CHANNEL ESTIMATION FOR TRANSFORM MODULATIONS IN MOBILE COMMUNICATIONS*

Meritxell Lamarca, Gregori Vázquez
Dept. of Signal Theory and Communications, Polytechnic University of Catalonia (UPC)
c/Gran Capità s/n, 08034-Barcelona (SPAIN)
Tel: +34-3-4016436; Fax:+34-3-4016447; e-mail: xell@gps.tsc.upc.es

ABSTRACT

This paper deals with data-aided channel estimation in systems using OFDM modulation. We formulate a pilot symbol-based channel estimator and compare it with the pilot-tone one proposed in [1]. Although this paper focuses in flat fading mobile channels, the results could easily be applied to OFDM systems operating in frequency-selective channels.

1. INTRODUCTION

Mobile communications are one of the most difficult and challenging scenarios of communications engineering due to signal distortion caused by multipath propagation and vehicle displacement ([2]). Depending on the transmission rate and vehicle speed, either frequency-selectivity or Doppler spectrum spreading becomes the major concern, but the application of transform-domain modulation techniques (mainly OFDM, Orthogonal Frequency Division Multiplex ([3])) is very promising in both cases.

Early applications of OFDM modulation restrained channel impairment using differential modulation and channel coding. Although this results in simple demodulation schemes, it usually requires a high EbNo due to coding redundancy and to the 3dB loss characteristic of differential demodulation. If the OFDM system performance is to be improved spatial diversity and/or equalization techniques must be applied, and in most cases both of them require a channel estimate prior to signal demodulation (e.g. maximal ratio combiner, MLSE, etc.).

A good channel estimation is of paramount importance, because a poor estimate would preclude adequate demodulator operation. However, channel estimation must be performed on the basis of a short observation due to the time varying nature of the mobile channel. Given that blind channel identification methods require long sets of data to converge, one must resort to schemes where the transmitted signal includes some kind of reference that speeds up the estimation process. For the case of frequency-flat fading communications, most methods for channel estimation reported in the literature can be classified as pilot-tone or pilot-symbol assisted techniques (e.g. [4] and [5]). Even though in most cases pilot symbols are preferred to pilot tones due to the problems that arise when implementing pilot tone techniques, both choices can be employed in OFDM systems because in this case these difficulties disappear. Moreover, both methods can also be used when OFDM modulation is applied to frequency-selective channels.

In [1] the authors presented a channel estimation procedure for OFDM systems operating in frequency non-selective mobile channels. Their proposal was based on the introduction of a pilot tone in the transmitted signal and was described in the frequency domain. As it will be shown, when the flat fading case is analyzed in the transformed domain, the multiplicative distortion becomes a convolution and so, one can apply well-known tools that were originally developed for frequency selective channels.

In this paper we introduce a new formulation for OFDM transmissions over mobile channels. We use it to rephrase pilot-tone estimators and to describe pilot symbol methods in the transformed domain; afterwards we analyze and compare the two identification techniques. In our analysis we will use z-transform (or equivalently polynomial) notation and we will apply polynomial congruence properties that proceed from number theory. Although this paper is centred in channels subject to multiplicative distortion, the results obtained could easily be employed in frequency-selective channels where symbols are extended cyclically to avoid ISI effects ([3]). Furthermore, if little modifications were introduced, it could also be used in single-carrier modulation schemes for frequency non-selective channels.

The proposed formulation is rather general and can be useful to analyze the performance of many estimators based on pilot symbols or pilot tones. In this paper a simple Least Squares channel estimator will be derived which arises naturally when using this formulation.

2. NOTATION AND PROPERTIES

2.1. Notation

Hereafter, the symbols $\otimes$ and $*$ will stand for circular convolution (CC) and linear convolution (LC) respectively. Data sequences will be denoted by square brackets, whereas the parentheses and boldface letters will be used for polynomials. Besides, we will use lower(upper) case letters for time(frequency) domain variables. Thus, if a[n] is a sequence, A[k] will be its DFT and $\mathbf{A}(z)$ will be the polynomial in z whose k^{th} power coefficient is A[k].

