MULTIPATH AND INTERFERENCE ERRORS REDUCTION IN GPS USING ANTENNA ARRAYS

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ABSTRACT

The Global Positioning System (GPS) is a worldwide satellite based positioning system that provides any user with tridimensional position, speed and time information. The measured pseudorange is affected by the multipath propagation, which probably is the major source of errors for high precision systems. After a presentation of the GPS and the basic techniques employed to perform pseudorange measurements, the influence of the multipath components on the pseudorange measurement is explained. Like every system the GPS is also exposed to the errors that can be caused by the interferences, and a lot of civil applications need robust receivers to interferences for reasons of safety. In this paper some signal array processing techniques for reducing the code measurement errors due to the multipath propagation and the interferences are presented. Firstly, a non-adaptive beamforming is used. Secondly, a variant of the MUSIC and the maximum likelihood estimator can be used to estimate the DOA of the reflections and the interferences, and then a weight vector that removes these signals is calculated. In the third place, a beamforming with temporal reference is presented; the reference is not the GPS signal itself, but the output of a matched filter to the code. An interesting feature of the proposed techniques is that they can be applied to an array of arbitrary geometry.

1. INTRODUCTION

The GPS is a satellite based navigation and positioning system that allows to estimate precisely the tridimensional position, the speed and the time in the receiver. It is a universal system, which works all the time independently of the meteorological conditions. Its applications are not restricted to navigation, but they comprise precise surveying, geodesy, fleet control, traffic control in harbors, etc. Nowadays there is an increasing interest about the GPS since it could mean a great progress in many sciences.

The GPS is based on the measurement of the distance between the satellites and the receiver by means of the signal propagation delay, so it is clear that the only signal of interest is the line-of-sight one. The propagation delay multiplied by the electromagnetic wave propagation speed is the GPS observable called pseudorange. The pseudorange does not coincide with the distance between the satellite and the receiver because of several errors, such as clock errors, ionospheric errors, etc. The other GPS observable is the phase of the line-of-sight signal. The pseudorange is the basic observable in most receivers, since a receiver can calculate its position and the GPS time from four pseudoranges. There are many phenomena that cause errors in the pseudorange and in the phase, but the multipath propagation is the most important source of errors for high precision systems (Ref. 1), such as landing systems and receivers calculating differential corrections, and it can not be corrected by differential techniques. Moreover, like every system the GPS is also exposed to the errors that can be produced by the interferences, whether they are intentioned or not; and a lot of civil applications need robust receivers against interferences for reasons of safety.

A strategy to combat the multipath effects consists on carrying out a temporal processing of the received signal. Some examples of this approach are the narrow spacing DLL (Ref. 2) and the Multipath Estimating Delay Locked Loop (MEDLL) (Ref. 3). The MEDLL performs measurements of the multipath characteristics and it exploits this knowledge to contend the multipath. In this way the multipath is partially canceled in a general scene, but it is not possible to eliminate the interferences. In this paper we propose some techniques that make use of antenna arrays in order to form a beam that has nulls in the directions of arrival (DOA) of the reflections and the interferences.

The paper is organized as follows. In section 2, we describe the signal model. In section 3, the way the pseudorange is measured is explained. In section 4, we describe the proposed methods. Finally, in section 5 we analyze the performance of each method using computer simulations and conclusions are drawn.

2. SIGNAL MODEL

Each satellite transmits two signals of frequencies 1575.42 MHz (L1) and 1227.60 MHz (L2). They are direct sequence spread spectrum (DS-SS) signals and DS-CDMA is used as the multiple access technique to the satellites. Each carrier is modulated by the 50 bps navigation message and by a pseudo-noise (PN) code. The navigation message contains the ephemerides (position of the satellite), clock correction parameters, ionospheric correction parameters, almanac, etc. The signal L1 is modulated by the C/A code and by the P code in quadrature. The signal L2 is only modulated by the P code, which has a chip rate of 10.23 Mchips/s and it is only available to authorized users. The C/A code is a Gold sequence of period 1023 chips and it has a chip rate of 1.023 Mchips/s. Civil users have only access to this code, therefore the study presented in this paper will be centered in the C/A code. The usual values of the ratio of signal power to noise spectral density (C/N_o) for the L1-C/A signal range between 35dB-Hz and 45dB-Hz for a $0dB_I$ right-hand polarized antenna.

The baseband line-of-sight received signal from the satellite *l* is:

$$s_o^l(t) = a_o^l \, e^{j\vartheta_o^l} \, e^{j2\pi f_{d,o}^l t} \sum_{k=-\infty}^{\infty} d^l(k) \; p(t - \tau_o^l - kT_b) \tag{1}$$

where $d^l(k)$ is the navigation message {1, -1} of the satellite *l*.

$f_{d,o}^l$ is the Doppler frequency

τ_o^l is the propagation delay that we want to measure.

$\alpha_o^l = a_o^l \, e^{j\vartheta_o^l}$ is the complex amplitude of the line-of-sight signal.

$p(t)$ represents 20 periods of the C/A code.

T_b is the bit time.

For simplicity sake's in the rest of the paper we will not write the superindex *l* because it is assumed that the signals from different satellites can be separated thanks to the quasi-orthogonality of the codes.

The multipath propagation is caused by the reflections of the emitted signal in reflectors that are in the vicinity of the receiver. The influence of these reflections depends on their power, delay and phase with which they arrive at the receiver with regard to the direct signal. The multipath component can be made up of a specular component and/or a diffuse component, depending on the physical environment. The first one is a collection of few reflections produced by smooth surfaces, such as the wings of a plane, the sea or the earth surface and certain buildings. The diffuse component is represented by a discrete or continuos summation of a large number of reflections with less power than the previous ones. In this case it is usual to consider that each reflection has a uniformly distributed phase over [0, 2π] and a Rayleigh envelope. The power delay profile is usually taken exponential:

$$P(\tau) = \frac{P_o}{d} e^{-\frac{\tau}{d}} \, u(\tau) \tag{2}$$

where d is the mean delay of the multipath component in relation to the direct signal and P_o is the total power in the diffuse component. The power of every reflection is lower than that of the line-of-sight signal, mainly due to the attenuation introduced by the reflector, but the total power in the diffuse component could be greater than the power of the direct signal.

