By trial and error, we have been able to define the new constant equivalent solid angle value on a triangle as the weighted average of the local solid angles over each edge (figure 2).

This correction approaches the MoM-MFIE results to the MoM-EFIE ones for any geometry and brings up information about edges and vertexes that was inherently ignored in the first raw MoM-MFIE formulation.

$$\begin{aligned}\left[Z^{H}_{mn}\right][I_n]-[D_{mn}][I_n] \\ =\left[Z^{H}_{mn}\right]_{PV}[I_n]+\frac{\Omega_{eq}}{4\pi}[D_{mn}][I_n]-[D_{mn}][I_n] \\ =\left[Z^{H}_{mn}\right]_{PV}[I_n]+\left(\frac{\Omega_{eq}}{4\pi}-1\right)[D_{mn}][I_n] \quad n,m=1..N\end{aligned} \tag{10}$$

This modified MoM-MFIE operator 10 works very precisely in any case as long as one testing point is used in MoM. The modified MoM-MFIE excels thanks to its simplicity, for only one field testing point is required, and to its accuracy, which is better than the non corrected MoM-MFIE for any geometry when compared to MoM-EFIE. One can assess its good behavior in sharp-cornered geometries as shown in figures 3, 4, 5, 6 and 7. In figure 7, the cone is overdiscretized along the polar direction so as to cancel the influence of a coarse discretization of the curvature on the EFIE solution, which thus remains as good reference to evaluate the solid angle correction of the cone basis effect on the MoM-MFIE.