The congruence operator and its result will be designated by *'mod'* and by a letter superscript respectively:

$$\mathbf{f}^{g}(z) \equiv \mathbf{f}(z) \mod \mathbf{g}(z) \Rightarrow \mathbf{f}(z) = \mathbf{q}(z)\cdot\mathbf{g}(z) + \mathbf{f}^{g}(z)$$

where $\mathbf{f}^{g}(z)$ verifies $\deg\{\mathbf{f}^{g}(z)\}<\deg\{\mathbf{g}(z)\}$.

Throughout the paper I[k], T[k], Y[k], W[k], C[k] and $\hat{C}[k]$ will represent the information, the transmitted and received signals, the AWGN term of variance σ^2, the channel response and its estimate. Their lengths will be called N, D, D, D, L and $\hat{L}$ respectively. Some additional conventions will be introduced later on.

* This work has been supported by the Spanish Research Council (PRONTIC/CICYT TIC95-1022-C05-01) and by CIRIT/ Generalitat de Catalunya GRQ93-3021

2.2. Properties

Although a deep analysis of our proposal requires the application of several polynomial congruence properties, only two of them are listed here; many other ones can be found in number theory references such as [6].

<u>*P1*</u>: **f**(z) mod **g**(z) = {**f**(z) mod **g**(z)·**h**(z)} mod **g**(z)

<u>*P2*</u>: $f[n] = \mathbf{F}(z)\big|_{z=e^{j\frac{2\pi}{N}n}} = \mathbf{F}(z) \bmod \left(z - e^{j\frac{2\pi}{N}n}\right)$

3. PROBLEM STATEMENT

In OFDM schemes, the modulation(demodulation) process can be shown to be equivalent to computing the IDFT(DFT) of the input data. Next diagram shows a discrete time model for an OFDM system subject to frequency non-selective fading[1]:

$$
\begin{array}{ccc}
T[k] & & Y[k] = T[k] \otimes C[k] + W[k] \\
\downarrow IDFT & & \uparrow DFT \\
t[n] & \xrightarrow[\text{Channel}]{\text{Transmission}} & y[n] = t[n] \cdot c[n] + w[n]
\end{array}
$$

We will assume that c[n] is a slowly varying multiplicative distortion and so, C[k] will only have a few significant coefficients corresponding to low frequency components (L<<D). Furthermore, we will also assume that **C**(z) has degree lower than L (in other words C[k]=0 for k>L), but we will release this constraint in section 4.3. Using polynomial notation, the received signal can be expressed as

$$\mathbf{Y}(z) \equiv \mathbf{T}(z) \cdot \mathbf{C}(z) + \mathbf{W}(z) \mod z^{D} - 1 \qquad (1)$$

3. TRANSMITTED SIGNAL FORMULATION

Next, pilot-symbol and pilot-tone-based channel estimator's policies are briefly summarized and a mathematical formulation is provided for their application to OFDM modulation schemes. The transmitted signal can be expressed in both techniques as

$$T[k] = U[k] + P[k] * I[k] \Leftrightarrow \mathbf{T}(z) = \mathbf{U}(z) + \mathbf{P}(z) \cdot \mathbf{I}(z) \qquad (2)$$

where U[k] and P[k] are the sequences which introduce the redundant symbols and which assist the estimator. As will be seen in section 4, the different performance of the two types of estimators is due to their different choice of U[k] and P[k].

3.1. Pilot tone-based estimators

Pilot tone channel estimators consist of the introduction of an unmodulated carrier in the transmitted signal. Then, the channel is estimated by observing the carrier spectrum spreading caused by the multiplicative distortion. Notice that the transmitted spectrum must be set to zero in a frequency band next to the pilot carrier, otherwise the channel response could not be observed. This procedure was first applied to OFDM in [5] and was first described in the frequency domain in [1].

If only one pilot tone is introduced at the beginning of the frame, the transmitted sequence can be expressed as

$$T[k] = \delta[k] + I\left[k - \hat{L} - 1\right] \Leftrightarrow \mathbf{T}(z) = 1 + z^{\hat{L}} \cdot \mathbf{I}(z) \qquad (3)$$

that is, referring to equation (2), **U**(z)=1 and **P**(z)=$z^{\hat{L}}$.