If M reflections arrive at the receiver the received signal can be expressed as:

$$x(t) = \sum_{i=0}^{M} s_i(t) + n(t) + i(t) \tag{3}$$

$$\text{with} \quad s_i(t) = \alpha_i \, e^{j2\pi f_{d,i} t} \sum_{k=-\infty}^{\infty} d(k) p(t - \tau_i - kT_b) \tag{4}$$

where i(t) represent the interferences, n(t) is the AWGN and the subindex 0 stands for the line-of-sight signal, $\tau_i > \tau_o$ para $i > 0$

Unlike the communication systems, where the reflections can cause temporal diversity that may be useful to increase the SNR and, therefore, improve the system performance, in GPS the reflections produce errors in the estimation of the pseudoranges. When an antenna array is used the received signal can be expressed in vectorial form:

$$\mathbf{x}(t) = \sum_{i=0}^{M} \mathbf{a}_i \, s_i(t) + \mathbf{i}(t) + \mathbf{n}(t) = \sum_{i=0}^{M} \mathbf{a}_i \, \alpha_i \, e^{j2\pi f_{d,i} t} \sum_{k=-\infty}^{\infty} d(k) p(t - \tau_i - kT_b) + \mathbf{i}(t) + \mathbf{n}(t) \qquad (5)$$

where $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_N(t)]^T$ is a column vector that contains the signal of each antenna, N is the number of antennas, $\mathbf{n}(t)$ is additive white (spatially and temporally) gaussian noise and $\mathbf{i}(t)$ represents the interferences. The vector $\mathbf{a}_i$ is called steering vector or DOA vector and it contains the carrier phase delay affecting the signal in each antenna:

$$\mathbf{a}_i = \left[e^{-j2\pi f_o \tau_{1,i}}, e^{-j2\pi f_o \tau_{2,i}}, \ldots, e^{-j2\pi f_o \tau_{N,i}} \right]^T = \left[e^{-j\mathbf{k}_i \mathbf{r}_1}, e^{-j\mathbf{k}_i \mathbf{r}_2}, \ldots, e^{-j\mathbf{k}_i \mathbf{r}_N} \right]^T \qquad (6)$$

f_o is the carrier frequency (1575.42 MHz), $\mathbf{k}_i$ is the wave vector of the signal i, $\tau_{n,i}$ is the propagation delay of the signal i between a reference point and the antenna n ($\tau_{n,i}$ should not be confused with τ_i, which is the propagation delay of the signal between the satellite and the receiver) and $\mathbf{r}_n$ is the position vector of the antenna n with regard to the reference point. If M_I interferences are received their contribution can be written as:

$$\mathbf{i}(t) = \sum_{n=1}^{M_I} \mathbf{a}_{I,n} \, i_n(t) \qquad (7)$$

It has to be noted that this notation is possible because the array works under narrow-band conditions given the carrier frequency and the bandwidth of the GPS signals. The received vector can also be expressed using matrices:

$$\begin{aligned} \mathbf{x}(t) &= \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \\ \mathbf{A} &= \left[\mathbf{a}_o, \mathbf{a}_1, \ldots, \mathbf{a}_M, \mathbf{a}_{I,1}, \ldots, \mathbf{a}_{I,M_I} \right] \\ \mathbf{s}(t) &= \left[s_o(t), s_1(t), \ldots, s_M(t), i_1(t), \ldots, i_{M_I}(t) \right] \end{aligned} \qquad (8)$$

The matrix $\mathbf{A}$ is called steering matrix. The signals of different antennas can be combined in order to cancel the reflections and the interferences. Since we have a narrow band array the beamforming is carried out by the weighted sum of the signals. If $\mathbf{w}$ is the weight vector the output of a beamformer is

$$z(t) = \mathbf{w}^H \mathbf{x}(t) \qquad (9)$$

3. MEASUREMENT OF THE PSEUDORANGE

The pseudorange is measured in a GPS receiver by evaluating the GPS signal propagation time delay from the satellite, based on the time information carried by the phase of the C/A code. Each channel of the GPS receiver generates a replica of the C/A code to synchronize the code of the input GPS signal from the satellite. Once the receiver locally generated PN code is matched with the received code, the input PN code phase is known, from which the satellite clock can be derived. The pseudorange time delay is estimated by calculating the time difference between the satellite clock and the receiver clock. So, it is clear that the pseudorange is measured by accurately tracking the PN code phase of the input GPS signal, and the most widely used system to do this is the non-coherent delay locked loop, which is shown in Figure 1.

Firstly a phase counter-rotation is performed in accordance with the estimated Doppler frequency of the line-of-sight signal. Next, the signal is multiplied by a delayed and an advanced versions of the code:

$$\begin{aligned} e(t) &= p(t - \hat{\tau}_o + \delta T_c) \\ l(t) &= p(t - \hat{\tau}_o - \delta T_c) \end{aligned} \qquad (10.a)$$

where $\hat{\tau}_o$ is the estimated delay of the line-of-sight signal, T_c is the chip time, $\delta \in [0, 1]$ is the normalized spacing between the two signals. It is said that the DLL has a $2\delta T_c$ correlator spacing. ξ is the normalized error in the estimation of the propagation delay:

$$\xi = \frac{\tau_o - \hat{\tau}_o}{T_c} \tag{10.b}$$

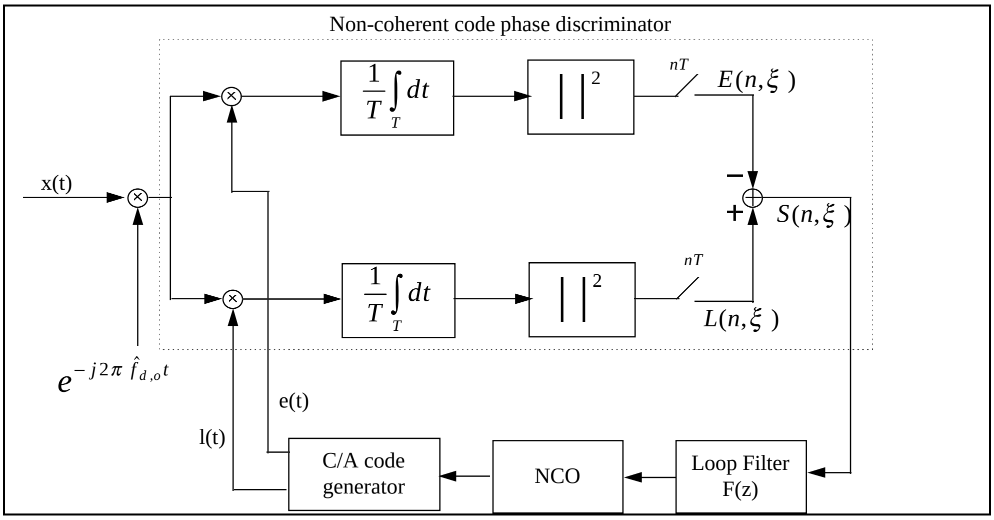

Figure 1.- Non-coherent DLL

After integration and squaring the envelope of the signals in each branch, they are subtracted and we obtain the value $S(n,\xi)$, which is the output of the code phase discriminator. The code phase discriminator is characterized by the so called s-curve or discriminator curve, that is:

$$S(\xi) = E\{S(n,\xi)\} \tag{11}$$

In the circuit the average is performed by the loop filter and by the loop itself. We can write some expressions that describe the circuit:

$$S(n,\xi) = L(n,\xi) - E(n,\xi) + n_s(n)$$
$$L(n,\xi) = \left| \sum_{i=0}^{M} \alpha_i \, e^{j\psi_i(n)} \, C_p\left(\tau_i - \hat{\tau}_o - \delta T_c\right) sinc\left(\left(f_{d,i} - \hat{f}_{d,o}\right)T\right) \right|^2 \tag{12}$$
$$E(n,\xi) = \left| \sum_{i=0}^{M} \alpha_i \, e^{j\psi_i(n)} \, C_p\left(\tau_i - \hat{\tau}_o + \delta T_c\right) sinc\left(\left(f_{d,i} - \hat{f}_{d,o}\right)T\right) \right|^2$$

where $n_s(n)$ is the equivalent noise component, $\psi_i(n) = 2\pi\,(f_{d,i} - \hat{f}_{d,o})nT$ and $C_p(t)$ is the C/A code autocorrelation. We have neglected the code self-noise because the integration interval T is much larger than the chip time T_c. The autocorrelation approximates for small delays the ideal autocorrrelation function:

$$C_p(t) \cong \Lambda\left(\frac{t}{T_c}\right) = \begin{cases} 1 - \dfrac{|t|}{T_c}, & |t| \le T_c \\ 0\,, & |t| > T_c \end{cases} \tag{13}$$

The value of S is fed into the loop filter, whose output controls the NCO and the C/A code generator in order to update the value of $\hat{\tau}_o$. The steady-state estimation of the delay of the line-of-sight signal is the value of $\hat{\tau}_o$ for which $S(\xi)=0$ with positive slope. In absence of multipath, $S(0)=0$, so there is no bias in the estimation of the delay and the noise only produces a jitter in the code phase error. The effect of the interferences is similar to that of the noise. The interferences can be viewed as a increase of the noise spectral density, hence they produce an increase of the jitter. However, interferences can be more harmful, if their power is very high they can cause the DLL to loose lock within the received code. On the other hand, the multipath signals make that $S(\xi_1)=0$ for some $\xi_1 \neq 0$, so they produce a bias in the estimation of the delay and, obviously, produce an error in the measurement of the pseudorange.

If there is not multipath propagation and ξ is small the code phase jitter can be obtained using the linear model of the DLL. The system equation is:

$$\frac{\hat{\tau}_o}{T_c} = D(z)F(z)S(z,\xi)$$
$$D(z) = \frac{z^{-1}}{1-z^{-1}} \tag{14}$$

D(z) is the transfer function of the NCO. For small ξ the discriminator function can be modeled by a liner function:

$$S(z,\xi) = K(\delta)\xi + n_s(z)$$
$$K(\delta) = a_o^2\, sinc^2\left(\left(f_{d,o} - \hat{f}_{d,o}\right)T\right)4(1-\delta) \tag{15}$$

$K(\delta)$ is the discriminator gain and the system equation stands:

$$\frac{\hat{\tau}_o}{T_c} = D(z)F(z)K(\delta)\left(\xi + \frac{n_s}{K(\delta)}\right) \tag{16}$$

which can be rewritten as:

$$\xi = H_\tau(z)\frac{\tau_o}{T_c} - H_n(z)\frac{n_s}{K(\delta)}$$
$$H_\tau(z) = \frac{1}{1 + D(z)F(z)K(\delta)} \tag{17}$$
$$H_n(z) = \frac{D(z)F(z)K(\delta)}{1 + D(z)F(z)K(\delta)}$$

Then, the code phase jitter produced by the input noise is:

$$\sigma_\xi^2 = \frac{\sigma_{n_s}^2}{K^2(\delta)}\frac{1}{2\pi}\int_0^{2\pi}\left|H_n(e^{jw})\right|^2 dw$$
$$\sigma_{n_s}^2 \approx 8a_o^2\, sinc^2((f_{d,o} - \hat{f}_{d,o})T)\frac{N_o}{T}\Lambda^2(\delta)\left(1-\Lambda(2\delta)\right) \tag{18}$$

4. DESCRIPTION OF THE PROPOSED METHODS

It is well know that spread spectrum systems have good rejection of interferences and multipath, but it is not enough to the precision wanted in GPS. Using an array each received signal appears at the output of the beamformer multiplied by the term $\mathbf{w}^H\mathbf{a}$. It is clear that a convenient choice of the weight vector can reduce considerably the amplitudes of the reflections and those of the interferences, at the same time that the line-of-sight signal is enhanced in front of the noise. An important characteristic of the proposed methods is that they are not restricted to a particular array geometry, so they can be applied to any array. The simplest solution is:

4.1. Non-adaptive beamformer

In this method a phase shift is applied to each antenna so that the beam has a maximum in the DOA of the desired signal. In other words, the selected weight vector is:

$$\mathbf{w} = \mathbf{a}_o \tag{19}$$

It is assumed that the steering vector of the line-of-sight signal is known since the receiver knows the GPS navigation message and its position with an error that do not affect the angle with which the satellite is observed. Any signal that arrives from the sidelobes is attenuated al least 12dB approximately in a linear or rectangular array and 7dB approximately in a circular array. This beamformer is not useful for the reflections and the interferences that fall in the main lobe or for very strong interferences although they fall in the side lobes. This is the reason why we propose the use of more sophisticated adaptive beamformers.