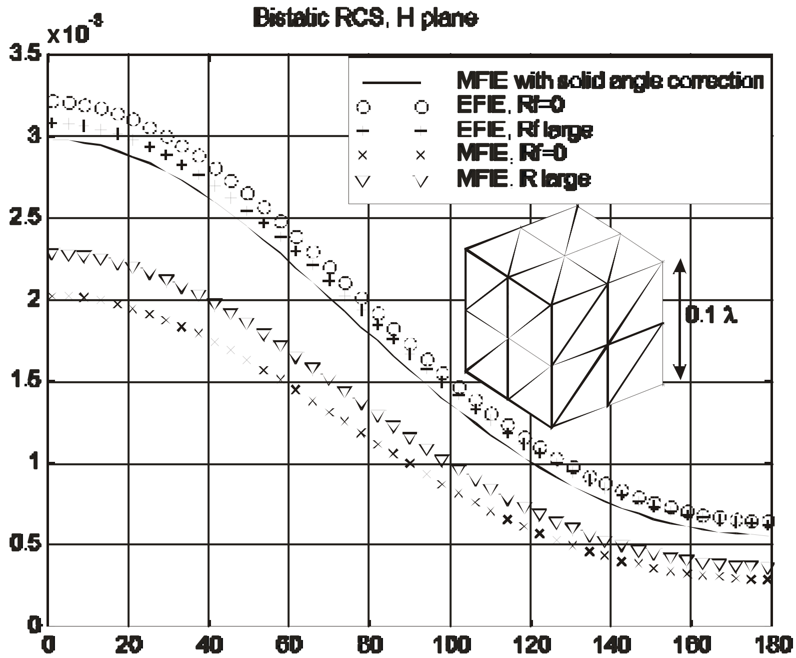

Figure 3: H plane bistatic RCS for a perfectly conducting cube with side of 0.1λ

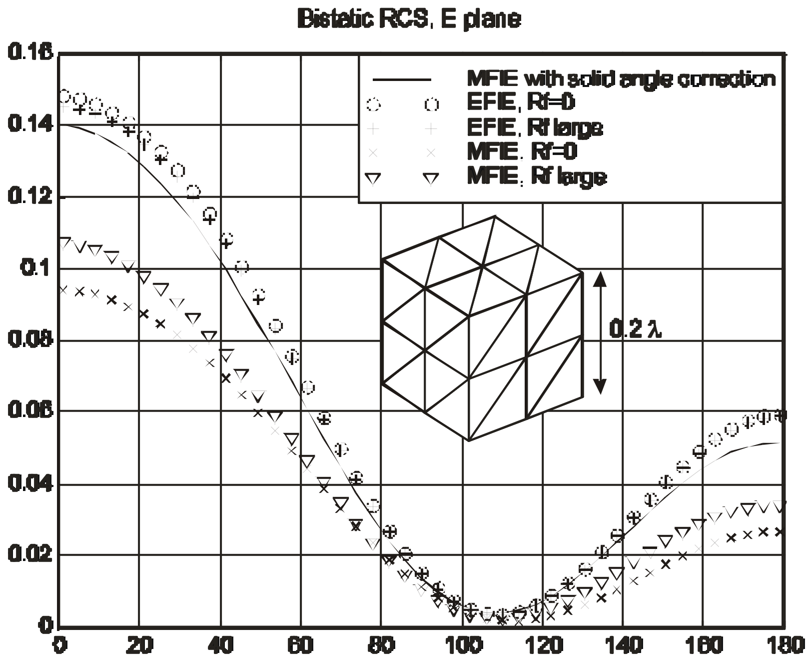

Figure 4: E plane bistatic RCS for a perfectly conducting cube with side of 0.2λ

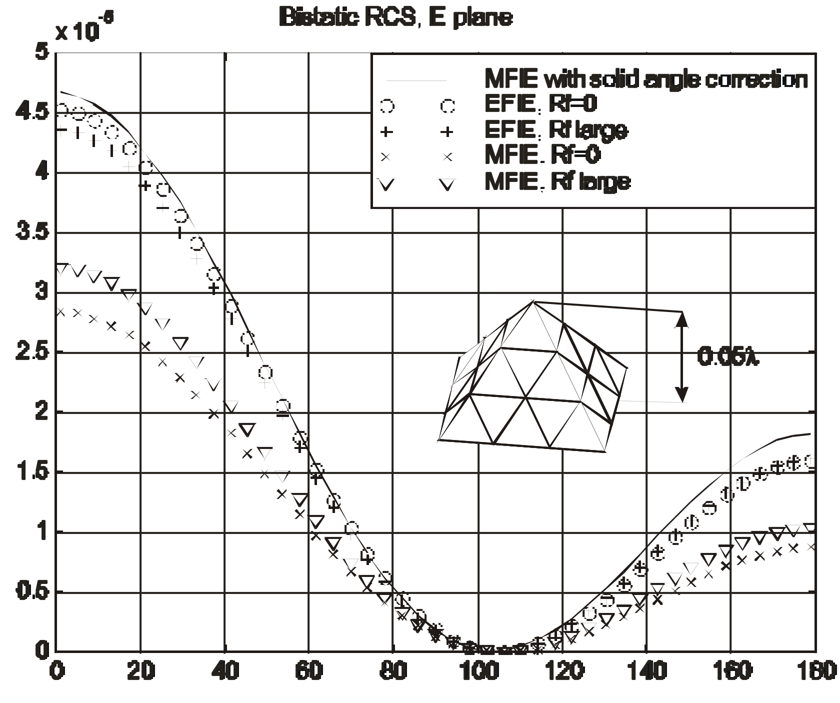

Figure 5: E plane bistatic RCS for a perfectly conducting pyramid with side of 0.07λ

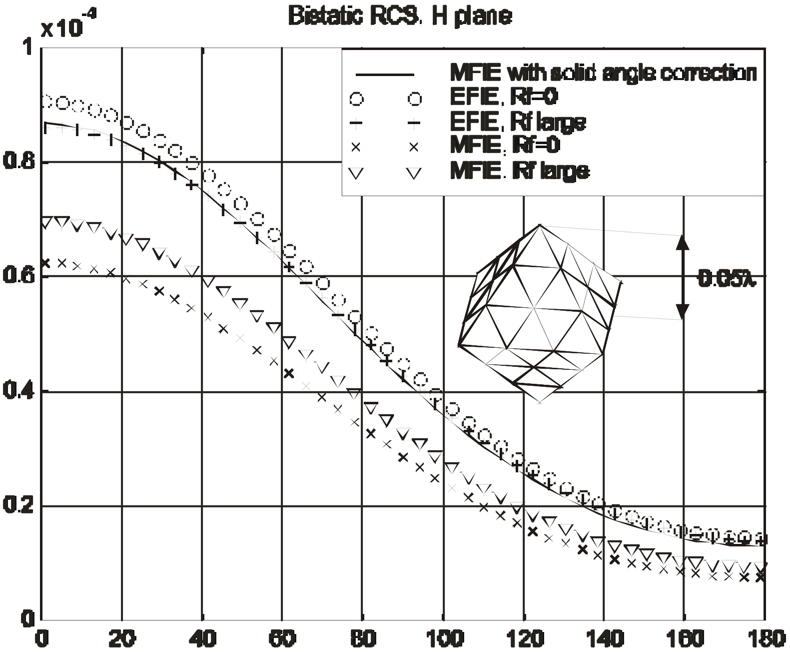

Figure 6: H plane bistatic RCS for a perfectly conducting octahedron with side of 0.07λ

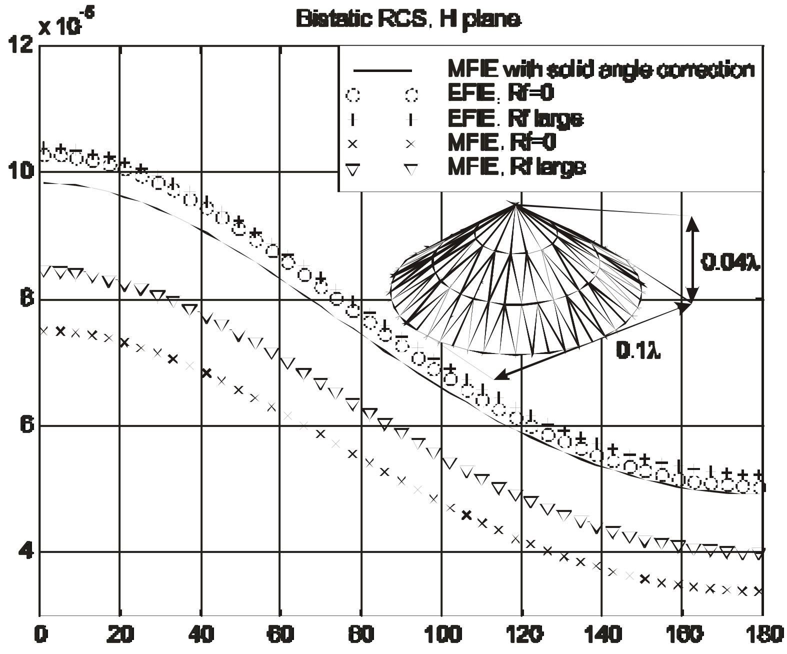

Figure 7: H plane bistatic RCS for a perfectly conducting cone with basis radius of 0.05λ and height of 0.04λ

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