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Teleparallel loop quantum cosmology in a system of intersecting branes

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ABSTRACT

Recently, some authors have removed the big bang singularity in teleparallel Loop Quantum Cosmology (LQC) and have shown that the universe may undergo a number of oscillations. We investigate the origin of this type of teleparallel theory in a system of intersecting branes in M -theory in which the angle between them changes with time. This system is constructed by two intersecting anti- $D8$ -branes, one compacted $D4$ -brane and a $D3$ -brane. These branes are built by joining $M0$ -branes which develop in decaying fundamental strings. The compacted $D4$ -brane is located between two intersecting anti- $D8$ branes and glues to one of them. Our universe is located on the $D3$ brane which wraps around the $D4$ brane from one end and sticks to one of the anti- $D8$ branes from the other one. In this system, there are three types of fields, corresponding to compacted $D4$ branes, intersecting branes and $D3$ -branes. These fields interact with each other and make the angle between branes oscillate. By decreasing this angle, the intersecting anti- $D8$ branes approach each other, the $D4$ brane rolls, the $D3$ brane wraps around the $D4$ brane, and the universe contracts. By separating the intersecting branes and increasing the angle, the $D4$ brane rolls in the opposite direction, the $D3$ brane separates from it and the expansion branch begins. Also, the interaction between branes in this system gives us the exact form of the relevant Lagrangian for teleparallel LQC.

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1. Introduction

Until now, many scientists have tried to propose a model which removes the big bang and predicts phenomenological events. One of the best theories which prevents this singularity by introducing a modification in Friedmann's equation is Loop Quantum Cosmology (LQC). The holonomy corrections in the flat Friedmann–Lemaître–Robertson–Walker (FLRW) geometry can be added to the classical Hamiltonian by replacing the Ashtekar connection $\vec{c} \equiv \gamma \vec{a}$, where γ is the Barbero–Immirzi parameter, by the function $\frac{\sin(2\vec{\mu}\vec{c})}{2\vec{\mu}}$, where $\vec{\mu} = \frac{3\sqrt{3}}{2}$ (see for instance [1]). Then, with the help of this new holonomy corrected Hamiltonian, one derives the

modified Friedman equation (an ellipse in the plane (H, ρ) , where H is the Hubble parameter and ρ the energy density of the universe) [2]. The same holonomy corrected Friedmann equation can be provided in the context of teleparallel gravity, considering a $F(\mathcal{T})$ -Lagrangian density – named as teleparallel LQC – where in the flat FLRW spacetime the scalar torsion is given by $\mathcal{T} = -6H^2$ [3]. From the viewpoint of teleparallel LQC, the universe evolves from the contracting phase to the expanding one passing through a non-singular bounce [4]. Moreover, this theory mixes the simple bounce obtained by holonomy corrections in LQC with the non-singular perturbation equations given by $F(\mathcal{T})$ gravity and derives a non-singular bounce scenario as a viable alternative to the inflationary paradigm [5,6]. In parallel, there are some models in string theory which replace the big bang singularity by the fundamental string and predict that the age of the universe is infinity [7–14]. In these models, firstly, N fundamental strings are excited and decay to N pairs of $D0$ -anti- $D0$ -branes in string theory or

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$M0$ -anti- $M0$ -branes in M -theory. Then, these branes stick to each other and construct a universe and an anti-universe in addition to one wormhole. This wormhole is a channel for flowing energy from extra dimensions into our universe and causes the evolution of the universe from inflation to late time acceleration.

Now, the question arises as to what is the relation between teleparallel LQC and cosmological models in string theory? We answer this question with a system of two intersecting anti- $D8$ -branes in which the angle between them changes with time. In this system, a compacted $D4$ -brane is located between two intersecting branes and glues to one of the anti- $D8$ -branes. The $D3$ brane sticks to one of the anti- $D8$ -branes from one end and wraps around the $D4$ brane from the other end. Also, there are three types of fields which live on the $D3$, anti- $D8$ and $D4$ branes and lead to oscillation of the angle between the intersecting branes. By decreasing the angle, two intersecting branes come close to each other, the $D4$ brane rolls, the $D3$ brane wraps around the $D4$ brane and the universe contracts. By increasing the angle, the two intersecting branes move apart from each other, the $D4$ brane rolls in the opposite direction, the $D3$ brane opens from it and the universe expands. This model gives us the exact form of the relevant action in teleparallel LQC.

The outline of the paper is as follows. In section 2, we will construct teleparallel LQC in a system of intersecting branes and obtain the relevant Lagrangian. In section 3, we will consider the origin of torsion in Einstein cosmology. The last section is devoted to a summary and conclusion.

The units used throughout the paper are: $\hbar = c = 8\pi G = 1$.

2. Teleparallel loop quantum cosmology in a system of intersecting branes

In this section, we will show that firstly, N fundamental strings decay to N pairs of $M0$ -anti- $M0$ -branes. Then these branes join to one another and form two intersecting anti- $D8$ branes, a compacted $D4$ -brane and a $D3$ -brane. We will show that the oscillation of the angle between the anti- $D8$ -branes leads to the emergence of teleparallel LQC and we re-obtain its Lagrangian in this system.