4.2. DOA Estimation

When the directions of arrival of the reflections and the interferences are known we can use a beamformer that set nulls in those directions and gain equal 1 in the direction of the line of the line-of-sight signal, minimizing the white noise at the output with these constraints:

$$\begin{aligned} &\mathbf{w} = \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{f}^* \\ &\mathbf{C} = \left[\mathbf{a}_o, \mathbf{a}_1, \ldots, \mathbf{a}_M, \mathbf{a}_{I,1}, \ldots, \mathbf{a}_{I,M_I}\right], \qquad \mathbf{f} = \left[1, 0, 0, \ldots, 0\right]^T \end{aligned} \tag{20}$$

To use this beamformer it is necessary to estimate the DOA of the reflections and the interferences. We must take into account that the signal scene is highly coherent since the modulus of the correlation coefficient between the line-of-sight signal and the reflections is very near 1, if only one sample per bit at the output of the matched filter is taken (this means that we use samples that are obtained correlating the received signal with only the prompt code). Therefore, a DOA algorithm that works with coherent signals must be used. One alternative is to apply the well known spatial smoothing, but it has two serious drawbacks: spatial smoothing reduces the array aperture (the resolution worsen) and it is restricted to arrays formed by several identical subarrays related by a translation (not rotated). We suggest the application of a variant of the MUSIC (Ref.5) and the maximum likelihood estimator by alternating projection (Ref. 4).

If the received vector **x**(t) is correlated with the code $p(t-\hat{\tau}_o)$ during an interval T we obtain the samples **y**(n)

$$\begin{aligned} &\mathbf{y}(n) \cong \sum_{i=0}^{M} \mathbf{a}_i\, \beta_i(n)\, d(n) C_p(\tau_i - \hat{\tau}_o) + \sum_{n=1}^{M_I} \mathbf{a}_{I,n}\, i_{f,n}(n) + \mathbf{n}_f(n) = \mathbf{A}\mathbf{s}_c(n) + \mathbf{n}_f(n) \\ &\beta_i(n) = \alpha_i\, sinc((f_{d,i} - \hat{f}_{d,o})T) e^{j\psi_i(n)} \end{aligned} \tag{21}$$

and the correlation matrix is:

$$\begin{aligned} &\mathbf{R}_{yy} = E\left\{\mathbf{y}(n)\mathbf{y}^H(n)\right\} = \mathbf{A}\mathbf{S}_{cc}\mathbf{A}^H + \sigma_f^2\mathbf{I} \\ &\mathbf{S}_{cc} = E\left\{\mathbf{s}_c(n)\mathbf{s}_c^H(n)\right\} \\ &\mathbf{s}_c(n) = \left[\beta_o(n) d(n) C_p(\tau_o - \hat{\tau}_o), \cdots, \beta_M(n) d(n) C_p(\tau_M - \hat{\tau}_o), i_{f,1}(n), \ldots, i_{f,M_I}(n)\right]^T \end{aligned} \tag{22}$$

We will assume that $\left|f_{d,i} - \hat{f}_{d,o}\right| << 1/T$, so $\beta_i(n) \approx \alpha_i$. The variant of the MUSIC that we propose is based on the eigendecomposition of $\mathbf{R}_{yy}$. The eigenvectors associated with the $M_I + 1$ highest eigenvalues span the following subspace:

$$span\left\{\mathbf{e}_1, \ldots, \mathbf{e}_{M_I+1}\right\} = span\left\{\mathbf{a}_{I,1}, \ldots, \mathbf{a}_{I,M_I}, \sum_{i=0}^{M} \mathbf{a}_i\, \alpha_i\, C_p(\tau_i - \hat{\tau}_o)\right\} = range\left\{\mathbf{A}\mathbf{S}_{cc}\mathbf{A}^H\right\} \neq range\left\{\mathbf{A}\right\} \tag{23}$$

The N-M_I-1 eigenvectors associated with the lowest eigenvalue (equal to σ_f^2) span the complementary subspace orthogonal to previous one. If we define[1]

$$\mathbf{P} = \sum_{i=M_I+2}^{N} \mathbf{e}_i \mathbf{e}_i^H$$
$$\mathbf{B} = \left[\mathbf{a}(\theta_0), \mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_M)\right], \quad \mathbf{c} = \left[1, c_1, \ldots, c_M\right] \tag{24}$$

then a natural extension of the MUSIC algorithm for arrival sets containing at the most (M+1)-dimensional coherent signal subsets is to project vectors of the form **Bc**. The spatial spectrum takes the form:

$$S(\theta_0, \theta_1, \ldots, \theta_M) = \left[\min_{\mathbf{c}} \frac{\mathbf{c}^H \mathbf{B}^H \mathbf{P} \mathbf{B} \mathbf{c}}{\mathbf{c}^H \mathbf{B}^H \mathbf{B} \mathbf{c}}\right]^{-1} \tag{25}$$

The dependence on the complex combining vector can be eliminated by realizing that

$$S(\theta_0, \theta_1, \ldots, \theta_M) = \left[\mathit{min\ eigenvalue\ of}\ \left(\mathbf{B}^H \mathbf{P} \mathbf{B}, \mathbf{B}^H \mathbf{B}\right)\right]^{-1} \tag{26}$$

The directions of arrival correspond to the peaks of this 2(M+1)-dimensional spatial spectrum. If we use a linear array the spectrum is M+1-dimensional. If the DOA of the line-of-sight signal is known we reduce the search in 2 or 1 dimensions for bidimensional or unidimensionals arrays, respectively. A particular case of special interest is when only one reflection is received, then a search in a 2 or 1- dimensional spectrum (for bidimensional or unidimensional arrays, respectively) gives the DOA of the reflection. For the interferences that arrive from only one direction their DOAs can be obtained using a conventional MUSIC. For a given interference that arrives from L (L≤M+1) directions the presented variant of the MUSIC can be used since the 2(M+1) (or M+1) -dimensional spectrum will present a peak at every point where L of the M+1 directions are those of the interference.