3.2. Pilot symbol-based estimators

These estimators insert pilot symbols whose value is known at the receiver in the transmitted signal (i.e. t[n] is known for some values of *n*). Given that the receiver can sample the channel response c[n] when those symbols are transmitted, if enough pilot symbols are inserted as required by Nyquist rate, the value of c[n] at any other time can be estimated by means of interpolation.

According to equation (2), if *M* pilot symbols are to be incorporated in the OFDM frame, then D=N+M and U[k] and P[k] must be selected as sequences of D and M+1 symbols respectively which verify

$$1 \equiv \mathbf{U}(z) \mod \mathbf{P}(z) \quad ; \quad 0 \equiv z^{D} - 1 \mod \mathbf{P}(z) \qquad (4)$$

We will next show that this choice of **U**(z) and **P**(z) sets to 1 certain samples of t[n] (those where pilot symbols are located) and so, equations (2) and (4) can be used for analyzing the performance of pilot symbol-based estimators.

The application of a D-point DFT to equation (2) provides the following result

$$t[n] = u[n] + p[n] \cdot i[n] \qquad (5)$$

whereas the equation (4) boils down to **P**(z) having its roots at the same position as M DFT frequency bins. Then, it is straightforward to see that u[n] and p[n] have M components equal to 1 and 0 respectively:

$$
\begin{aligned}
&\mathbf{P}(z) = \prod_{i=1}^{M}\left(z - e^{j\frac{2\pi}{D}n_i}\right) \quad n_i \in [0, D-1] \Rightarrow p[n_i] = 0 \quad i = 1 \ldots M \\
&u[n_i] \equiv \mathbf{U}(z) \mod (z - e^{j\frac{2\pi}{D}n_i}) \\
&\qquad \equiv \mathbf{U}^{p}(z) \mod (z - e^{j\frac{2\pi}{D}n_i}) \Rightarrow u[n_i] = 1 \quad i = 1 \ldots M
\end{aligned}
\qquad (6)
$$

Hence, substituting (6) into (5) it turns out that

$$t[n_i] = 1 \quad i = 1 \ldots M \qquad (7)$$

and so, the transmitted signal has M pilot symbols of unitary value.

Notice that the pilot symbol positions are determined by the selection of **P**(z). It is clear that this formulation allows for any distribution of the pilot symbols, though the most interesting case occurs when D is multiple of M and

$$\mathbf{P}(z) = z^{M} - e^{j\frac{2\pi}{D}M n_o} \qquad (8)$$

being n_o integer: in this case the pilot symbols are evenly distributed along the transmitted signal. Regarding **U**(z), the selection of a polynomial verifying (4) only concerns the relative power of the transmitted symbols that contain the information i[n]. Both **U**(z) and **P**(z) introduce and offset and a scaling factor in t[n] for $n \neq n_i$ and, so, their election may result in unequal error protection of the transmitted symbols t[n] in front of channel impairments (noise and multiplicative distortion). Although it is not shown here due to length constraints, a small change in the proposed format would allow to have constant power transmitted and other pilot symbol values different from 1 (as would be the case of pseudo-random sequences of pilot symbols).

---

[1] Throughout the paper the D or 1/D factors associated to the DFT/IDFT transform will be ommited for the sake of simplicity.

4. CHANNEL ESTIMATION ALGORITHMS

4.1. Pilot tone-based estimator

If equations (1) and (3) are combined, the received signal can be expressed as

$$\mathbf{Y}(z) \equiv \mathbf{C}(z) + z^{\hat{L}} \cdot \mathbf{I}(z) \cdot \mathbf{C}(z) + \mathbf{W}(z) \mod z^{D} - 1 \qquad (9)$$

and the pilot tone-based channel estimate (called $\hat{C}_T[k]$) is obtained as the first samples of Y[k]

$$\hat{\mathbf{C}}_T(z) \equiv \mathbf{Y}(z) \mod z^{\hat{L}+1} \qquad (10)$$

Several observations can be made from equation (10):

- For the estimator to work properly, the CC must operate as a LC, in other words: $D>N+L+\hat{L}-2$. Therefore, at least 2L-1 redundancy symbols must be introduced to estimate a channel response of length L.
- If Q pilot tones are transmitted, the estimate is corrupted by white Gaussian noise of variance σ^2/Q.
- The estimation $\hat{\mathbf{C}}_T(z)$ will only be free from distortion due to the information I[k] if the channel response length is really bounded to at most $\hat{L}$ symbols.