Firstly, let us introduce the relevant Lagrangian of teleparallel LQC [4–6]:

$$L = VF(\mathcal{T}) - V\rho, \tag{1}$$

where $V = a^3$ is the volume, a being the scale factor of universe. Also, when the universe is filled by a barotropic fluid, the energy density of the universe ρ has to be understood as a function of the volume V , and the Hubble parameter can be expressed as $H = \frac{1}{3V} \frac{dV}{dt}$. On the other hand, the gravitational part of the Lagrangian is given by

$$F_{\pm}(\mathcal{T}) = \pm \sqrt{-\frac{\mathcal{T}\rho_c}{2}} \arcsin\left(\sqrt{-\frac{2\mathcal{T}}{\rho_c}}\right) + \frac{\rho_c}{2} \left(1 \pm \sqrt{1 + \frac{2\mathcal{T}}{\rho_c}}\right), \tag{2}$$

where $\mathcal{T} = -6H^2$ is the scalar torsion in the flat FLRW space-time and ρ_c is the critical energy density (the maximum value of the energy density), i.e., the energy density of the universe at the bouncing time.

Then, the Hamiltonian constraint in teleparallel gravity

$$\mathcal{H} \equiv \dot{V} \partial_V L - L = (2TF'(\mathcal{T}) - F(\mathcal{T}) + \rho)V = 0 \tag{3}$$

leads to the modified Friedmann equation

$$H^2 = \frac{\rho}{3} \left(1 - \frac{\rho}{\rho_c}\right) \iff \rho = G_{\pm} = \frac{\rho_c}{2} \left(1 \pm \sqrt{1 + \frac{2\mathcal{T}}{\rho_c}}\right), \tag{4}$$

which depicts an ellipse in the plane (H, ρ) .

We will construct this Lagrangian in a system of intersecting branes and show that the universe oscillates by increasing and decreasing the angle between branes such as (F_+, G_+) which correspond to the upper branch of the ellipse and (F_-, G_-) which is the lower one. In our system, there are three fields that live on the anti- $D8$, $D4$ and $D3$ branes, named, respectively, A , Θ and E . We will show that by approaching the intersecting branes, these fields gain negative mass and transit to a tachyon T . In these conditions, the tachyonic action of the $D3$ brane can be written as [15]:

$$S_{DBI-D3} = -T_3 \int d^4\sigma V(T) \sqrt{-det D}, \tag{5}$$

where

$$D_{ab} = P \left[g_{ab} - \frac{T^2}{2\pi\alpha' Q} g_{ai} l^i l^j g_{bj} \right] + 2\pi\alpha' F_{ab} + \frac{1}{Q} \left(\pi\alpha' (D_a T (D_b T)^*) + D_b T (D_a T)^* \right) + \frac{i}{2} (g_{ai} + \partial_a X^j g_{ji}) l^i (T (D_b T)^* - T^* (D_b T)) + \frac{i}{2} (T (D_a T)^* - T^* (D_a T)) l^i (g_{bi} + \partial_b X^j g_{ji}),$$

$$Q = 1 + \frac{T^2}{2\pi\alpha'} l^i l^j g_{ij} \quad D_a T = \partial_a T - i E_a T,$$

$$V(T) = \frac{1}{\cosh(\sqrt{\pi} T)} \quad F_{ab}^k = \varepsilon^{ijk} E_a^i E_b^j. \tag{6}$$

Here, g_{ij} is the metric of the $D3$ -brane and E_b^j is the field that lives on the $D3$ -brane. This field has a direct relation with the metric of the $D3$ -brane ($\langle E_a, E_b \rangle = g_{ab}$). Our universe is located on the $D3$ -brane and thus these fields can be written in terms of the scale factor of the universe ($E^a = (-1, a, a, a)$, $E_a = (1, a, a, a)$, $g^{ab} = (-1, a^2, a^2, a^2) = F^{ab}$ where q is the scale factor of the universe and F is the field strength on the brane). Also, T_3 is the tension of the $D3$ -brane, T is the tachyon, α' is the string coupling, a, b are related to the tangent directions of the D -branes, while i, j correspond to the transverse one. On the other hand, l^i is the separation between two anti- $D8$ -branes, $P = \eta_{MN} \partial_a X^M \partial_b X^N$ and $M, N = 0, 1, 2, \dots, 9$. The world-volume coordinates of the $D3$ -brane are (x_0, x_1, x_2, x_3) , $l^i = r\Theta$ and r is the length of the anti- $D8$ -brane. For simplicity, we assume that all fields depend only on time and obtain the relevant tachyonic action of the $D3$ brane [15]:

$$S_{DBI-D3} = \frac{-2T_3}{g_s} \int d^4 V(T) \times \sqrt{1 + \frac{r^2}{4} \Theta'^2 + 2\pi\alpha' T'^2 + \frac{r^2}{2\pi\alpha'} \Theta^2 T^2 + F_{ab}^2}. \tag{7}$$