Another method that works with coherent signals is the maximum likelihood estimator (MLE) by alternating projection. The MLE is the estimator that presents the minimum error in the estimation. However, it did not become popular because of the high computational load of the multivariate nonlinear maximization problem involved. If Q is the total number of received sources (including the signal reflections, the interferences and their reflections) it is necessary to calculate the maximum of certain function in a Q or 2Q-dimensional space. To solve this drawback the “Alternating Maximization” (AM) is used, that transforms the multidimensional problem into a sequence of much simpler one or two-dimensional maximization problems, so the computational load is similar to that of MUSIC, linear prediction, minimum variance... The MLE has several advantages over the previous methods, for instance: its lower variance of the error (especially for low SNR or for few snapshots), it can be used with few snapshots (even only one), it works with coherent sources and it does not impose any restrictions to the antennas or to the array geometry, only that the array manifold $\mathbf{a}(\theta)$ has to be know.

The matrix **D** represents the DOAs of the Q sources that impinge on the array.

$$\mathbf{D} = \left[\theta_1, \theta_2, \ldots, \theta_Q\right] \tag{27}$$

Considering that the noise is spatially and temporally white, and that the signals are deterministic but unknown, the ML estimator is obtained by solving the following minimization problem:

$$\min_{\mathbf{D}, \mathbf{s}_c(n)} \left\{ \sum_{n=1}^{N_{sum}} \left\| \mathbf{y}(n) - \mathbf{A}(\mathbf{D})\, \mathbf{s}_c(n) \right\|^2 \right\} \tag{28}$$

that coincides with the Least Squares (LS) criterion. First, we fix **D** and minimize with respect to **s,** yielding

$$\mathbf{s}_c(n) = \left(\mathbf{A}^H \mathbf{A}\right)^{-1} \mathbf{A}\, \mathbf{y}(n)$$
$$\min_{\mathbf{D}} \left\{ \sum_{n=1}^{N_{sum}} \left\| \mathbf{y}(n) - \mathbf{P}_{\mathbf{A}}\, \mathbf{y}(n) \right\|^2 \right\}, \quad \mathbf{P}_{\mathbf{A}} = \mathbf{A}\left(\mathbf{A}^H \mathbf{A}\right)^{-1} \mathbf{A}^H \tag{29}$$

[1] The vector **θ** means the direction of arrival of one wave, it can be a scalar (angle with the normal of the array) if the array is linear or it can have two components (azimuth and zenithal angle) in bidimensional arrays .

$\mathbf{P_A}$ is the projection operator onto the space spanned by the columns of the matrix **A**. Thus, the ML estimation of the DOAs is obtained by maximizing:

$$L(\mathbf{D}) = \frac{1}{N_{sum}} \sum_{i=1}^{N_{sum}} \left\| \mathbf{P}_{\mathbf{A}(\mathbf{D})}\, \mathbf{y}(n) \right\|^2 = tr\left\{ \mathbf{P}_{\mathbf{A}(\mathbf{D})}\, \mathbf{R}_{yy} \right\} \tag{30}$$

This maximization is computationally very expensive. This is why the "Alternating Maximization" is applied, which is a iterative technique; at each iteration a maximization is performed with respect to a single source while the estimated DOAs of the other sources are held fixed. That is, the value of $\boldsymbol{\theta}_\mathbf{i}$ at the (k+1) iteration is obtained by solving the following problem:

$$\hat{\theta}_i^{k+1} = \arg \max_{\theta_i} tr\left\{ \mathbf{P}_{\left[\mathbf{A}(\hat{\mathbf{D}}_i^k),\mathbf{a}(\theta_i)\right]} \mathbf{R}_{yy} \right\} \tag{31}$$

where $\hat{\mathbf{D}}_i^k$ contains the estimated DOAs of all the sources except for the source i. The drawback of this technique is that the algorithm converges to a local maximum. Depending on the initial condition, the local maximum may or may not be the global one. The initialization algorithm we have chosen is very simple, and though it does not assure the convergence to the global maximum, in nearly all the simulations we have performed it has been attained. It is based on: the initial value of DOA for the first source is calculated with (31) considering that only one source is received, the initial value of the DOA for the second source is calculated with (31) using only the initial value of the first one, for the third source only the two previous values are used, and so on.

The AM technique reduces the dimension of the maximization problem, but the computational load at each iteration is still high because of the calculation of the matrix **P**. This can be solved using a property of projection matrices, called the projection-matrix update formula. Let **B** and **C** be two arbitrary matrices with the same number of rows, then

$$\begin{aligned} \mathbf{P}_{[\mathbf{B},\mathbf{C}]} &= \mathbf{P}_\mathbf{B} + \mathbf{P}_{\mathbf{C}_\mathbf{B}} \\ \mathbf{C}_\mathbf{B} &= \left(\mathbf{I} - \mathbf{P}_\mathbf{B}\right)\mathbf{C} \end{aligned} \tag{32}$$

Thanks to this formula equation (31) may be rewritten as

$$\hat{\theta}_i^{k+1} = \arg \max_{\theta_i} tr\left\{ \mathbf{P}_{\mathbf{a}(\theta_i)_{\mathbf{A}(\hat{\mathbf{D}}_i^k)}} \mathbf{R}_{yy} \right\} \tag{33}$$

which can be further simplified, resulting:

$$\begin{aligned} \hat{\theta}_i^{k+1} &= \arg \max_{\theta_i} \left\{ \mathbf{b}^H\left(\theta_i, \hat{\mathbf{D}}_i^k\right) \mathbf{R}_{yy} \mathbf{b}\left(\theta_i, \hat{\mathbf{D}}_i^k\right) \right\} \\ \mathbf{b}\left(\theta_i, \hat{\mathbf{D}}_i^k\right) &= \frac{\mathbf{a}(\theta_i)_{\mathbf{A}(\hat{\mathbf{D}}_i^k)}}{\left\| \mathbf{a}(\theta_i)_{\mathbf{A}(\hat{\mathbf{D}}_i^k)} \right\|} \end{aligned} \tag{34}$$

When the DOA of the line-of-sight signal ($\boldsymbol{\theta}_\mathbf{o}$) is known no iteration is performed for this signal, the computational load reduces and the DOA estimation error and convergence velocity for the other signals improves.