4.2. Pilot symbol-based estimators

In these schemes the receiver provides the estimator with noisy observations of channel response samples: using equation (7)

$$y[n_i] = c[n_i] + w[n_i] \quad i = 1 \ldots M \qquad (11)$$

and an estimate $\hat{C}[k]$ is obtained from them by means of filtering and interpolation. In this case, the following statements hold no matter what is the procedure for estimating $\hat{C}[k]$ from the pilot symbols:

- If a channel of length L is to be estimated, at least L pilot symbols are required (as opposed to 2L-1 for pilot tone techniques). Therefore, the polynomial $\mathbf{P}(z)$ must be of degree not lower than L. In fact, this bound is equivalent to the Nyquist sampling rate.
- The estimation $\hat{C}[k]$ is always independent of the information sequence I[k].

Next two estimators called $\hat{C}_C[n]$ and $\hat{C}_{LS}[n]$ are described.

4.2.1. Congruence channel estimator

Although many types of interpolators can be applied to this problem, a particular one arises naturally when using this formulation. Notice that if equation (6) holds, when the congruence of $\mathbf{Y}(z)$ modulo $\mathbf{P}(z)$ is evaluated at the receiver, the application of *P1* in equation (1) yields[2]

$$\mathbf{Y}^p(z) = \mathbf{C}^p(z) + \mathbf{W}^p(z) \qquad (12)$$

[2]Notice that property *P1* could not be applied if $\mathbf{P}(x)=z^{\hat{L}}$. This can be seen as a mathematical explanation of the fact that pilot tone methods require more redundancy than pilot symbol ones.

Furthermore, if enough pilot symbols are transmitted ($M \geq L$), then $\mathbf{C}^p(z)=\mathbf{C}(z)$ and the congruence $\mathbf{Y}^p(z)$ can be used to provide an unbiased estimate of the mobile channel that is only corrupted by Gaussian noise:

$$\hat{\mathbf{C}}_C(z) = \mathbf{Y}^p(z) = \mathbf{C}(z) + \mathbf{W}^p(z) \qquad (13)$$

Actually, the congruence operator in equation (12) is equivalent to the Lagrange interpolation formula and so, the estimator in (13) interpolates $\hat{c}[n]$ exactly through the M noisy samples of the channel response $y[n_i]$. No matter what the channel response length L is, the estimate has as many coefficients as pilot symbols have been introduced ($\hat{L}=M$). If M<L the estimate suffers the same effects as any signal that is sampled under the Nyquist rate, whereas if M>L the equation yields an estimate longer than the channel response.

4.2.2 Least Squares estimator

In the case of M>L the receiver, rather than increasing $\hat{L}$ should utilize the additional redundancy to combat noise and reduce the estimation variance. This can be achieved by Least Squares fitting a $\hat{L}$-coefficient estimate to the $M>\hat{L}$ noisy channel samples. Thus, defining the vectors and matrices

$$\underline{y}_{M\times 1} = \left\{ y[n_i] \right\}_{i=1\ldots M} \quad ; \quad \underline{\underline{F}}_{M\times \hat{L}} = \left\{ e^{j\frac{2\pi}{D}k\,n_i} \right\}_{i=1\ldots M,\, k=0\ldots \hat{L}-1} \qquad (14)$$
$$\underline{C}_{\hat{L}\times 1} = \left\{ C[k] \right\}_{k=0\ldots \hat{L}-1} \quad ; \quad \underline{w}_{M\times 1} = \left\{ w[n_i] \right\}_{i=1\ldots M}$$

equation (11) can be written as

$$\underline{y} = \underline{\underline{F}}\,\underline{C} + \underline{w} \qquad (15)$$

and so the Least Squares estimate can be obtained as follows

$$\left.\begin{array}{l} \underline{\underline{F}}^{+} = \left(\underline{\underline{F}}^{H} \underline{\underline{F}} \right)^{-1} \underline{\underline{F}}^{H} \\ \underline{\hat{C}}_{LS\,\hat{L}\times 1} = \left\{ \hat{C}[k] \right\}_{k=0\ldots \hat{L}-1} \end{array}\right\} \Rightarrow \quad \underline{\hat{C}}_{LS} = \underline{\underline{F}}^{+}\underline{y} \qquad (16)$$