Assuming that the fields are very small and $V(T) = 1$, we can rewrite the above action as follows:

$$S_{DBI-D3} = -\frac{2T_3}{g_s} \int d^4 \left(1 + \frac{r^2}{4} \Theta'^2 + 2\pi\alpha' T'^2 + \frac{r^2}{2\pi\alpha'} \Theta^2 T^2 + F_{ab}^2 \right). \tag{8}$$

When intersecting branes move away from each other, the tachyon is replaced by the usual fields which live on the $D8$ brane (A^i). In

fact, by approaching branes and anti-branes towards each other, these fields gain negative mass and transit to tachyons. By rebounding branes, these fields return to their usual state. Under these conditions, the action (8) can be rewritten as:

$$S_{DBI-D3} = \frac{-2T_3}{g_s} \int d^4x \left(1 + \frac{r^2}{4} \Theta'^2 + 2\pi\alpha' \left(\frac{dA^i}{dt} \right)^2 + \frac{r^2}{2\pi\alpha'} \Theta^2 (A^i)^2 + F_{ab}^2 \right). \quad (9)$$

We can show that this action is built by summing over the actions of M0-branes. To do this, we will use the method [7]. In that mechanism, a fundamental string is excited and transits to a pair of M0-anti-M0-branes in addition to some extra energy (V) [7]:

$$S_{F-string} = S_{M0} + S_{anti-M0} + 2V(extra). \quad (10)$$

Here, $V_{Extra} = -6T_{M0} \int dt \Sigma_{M,N,L,E,F,G=0}^9 \epsilon_{MNLDEFG}^D X^M X^N X^L X^E \times X^F X^G$ and the action of the M0-branes is given by [7,16–25]:

$$S_{M0} = S_{anti-M0} = T_{M0} \int dt Tr(\Sigma_{M,N,L=0}^{10} \langle [X^M, X^N, X^L], [X^M, X^N, X^L] \rangle), \quad (11)$$

where T_{M0} is the brane tension, X^m are transverse scalars, $X^M = X_\alpha^M T^\alpha$, and

$$\begin{aligned} [T^\alpha, T^\beta, T^\gamma] &= f_\eta^{\alpha\beta\gamma} T^\eta \\ \langle T^\alpha, T^\beta \rangle &= h^{\alpha\beta} \\ [X^M, X^N, X^L] &= [X_\alpha^M T^\alpha, X_\beta^N T^\beta, X_\gamma^L T^\gamma] \\ \langle X^M, X^M \rangle &= X_\alpha^M X_\beta^M \langle T^\alpha, T^\beta \rangle. \end{aligned} \quad (12)$$

This equation shows that the relevant action of M0-branes has a three dimensional Nambu–Poisson bracket with the Li-3-algebra [22–25]. To obtain the relevant action for D3-branes (equation (9)), we use the following mappings [7,16–25]:

$$\begin{aligned} X^a &= E^a \quad X^i = A^i \quad X^j = r\Theta^j, \\ E^a &= (-1, a, a, a) \quad E_a = (1, a, a, a) \\ F^{ab} &= (-1, a^2, a^2, a^2) = g^{ab}, \\ E_\alpha^a E_b^\beta &= \delta_\alpha^b \delta_\alpha^\beta \quad \langle E_a, E_b \rangle = F_{ab} \\ \langle [E_a, X^b, \Theta^j], [E^b, X^a, \Theta^i] \rangle &= \delta_a^b \partial_a \Theta^i \partial_b \Theta^j, \\ \langle [E_a, E^b, E^c], [E^a, E^b, E^c] \rangle &= \delta_a^a (F^{bc})^2 \\ \langle [E_a, X^b, \Theta^j], [E^b, X^a, A^i] \rangle &= \delta_a^b 2\pi\alpha' \partial_a A^i \partial_b A^j, \\ \langle [E_a, A^i, \Theta^j], [E^b, A^j, \Theta^i] \rangle &= \frac{\delta_a^b}{2\pi\alpha'} A^i A^j \Theta^i \Theta^j \\ \langle [E_a, E_b, E_c], [E^a, E^b, E^c] \rangle &= \delta_a^a \delta_b^b \delta_c^c, \\ \Sigma_m &\rightarrow \frac{1}{(2\pi)^p} \int d^{p+1} \sigma \Sigma_{m-p-1} \\ i, j &= p+1, \dots, 10 \quad a, b = 0, 1, \dots, p \quad m, n = 0, \dots, 10, \end{aligned} \quad (13)$$

where a is the scale factor of our universe which is located on the D3-brane and g^{ab} is the metric of the universe and its related D3-brane. Also, E is the field which lives on the D3 brane and A and Θ are fields which live on the D8 and D4 branes. Summing over the action of the M0-brane in equation (11) and applying mappings in equation (13) yields:

$$\begin{aligned} S_{Sum} &= \Sigma_{a=0}^3 S_{M0} \\ &= -\Sigma_{a=0}^3 T_{M0} \int dt Tr \left(\Sigma_{m=0}^9 \langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle \right) \\ &= -\frac{2T_3}{g_s} \int d^4x \left(1 + \frac{r^2}{4} \Theta'^2 + 2\pi\alpha' \left(\frac{dA^i}{dt} \right)^2 + \frac{r^2}{2\pi\alpha'} \Theta^2 (A^i)^2 + F_{ab}^2 \right) = S_{DBI-D3}, \end{aligned} \quad (14)$$

where we have made use of the fact that $T_{M0} = \frac{2T_3}{(2\pi)^3 g_s}$. This equation shows that the D3-brane is constructed from joining N M0-branes. To obtain the explicit form of this action, we should calculate Θ and T in terms of time. For this reason, we consider the effect of other branes by deriving their actions and relations between fields on each brane. The effective tachyonic action for the D4-brane can be written as [15]:

$$\begin{aligned} S_{D4} &= -\frac{2T_4}{g_s} \int d^5 \sigma V(T) \sqrt{-det D}, \\ D_{ab} &= \left(g_{MN} - \frac{T^2 l^2}{Q} g_{M4} g_{N4} \right) \partial_a X^M \partial_b X^N + (F^{MN})^2 \\ &\quad + \frac{1}{2Q} \left((D_a T)(D_b T)^* + (D_a T)^*(D_b T) \right) \\ &\quad + i l \left(g_{a4} + \partial_a X^i g_{44} \right) \left(T(D_b T)^* - T^*(D_b T) \right) \\ &\quad + i l \left(T(D_a T)^* - T^*(D_a T) \right) \left(g_{b4} + \partial_b X^i g_{44} \right), \\ Q &= 1 + \frac{T^2}{2\pi\alpha'} l^i l^j g_{ij} \quad D_a T = \partial_a T - i E_a T, \\ V(T) &= \frac{1}{\cosh(\sqrt{\pi T})}, \end{aligned} \quad (15)$$

where $i, j = 5, \dots, 10$ and $a, b = 0, 1, 2, 3, 4$. When intersecting branes are away from each other, the tachyon (T) is replaced by A^i . Using this, and assuming that the fields are small, we can rewrite the action of the D4-brane as:

$$\begin{aligned} V(T) &= 1 \quad F^{ij} = E^i E^j, \\ E_i E^j &= \delta_i^j \quad X^M = E^M \quad l_i = r\gamma\Theta, \\ X^N &= \Theta^N \quad T = A^i \quad 2\pi\alpha' = 1, \\ g_{MN} &= F_{MN} = E_M E_N \\ T &= T^* \rightarrow T(D_a T)^* - T^*(D_a T) = 2i E_a T^2, \\ &\rightarrow T(D_b T)^* - T^*(D_b T) = 2i E_b T^2, \\ S_{D4} &= -\frac{2T_4}{g_s} \int d^5 \sigma \sqrt{U}, \\ U &= 1 + \partial_a F^{ij} \partial_b F^{ij} + \partial_a A^j \partial_b A^i - (A^j)^2 - (F^{ij})^2 \\ &\quad - 4E_i E_j \partial_a E^j \partial_b E^i + r^2 \partial_a (\Theta^2) \partial_b (\Theta^2). \end{aligned} \quad (16)$$

We can re-obtain the relevant action for D4-branes in the background of the D3 and anti-D8-branes by summing over the action of the M0-branes and apply the following mappings [7,16–25]:

$$\begin{aligned} X^a &= \Theta^a \quad X^i = E^i \quad X^j = A^j \\ \langle [X^a, \Theta^b, \Theta^c], [X^b, \Theta^b, \Theta^c] \rangle &= \partial_a \Theta^2 \partial_b \Theta^2, \\ E_\alpha^i E_j^\beta &= \delta_\alpha^j \delta_\alpha^\beta \quad \langle E_i, E_j \rangle = F_{ij} \\ \langle [X^a, X^b, F^{ij}], [X^a, X^b, F^{ij}] \rangle &= \partial_b (E_i \partial_a E^j) \partial_a (E_j \partial_b E^j), \end{aligned}$$

$$\begin{aligned}
 \langle [X^a, E^i, E^j], [X^b, E^i, E^j] \rangle &= \partial_a F^{ij} \partial_b F^{ij} \\
 \langle [E_k, E^i, E^j], [E^k, E^i, E^j] \rangle &= -\delta_k^j F^{ij} F^{ij}, \\
 \langle [E_i, X^a, A^j], [E^j, X^b, A^j] \rangle &= \delta_i^j \partial_a A^j \partial_b A^i \\
 \langle [E_i, E^i, A^j], [E_j, E^j, A^j] \rangle &= -\delta_i^j \delta_j^i A^j A^i, \\
 \langle [E_i, X^b, E^j], [E_j, X^b, E^i] \rangle &= -E_i E_j \partial_a E^j \partial_b E^i \\
 \langle [E_i, E_j, E_k], [E^i, E^j, E^k] \rangle &= \delta_i^j \delta_j^k \delta_k^i, \\
 \Sigma_m &\rightarrow \frac{1}{(2\pi)^p} \int d^{p+1} \sigma \Sigma_{m-p-1} \\
 i, j &= p+1, \dots, 10 \quad a, b = 0, 1, \dots, p \quad m, n = 0, \dots, 10. \quad (17)
 \end{aligned}$$