5. SIMULATIONS

Firstly we are going to describe the parameters of the DLL and the arrays used in the simulations. The code phase delay is estimated by means of a non-coherent DLL with:

- Early-late spacing : T_c
- Sampling frequency : 6.5374 MHz
- Correlation time: T=20 ms
- Order :1^{st} ($F(z)=g_1$)
- Loop gain: $G_1=K(\delta)g_1=0.4$ (in ideal conditions, no CAG is used)

The incoming baseband equivalent signal is filtered with a second-order Butterworth low-pass filter with 3.25 MHz one-side passband. Next, the signal is passed through a filter that is matched to the previous one. We use two arrays:

- A uniform linear array (ULA) with 8 isotropic antennas spaced 0.5λ.
- A uniform circular array (UCA) with 8 isotropic antennas and a radius equal to 0.8λ.

The proposed methods can be applied to any array, but in many simulations we use a the linear array because it is easier to draw the array beam pattern and extract conclusions from it.

The two signals that are always present at the signal environment are the line-of-sight signal and the noise, with C/N_o=40 dB-Hz. The DOA of the line-of-sight signal (LOSS) is φ=30º for the linear array, and ϕ=180º , θ=30º for the circular array. The noise is spatially white. Besides, there can be one or more of the signals described in the Table 1. The interference is narrow-band and is shifted 10 KHz from the GPS L1 carrier frequency. The diffuse reflection is modeled by discrete reflections spaced T_c/10 away one each other.

Signal type and name	Power in relation to the power of the LOSS (dB)	Phase difference with the LOSS (º)	DOA for the linear array (φ)	DOA for the circular array (ϕ,θ)	Delay / T_c with regard to the LOSS
Specular reflection S1	-1.6 dB	0º	38º	(5º,80º)	0.75
Specular reflection S2	-3 dB	180º	22º	(160º,20º)	0.3
Diffuse reflection D1	-4.5, -5, -5.5, -6, -6.5, -7, -7.5, -8, -8.5, -9, -9.5, -10 dB	20, 40, 70, 0, 5, 10, -30, 0, -120, 150, 10, -50º	-50, 80, -40, 24, 35, 37, -20, 8, 40, -6.5, 60, 23º	(50,50), (300,50), (180,75), (0,80), (350, 85), (8,85), (200,36), (155,20), (165,40), (210,18), (0,75), (130º,80º)	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.1, 1.2
Interference I1	40 dB	------	20º	(200º,40º)	-------

Table 1.- Description of the signal environment.

The results presented in Table 2 and in Figure 3 show the errors produced by the multipath and by the interference when an array is not used, that is to say, there is only one antenna. The multipath propagation distorts the S-curve and it is responsible for the mean code phase estimation error. The interference increase the jitter (root mean square error) in the code phase estimation error. It has to be noted that when the code phase estimation error (ξ) is negative the estimated code phase delay ($\hat{\tau}_o$) is greater than the input code phase delay (τ_o) (10.b). In these simulations the received signal is composed by the LOSS and the noise plus some possible reflections and interferences, as it is indicated in the table and in the graphs.

Received signals (besides LOOS and noise)	Mean normalized code phase estimation error $\bar{\xi}$	Jitter in the normalized code phase estimation error $\sqrt{\overline{(\xi-\bar{\xi})^2}}$
None	0 --> 0 meters	0.017 --> 5 m
Specular reflection S1	-0.3415 --> -100.1m	0.026 --> 7.6 m
Specular reflection S2	0.3164 --> 92.8 m	0.009 --> 2.6 m
Diffuse Reflection D1	-0.3210 --> -94.1 m	unstable
Interference I1	0 --> 0 m	0.081 --> 24 m

Table 2.- Errors without beamformer

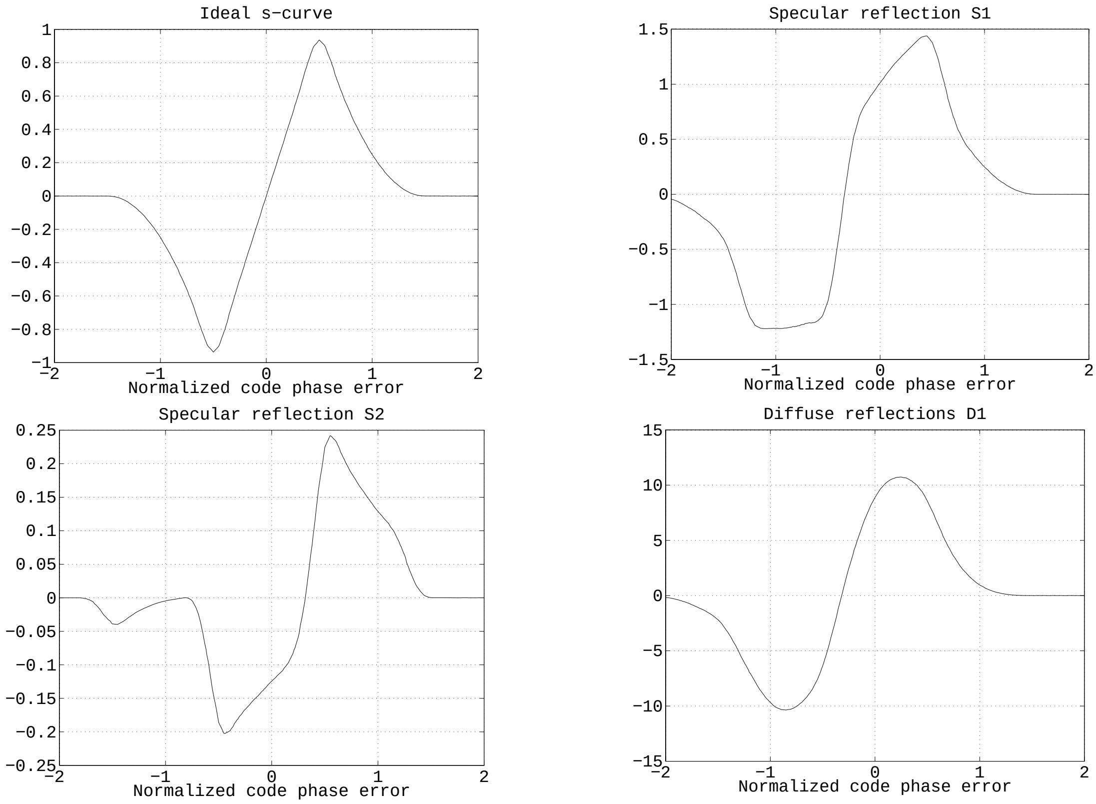

Figure 3.- S-curves without beamformer

Next, we simulate the first method we have proposed; in others words, a beamformer that applies a phase shift to each antenna in order to sum in-phase the contributions of the LOSS is used. The results (Table 3, Figures 4 and 5) show that little improvement is achieved, in some cases, because the reflections and the interferences fall in the main lobe or in a high side lobe of the array beam pattern. The jitter produced by the thermal noise is reduced with the use of the array because the SNR increases by 9 dB. These results make clear that other more sophisticated array processing techniques should be applied.