As it is well known, if $L \leq \hat{L}$ equation (16) provides an unbiased estimate of C[k] with covariance matrix $\sigma^2 \underline{\underline{F}}^{+} (\underline{\underline{F}}^{+})^{H}$.

Although in the general case of arbitrary pilot symbol distribution (16) obtains a lower estimation variance than (113), it can be seen that for the mostly used case of uniformly distributed pilot symbols ($n_i = n_o + D \cdot i/M$) the first coefficients of the two estimators coincide:

$$\hat{C}_{LS}[k] = \hat{C}_C[k] \qquad k = 0 \ldots \hat{L} - 1 \qquad (17)$$

and therefore they are equivalent.

4.3. Other pilot-symbol methods

Here, two variants of the basic procedure for pilot symbol-based estimators will be outlined which achieve a better performance without increasing the number of pilot symbols M.

4.3.1. Data shifting

So far, it has been assumed that for slowly varying channels, the DFT C[k] had only a few significant terms which were located at the first coefficients 0≤k<L. However, in fact those terms will be located in those DFT coefficients associated to low frequency components, i.e., 0≤k<L/2 and D-L/2≤k<D. Therefore, these are the index *k* which must be used in the Least Squares formulation in (17). For the congruence channel estimator, equation (13) must be replaced by

$$\mathbf{V}(z) \equiv z^{M/2} \cdot \mathbf{Y}(z) \mod \mathbf{P}(z) \qquad (18)$$

$$\hat{C}_C(z) \equiv z^{D-M/2} \cdot \mathbf{V}(z) \mod z^D - 1 \qquad (19)$$

4.3.2. Even symmetry extension

The DFT operator implicitly assumes that the sequence to which it is applied is periodic. However, in the case of OFDM modulation, the DFT is applied to a sequence c[n] originated from the truncation of the time varying channel multiplicative distortion. This results in discontinuities at the borders of the periodically extended sequence and, therefore, it results in channel responses C[k] with higher frequency components than expected. In order to ameliorate this effect we propose to expand the received sequence up to length 2D by even symmetry:

$$y_e[n] = \begin{cases} y[n] & 0 \le n < D \\ y[2D-1-n] & D \le n < 2D \end{cases} \qquad (20)$$

and then perform channel estimation based on the extended sequence.

4.4. Channel Estimators Comparison

In the introduction it was said that both types of estimators (pilot tone and pilot symbol techniques) were easy to implement in OFDM systems, so the two of them had to be analysed. However, many drawbacks arise when pilot tone-based estimators are compared with pilot symbol ones. First, it has been shown that they require more redundancy. Second, in pilot tone techniques the transmitter needs to know the channel length L in order to achieve an optimum use of the redundancy symbols, while in pilot symbol-based estimators this is not required. Third, pilot-tone method's estimates are always corrupted by the information symbols, given that practical channels never fulfill the $L \le \hat{L}$ hypothesis. Finally, it also must be taken into account that the even extension of the received data proposed in section 4.3.2 only applies to pilot-symbol methods and, therefore, this tool cannot be used to reduce DFT end effects in pilot tone-based estimators.

5. SIMULATION

Figure 1 shows the modulus and phase of a multiplicative channel response with Rayleigh statistics, as well as its estimated values using the pilot symbol-based estimator. Simulation parameters were: D=480, M=60, signalling rate: 1Kbps, carrier frequency: 900 MHz, vehicle speed:30 Km/h, EsNo=16dB, **P**(z) was selected according to eq.(8) with n_o=39, and a LS estimator of length $\hat{L}$=80 was applied to the shifted and symmetrically extended signal.