Using the above equations, the action of the compacted D4-brane in the background of the D3 and anti-D8 branes can be derived as [7,16–25]:

$$\begin{aligned}
 S_{compact-D4} &= \Sigma_{a=0}^4 S_{M0} \\
 &= -\Sigma_{a=0}^4 T_{M0} \int dt Tr(\Sigma_{m=0}^9 \langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle) \\
 &= -\frac{2T_4}{g_s} \int d^5 \sigma Tr(\Sigma_{a,b,c=0}^4 \Sigma_{i,j,k=5}^{10} \{1 + \partial_a F^{ij} \partial_b F^{ij} \\
 &\quad + \delta_i^j \partial_a A^j \partial_b A^i + \partial_b (E_i \partial_a E^j) \partial_a (E_j \partial_b E^i) \\
 &\quad - \delta_i^j \delta_j^i A^j A^i - \delta_k^k (F^{ij})^2 - 4E_i E_j \partial_a E^j \partial_b E^i \\
 &\quad + r^2 \partial_a \Theta^2 \partial_b \Theta^2 \}), \quad (18)
 \end{aligned}$$

where we have made use of the fact that $T_{M0} = \frac{2T_4}{(2\pi)^4 g_s}$. This action is the same action as in (16) and includes extra terms due to the compactification of the D4 brane. Now, we assume that our universe is located on the D3-brane and interacts with the D4-brane via F^{ij} and this field plays the role of the graviton for it. Thus, this field has a direct relation with the metric on D3 ($g^{ij} = \eta^{ij} + F^{ij}$) and E^i can be written as a function of the scale factor of the universe ($E^i = (-1, a, a, a)$). On the other hand, the coordinates of the compacted D4 brane are $(X^0, X^1, X^2, X^3, \theta)$. Assuming that fields are small and only a function of θ , we can write the action of the compacted D4 brane as follows:

$$\begin{aligned}
 S_{compact-D4} &= -T_{D4} \int d^5 \sigma Tr \left(\Sigma_{i,j=5}^{10} \left(1 + \left(\frac{\partial F^{ij}}{\partial \theta} \right)^2 + \left(\frac{\partial A^i}{\partial \theta} \right)^2 \right. \right. \\
 &\quad \left. \left. - (A^j)^2 - (F^{ij})^2 + \left(\frac{\partial}{\partial \theta} (E_i \frac{\partial E^j}{\partial \theta}) \right)^2 - 4E_i^2 \left(\frac{\partial E^i}{\partial \theta} \right)^2 \right. \right. \\
 &\quad \left. \left. + r^2 \left(\frac{\partial \Theta^2}{\partial \theta} \right)^2 \right) \right). \quad (19)
 \end{aligned}$$

The equations of motion obtained from the above equation are:

$$\begin{aligned}
 \frac{\partial^2 F^{ij}}{\partial \theta^2} + F^{ij} &= 0 \Rightarrow F^{ij} = M \sin(\theta) + N \cos(\theta), \\
 \frac{\partial^2 A^i}{\partial \theta^2} + A^i &= 0 \Rightarrow A^i = M' \sin(\theta) + N' \cos(\theta) \\
 \frac{\partial^2}{\partial \theta^2} (E_i \frac{\partial E^j}{\partial \theta}) + 4E_i \frac{\partial E^j}{\partial \theta} &= 0 \\
 \Rightarrow E_i \frac{\partial E^j}{\partial \theta} &= M'' \sin(2\theta) + N'' \cos(2\theta),
 \end{aligned}$$

$$r = constant \quad \frac{\partial^2 \Theta^2}{\partial \theta^2} = 0 \Rightarrow \frac{\partial \Theta^2}{\partial \theta} = c \Rightarrow \Theta^2 = c\theta + c'. \quad (20)$$

This equation shows that all fields depend on θ and oscillate with decreasing or increasing the angle between intersecting branes. Now, we obtain the angle as a function of time by calculating the action of the rotating anti-M8 brane and deriving the equation of motion. To this end, we can use the following action:

$$\begin{aligned}
 S_{anti-D8} &= -\frac{2T_8}{g_s} \int d^9 \sigma \sqrt{-\det(g_{MN} \partial_a X^M \partial_b X^N) - r^2 \omega^2 \Theta^2} \\
 X^0 = t \quad X^M = r \Theta^M \quad X^N = A^N \quad g_{MN} = E_M E_N = F_{MN} \\
 \Rightarrow S_{anti-D8} &= -\frac{2T_8}{g_s} \int d^9 \sigma \\
 &\quad \times \sqrt{1 + r^2 (\partial_t \Theta)^2 - \omega^2 r^2 \Theta^2 + (\partial_t F^{ij})^2 + (\partial_t A^i)^2}. \quad (21)
 \end{aligned}$$