Received signals (besides LOOS and noise)	Mean normalized code phase estimation error $\overline{\xi}$		Jitter in the normalized code phase estimation error $\sqrt{\overline{(\xi-\overline{\xi})^2}}$	
	Linear array	Circular array	Linear array	Circular array
none	0 -->0m	0 --> 0m	0.006-->1.76m	0.006-->1.76m
Specular reflection S1	-0.2016-->-59m	-0.2314-->-68m	0.0077-->2.25m	0.0077-->2.25m
Specular reflection S2	-0.043-->-12.6m	0.2485-->72.9m	0.0067-->2m	0.0028-->0.8m
Diffuse Reflection D1	-0.2249-->-65.9m	-0.3023-->-88.6m	0.0087-->2.55m	0.0419-->12.3m
Interference I1	0 --> 0 m	0 --> 0m	0.012-->3.5m	0.0325-->9.5m

Table 3.- Errors with a non-adaptive beamformer

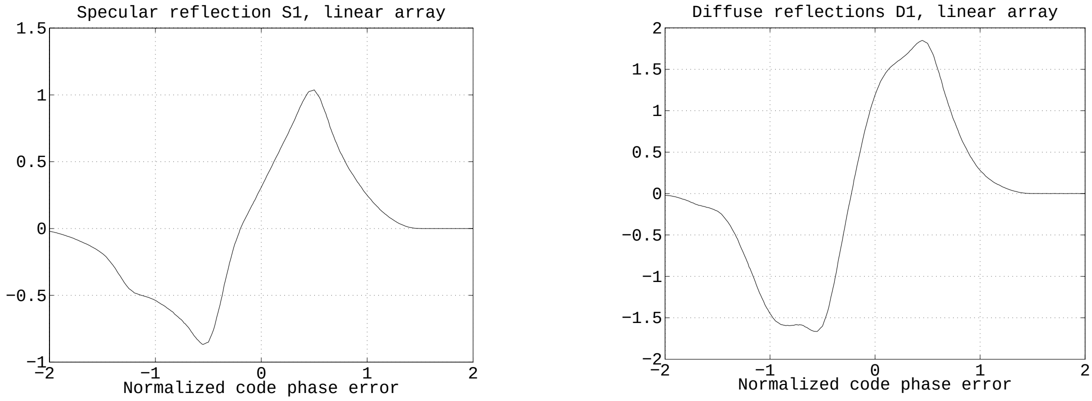

Figure 4.- S-curves with a non-adaptive beamformer, linear array

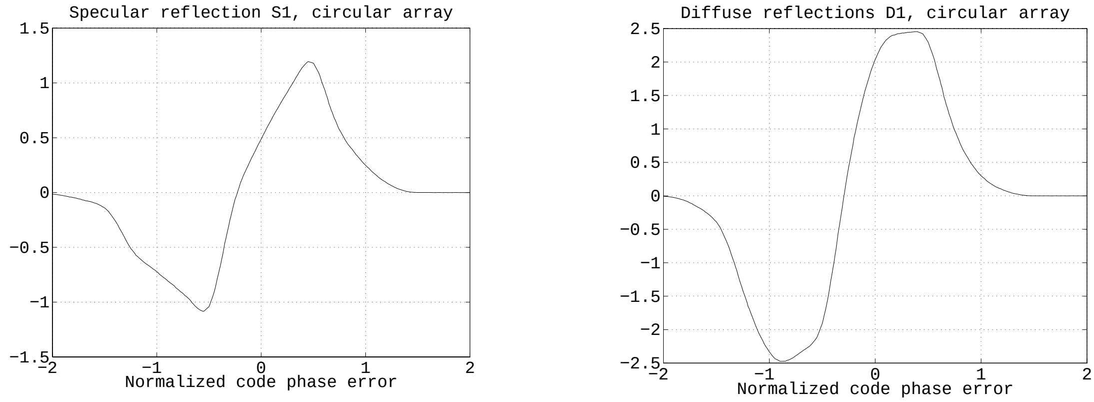

Figure 5.- S-curves with a non-adaptive beamformer, circular array

Now, we are going to use the beamformer described by (20) based on the DOA estimations carried out with the two methods explained in 4.2. The autocorrelation matrix is estimated by averaging 25 despreaded vector samples, that is to say, the needed time to calculate the autocorrelation matrix is 0.5 s. The despreading is performed with the code delay estimations obtained with the non-adaptive beamformer (Table 3).

Firstly, we apply the modified MUSIC. The specular reflection S1 and the interference I1 are received, and the linear array is used (similar results are obtained with the circular array, except for a increased computational load). Figure 6.a shows the conventional MUSIC spectrum, in which only the interference appears, and not the reflection nor the LOSS since they are highly correlated. This is solved by means of the modified MUSIC. Figure 6.b shows several realizations the 1D spectrum obtained with the modified MUSIC; the DOA of the LOSS has been supposed to be known. In this spectrum the DOA of the reflection and the interference can be determined; and in a certain realization the DOA estimations are 20.05º and 39.1º, respectively. In Figure 6.c it is presented the beam pattern associated to the weight vector obtained with (20) and the previous DOA estimations. With this beamformer the code phase jitter is $0.0106T_c$ (3.1 meters) and the mean code error is $-0.021T_c$ (-6.15 meters), it was -59m with the non-adaptive beamformer. The code jitter is also lower than the jitter obtained with the non-adaptive beamformer and it is very much lower than that obtained without beamformer (24 m). These results reveal that the reflection and the interference are greatly reduced by the beamformer. The S-curve is shown in Figure 6.d., which is nearly the same as the ideal one.