6. CONCLUSIONS

In this paper we have introduced a new formulation for the introduction of pilot symbols and pilot tones in the OFDM transmitted signal and have described the pilot symbol-based estimators in the frequency domain. We have used it to show that, even though the implementation complexity is similar in both of them, pilot symbol-based methods are preferred to pilot-tone ones. We have proposed two pilot symbol-based estimators (the Lagrange interpolator and the Least Squares estimate) and we have shown that both are equivalent for the most common case of equi-spaced pilot symbols.

7. REFERENCES

[1] J.Ahn and H.S.Lee, 'Frequency domain equalisation of OFDM signals over frequency nonselective Rayleigh fading channels' , *Electronic Letters*, vol.29, No.16, 5th August 1993, pp.1476-1477

[2] R.Steele, *Mobile Radio Communications*, Pentech Press, 1992

[3] B.LeFloch,, R.Halbert-Lassalle and D.Castelain 'Digital Sound Broadcasting to Mobile Receivers', *IEEE Trans. on Consumer Electronics*, Vol.35, No.3, August 1989, pp.494-503

[4] M.Moher and J.H.Lodge, 'TCMP-A Modulation and Coding Strategy for Rician Fading Channels', *IEEE Journal on Sel. Areas in Comm.*, Vol.7, No.9, December 1989, pp.1347-1355

[5] L. Cimini, 'Analysis and Simulation of a Digital Mobile Channel Using Orthogonal Frequency Division Multiplexing', *IEEE Trans. on Communications*, vol.COM-33, No.7, July 1985, pp.665-675

[6] J.H.McClellan and C.M.Rader, *Number Theory in Digital Signal Procesing*, Prentice Hall, 1979

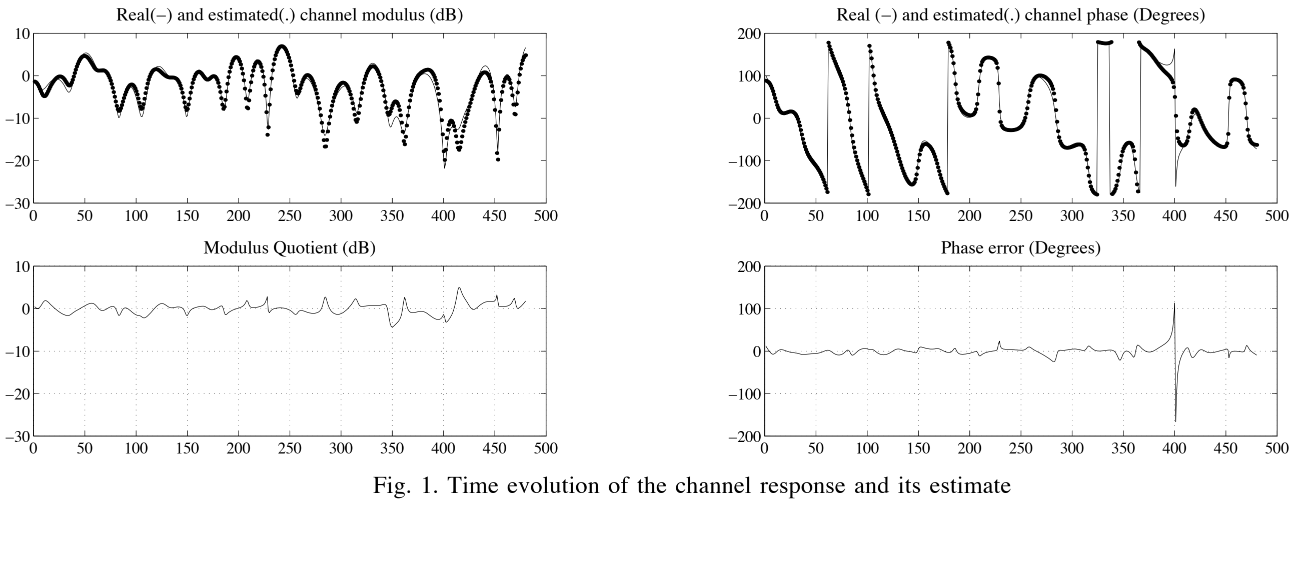

Fig. 1. Time evolution of the channel response and its estimate