We can re-obtain the action of the anti-M8-brane in the background of other branes by using the following mappings: [7, 16–25]:

$$\begin{aligned}
 \langle [X^0, E^i, E^j], [X^0, E^i, E^j] \rangle &= (\partial_t F^{ij})^2 \\
 \langle [A_k, E^i, E^j], [A_k, E^i, E^j] \rangle &= F^{ij} F^{ij} A_k A_k, \\
 \langle [E_i, X^a, A^j], [E_j, X^b, A^j] \rangle &= \delta_i^j \partial_a A^j \partial_b A^i \\
 \langle [E^i, A^i, A^i], [E^j, A^j, A^j] \rangle &= F^{ij} (A^i)^4, \\
 \langle [E_i, E_j, E_k], [E^i, E^j, E^k] \rangle &= \delta_i^j \delta_j^k \delta_k^i \\
 \langle [E_i, X^a, \Theta^j], [E^j, X^b, \Theta^j] \rangle &= \delta_i^j \partial_a \Theta^j \partial_b \Theta^i, \\
 \langle [E_i, E^i, \Theta^j], [E_j, E^j, \Theta^j] \rangle &= -\omega^2 \delta_i^j \delta_j^i \Theta^j \Theta^i \\
 \Sigma_m &\rightarrow \frac{1}{(2\pi)^p} \int d^{p+1} \sigma \Sigma_{m-p-1} \\
 i, j &= p+1, \dots, 10 \quad a, b = 0, 1, \dots, p. \quad (22)
 \end{aligned}$$

Using the above mappings, we can obtain the relevant action for the anti-D8-brane in the background of the D3 and D4 branes:

$$\begin{aligned}
 S_{anti-D8} &= \Sigma_{a=0}^8 S_{M0} \\
 &= -\Sigma_{a=0}^8 T_{M0} \int dt Tr \left(\Sigma_{m=0}^9 \langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle \right) \\
 &= -\frac{2T_{D8}}{g_s} \int d^9 \sigma Tr \left(\Sigma_{a,b,c=0}^8 \Sigma_{i,j,k=9}^{10} \left\{ 1 + r^2 (\partial_t \Theta)^2 \right. \right. \\
 &\quad \left. \left. - \omega^2 r^2 \Theta^2 + (\partial_t F^{ij})^2 + (\partial_t A^i)^2 \right. \right. \\
 &\quad \left. \left. + F^{ij} (A^i)^4 + (F^{ij})^2 (A^i)^2 \right\} \right), \quad (23)
 \end{aligned}$$

which is the same as that of action (21) in addition to some extra terms which emerge due to the interaction between branes. This action gives the following equations of motion:

$$\begin{aligned}
 r = constant \rightarrow \frac{\partial^2 \Theta}{\partial t^2} + \omega^2 \Theta &= 0 \Rightarrow \Theta = D \cos(\omega t) + D' \sin(\omega t) \\
 \text{and } \Theta^2 = c\theta + c' &\Rightarrow \\
 D^2 + (D')^2 + DD' \sin(2\omega t) &= c\theta + c' \Rightarrow \\
 D = D' = \sqrt{\pi} \quad c' = 2\pi \quad c = \frac{1}{2} &\rightarrow \theta = 2\pi \sin(2\omega t) \quad (24) \\
 V(A) = -(F^{ij} (A^i)^4 + (F^{ij})^2 (A^i)^2) &\Rightarrow \\
 m^2 = -\frac{\partial^2 V}{\partial A^2} = 12F^{ij} (A^i)^2 + 2(F^{ij})^2 &
 \end{aligned}$$

$$F^{ij} = N \cos(\theta) + M \sin(\theta) \quad A^i = N' \cos(\theta) + M' \sin(\theta)$$

$$\frac{1}{c} = 2, M = 1, N = 0, M' = N' = 1 \Rightarrow$$

$$F^{ij} = \sin(2\pi \sin(2\omega t))$$

$$A^i = \sin(2\pi \sin(2\omega t)) + \cos(2\pi \sin(2\omega t)) \Rightarrow$$

$$m^2 = 12 \sin(2\pi \sin(2\omega t)) (\sin(2\pi \sin(2\omega t))$$

$$+ \sin(2\pi \sin(2\omega t)))^2 + 2 \cos^2(2\pi \sin(2\omega t))$$

$$m^2 = 12 \sin(2\pi \sin(2\omega t)) (1 + \sin(4\pi \sin(2\omega t)))$$

$$+ 2 \sin^2(2\pi \sin(2\omega t)) \Rightarrow$$

$$\theta \leq 0 \Rightarrow \frac{T}{4} \leq t \leq \frac{T}{2} \Rightarrow m^2 \leq 0 \Rightarrow A \rightarrow T$$