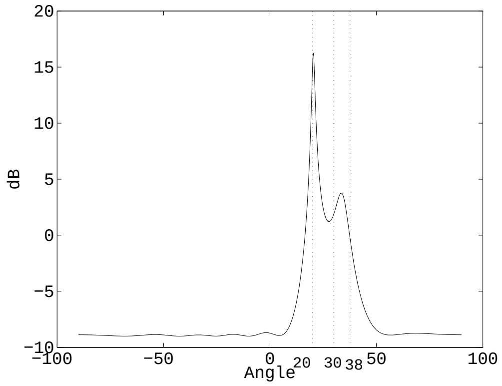

Figure 6.a.- Conventional MUSIC spectrum

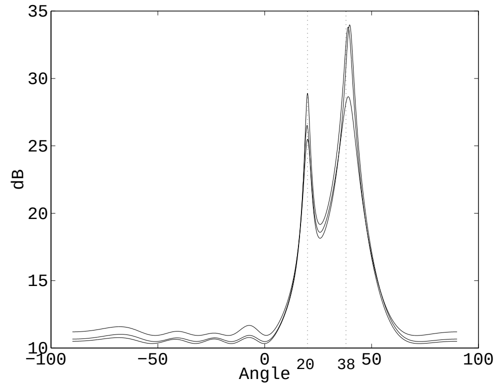

Figure 6.b.- Modified MUSIC 1D-spectrum

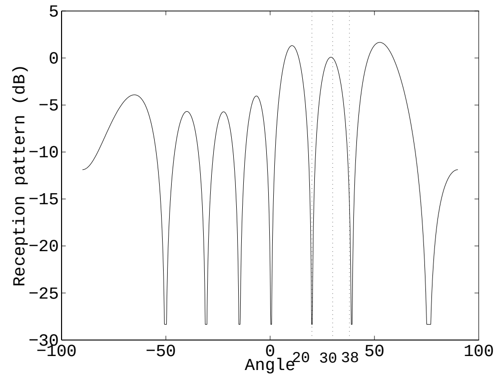

Figure 6.c.- Reception pattern

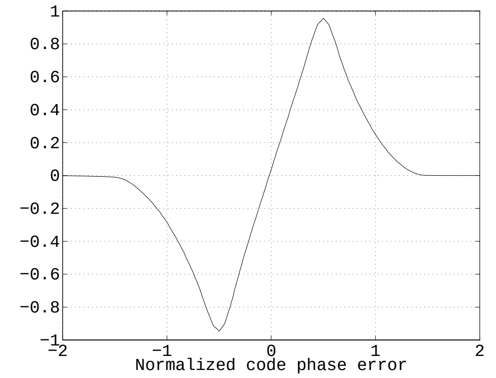

Figure 6.d.- S-curve obtained with the previous reception pattern

If the DOA of the LOSS is not known, the modified MUSIC can also be applied to estimate it, together with the DOA of the reflection and the interference. In this case, the search is performed in a 2D spectrum (Figure 7). The sharp peaks of this spectrum correspond to the coherent pair LOSS+reflection, and the sharp ridges correspond to the interference. The spectrum is symmetric, so it is only necessary to search peaks and ridges in a half of it. The performance obtained with the weight vector associated to these estimations is the same as that obtained with the previous one.

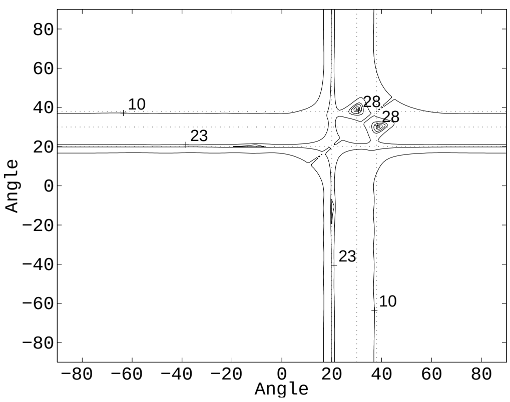

Figure 7.- Modified MUSIC 2D-spectrum

Secondly, we apply the maximum likelihood estimator (MLE) by alternating projection. The specular reflection S1 and the interference I1 are received, and the circular array is used (similar results are obtained with the linear array). We assume that the DOA of the LOSS is not known; if it is known the estimation process improves its velocity and accuracy, but really similar results in the jitter and mean error are obtained. Table 4 shows the successive estimations that are got at each iteration. The final values are (ϕ, θ)=(179.9º, 30.1º), (4.8º, 79.8º), (200.6º, 40.4º). With these values and (20) the weight vector of the beamformer is generated, which produces a code error jitter equal to $0.01T_c$ (2.93 meters) and mean code error equal to $-0.0016T_c$ (-0.46 meters) (see Figure 8). Thus, it is clear that a great improvement is achieved in relation to the non-adaptive beamformer. It can be said that the reflection and the interference are completely canceled, and the resulting jitter is only caused by the thermal noise, that is, obviously, slightly greater than the thermal noise in the non-adaptive beamformer.

Iteration	ϕ of LOSS	ϕ of S1	ϕ of I1	θ of LOSS	θ of S1	θ of I1
1	172	3	210	28	78	39.3
2	176.5	4.7	200.8	28.5	79.5	39.7
3	179.4	4.7	200.5	29.1	79.9	39.8
4	179.9	4.7	200.5	29.5	79.9	40
5	179.8	4.8	200.6	29.9	79.8	40.1
6	179.9	4.8	200.6	30	79.8	40.2
7	179.9	4.8	200.6	30.1	79.8	40.4

Table 4.- Estimated DOAs

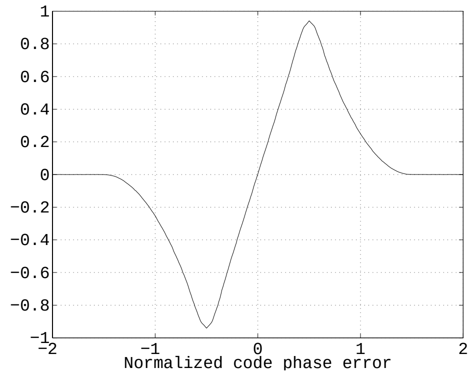

Figure 8.- S-curve obtained with the generated beamformer

The study carried out in this paper reveals that the use of adaptive antenna arrays is a effective technique to combat the errors produced by the multipath propagation and the interferences in GPS. The presented approach is based on the DOA estimation of the reflections and interferences.

6. REFERENCES

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