$$T \rightarrow A \Rightarrow \theta = \theta + \pi \Rightarrow m^2 \geq 0$$

$$E^j = a(\theta); \quad E_j = a^{-1}(\theta);$$

$$\theta = 2\pi \sin(2\omega t) \rightarrow d\theta = 4\pi \omega \cos(2\omega t) dt \Rightarrow$$

$$E_i \frac{\partial E^j}{\partial \theta} = E_i \frac{\partial E^j}{4\pi \omega \cos(2\omega t) \partial t} = \frac{\frac{da}{dt}}{4\pi \omega \cos(2\omega t) a}$$

$$= \frac{H}{4\pi \omega \cos(2\omega t)} = M'' \sin(2\theta) + N'' \cos(2\theta) \Rightarrow$$

$$H = (4\pi \omega \cos(2\omega t)) (M'' \sin(2\theta) + N'' \cos(2\theta))$$

$$\dot{H} = -(8\pi \omega^2 \sin(2\omega t)) (M'' \sin(2\theta) + N'' \cos(2\theta))$$

$$+ \dot{\theta} (4\pi \omega \cos(2\omega t)) (M'' \cos(2\theta) + N'' \sin(2\theta)) \quad (26)$$

Equation (24) shows that the angle between intersecting branes oscillates with time and causes the increase and decrease in the values of the fields which live on the $D3$, $D4$ and anti- $D8$ branes. As can be seen from equation (25), with the passage of time and approaching branes towards each other ($\theta \rightarrow 0$) or moving away from each other ($\theta \rightarrow \pi$), fields on the intersecting branes (A^i) gain negative mass ($m^2 \rightarrow -m^2$) and become tachyons. To remove these states, intersecting branes rebound, ($\theta \rightarrow \theta + \pi$) and ($T \rightarrow A^i$). It is clear from equation (26) that the Hubble parameter (H) depends on time and is positive for $t \leq \frac{T}{4}$ and negative between $\frac{T}{4}$ and $\frac{T}{2}$ which is a signature of the contracting branch of the universe. Substituting the equations (20), (24), (25), (26) in action (9), we get:

$$S_{DBI-D3} = \frac{-2T_3}{g_s} \int d^4x \left(1 + \frac{r^2}{4} \Theta'^2 + 2\pi \alpha' \left(\frac{dA^i}{dt} \right)^2 + \frac{r^2}{2\pi \alpha'} \Theta^2 (A^i)^2 + F_{ab}^2 \right) \\ = \frac{-2T_3}{g_s} \int d^4x \left(1 + \frac{r^2}{8} \frac{\theta'^2}{\theta} + 2\pi \alpha' \dot{\theta}^2 (-N' \sin(\theta) + M' \cos(\theta))^2 + \frac{r^2}{2\pi \alpha'} \theta (N' \cos(\theta) + M' \sin(\theta))^2 + (N \cos(\theta) + M \sin(\theta))^2 \right)$$

$$M = 1, N = 0, M' = N' = 1 \Rightarrow S_{DBI-D3}$$

$$= \frac{-2T_3}{g_s} \int d^4x \left(1 + \frac{r^2}{8} \frac{\theta'^2}{\theta} + 2\pi \alpha' \dot{\theta}^2 (-\sin(\theta) + \cos(\theta))^2 + \frac{r^2}{2\pi \alpha'} \theta (\cos(\theta) + \sin(\theta))^2 + \sin^2(\theta) \right) \Rightarrow$$

$$S_{DBI-D3} = \frac{-2T_3}{g_s} \int d^4x \left(1 + \frac{r^2}{8} \frac{\theta'^2}{\theta} + 2\pi \alpha' \dot{\theta}^2 (1 - \sin(2\theta)) + \frac{r^2}{2\pi \alpha'} \theta (1 + \sin(2\theta)) + \sin^2(\theta) \right)$$

$$N'' = 0, M'' = 1 \Rightarrow \sin(2\theta) = \frac{H}{(4\pi \omega \cos(2\omega t))} = \frac{H}{\lambda^2(t)} \\ = 4\pi \omega \sin(2\omega t)$$

$$2 \sin^2(\theta) = 1 - \cos(2\theta) = 1 - \sqrt{1 - \sin^2(2\theta)} = 1 - \sqrt{1 - \frac{H^2}{\lambda^2(t)}}$$

$$\Rightarrow S_{DBI-D3} = \frac{-2T_3}{g_s} \int d^4x \left(1 + \frac{r^2}{8} \frac{\theta'^2}{\theta} + 2\pi \alpha' \dot{\theta}^2 \left(1 - \frac{H}{\lambda(t)} \right) + \frac{r^2}{2\pi \alpha'} \theta \left(1 + \frac{H}{\lambda(t)} \right) + \frac{1}{2} \left(1 - \sqrt{1 - \frac{H^2}{\lambda^2(t)}} \right) \right)$$

$$\Rightarrow S_{DBI-D3} = S_{torsion} + S_{remain}$$

$$S_{torsion} = \frac{-2T_3}{g_s} \int d^4x \left(\frac{r^2}{2\pi \alpha'} \frac{H}{\lambda(t)} \theta + \frac{1}{2} \left(1 - \sqrt{1 - \frac{H^2}{\lambda^2(t)}} \right) \right)$$

$$= \frac{-2T_3}{g_s} \int d^4x \left(\frac{r^2}{2\pi \alpha'} \frac{H}{\lambda(t)} \arcsin \left(\frac{H}{2\lambda(t)} \right) \right)$$

$$+ \frac{1}{2} \left(1 - \sqrt{1 - \frac{H^2}{\lambda^2(t)}} \right)$$

$$= \frac{-2T_3}{g_s} \int d^4x \left(\frac{r^2}{2\pi \alpha'} \frac{\sqrt{-\mathcal{T}}}{\sqrt{6}\lambda(t)} \arcsin \left(\frac{\sqrt{-\mathcal{T}}}{2\sqrt{6}\lambda(t)} \right) \right)$$

$$+ \frac{1}{2} \left(1 - \sqrt{1 + \frac{\mathcal{T}}{6\lambda^2(t)}} \right)$$

$$M = 1, N = 0 \Rightarrow L_{tele-LQC} = -\frac{r^2}{2\pi \alpha'} \frac{\sqrt{-\mathcal{T}}}{\sqrt{6}\lambda(t)} \arcsin \left(\frac{\sqrt{-\mathcal{T}}}{2\sqrt{6}\lambda(t)} \right)$$

$$+ \frac{1}{2} \left(1 - \sqrt{1 + \frac{\mathcal{T}}{6\lambda^2(t)}} \right)$$

$$M = 0, N = 1 \Rightarrow L_{tele-LQC} = \frac{r^2}{2\pi \alpha'} \frac{\sqrt{-\mathcal{T}}}{\sqrt{6}\lambda(t)} \arcsin \left(\frac{\sqrt{-\mathcal{T}}}{2\sqrt{6}\lambda(t)} \right)$$

$$+ \frac{1}{2} \left(1 + \sqrt{1 + \frac{\mathcal{T}}{6\lambda^2(t)}} \right) \quad (27)$$

When $\frac{r^2}{2\pi \alpha'} = 1$, the Lagrangian in Eq. (27) is practically the same as defined in Eqs. (1) and (4) for teleparallel Loop Quantum Cosmology (LQC). The difference is that, in that case, the critical density given by $12\lambda^2(t)$ is time dependent. In fact, we find a new concept for the Ashtekar connection and fields that have been introduced in that model. These results show that the reason for torsion occurring is the wrapping and opening of the $D3$ brane around the $D4$ brane. Also, the interaction between fields on the brane causes the change of angle between the intersecting branes and oscillating of this angle leads to LQC.

3. The origin of torsion in Einstein cosmology

So far, we have proposed a new model which allows us to consider the origin of teleparallel LQC in a system of oscillating branes. Now, we show that by ignoring the effect of other branes, our

1 model produces usual teleparallel theory in Einstein cosmology.
 2 This type of cosmology is obtained from the following Lagrangian
 3 in teleparallel theory [4]:

$$4 \quad L = \frac{1}{2} \mathcal{T}V - \rho V, \quad (28)$$

7 where

$$9 \quad \mathcal{T} = S_\gamma^{\mu\nu} \mathcal{T}^\gamma_{\mu\nu},$$

$$10 \quad \mathcal{T}^\gamma_{\mu\nu} = \Gamma^\gamma_{\mu\nu} - \Gamma^\gamma_{\nu\mu},$$

$$11 \quad K^{\mu\nu}_\gamma = -\frac{1}{2} (\mathcal{T}^{\mu\nu}_\gamma - \mathcal{T}^{\nu\mu}_\gamma - \mathcal{T}_\gamma^{\mu\nu}),$$

$$12 \quad S_\gamma^{\mu\nu} = \frac{1}{2} (K^{\mu\nu}_\gamma - \delta_\gamma^\mu \mathcal{T}^{\theta\nu}_\theta - \delta_\gamma^\nu \mathcal{T}^{\theta\mu}_\theta),$$

$$13 \quad \Gamma^\gamma_{\mu\nu} = E^\alpha_\nu \partial_\nu E^\alpha_\mu. \quad (29)$$

18 One can show that the above Lagrangian can be obtained from the
 19 following action in string theory:

$$21 \quad S_{D3} = -\frac{2T_3}{g_s} \int d^4x \sqrt{-\det D},$$

$$22 \quad D_{ab} = g_{MN} \partial_a X^M \partial_b X^N + \frac{1}{2} F_{\mu\nu} \left(\frac{3}{2} F^{\mu\nu} - F^{\theta\nu} - F^{\theta\nu} \right),$$

$$23 \quad F_{\mu\nu} = \partial_\mu E_\nu - \partial_\nu E_\mu. \quad (30)$$

28 Now, we obtain the relation between torsion and field strength
 29 and substitute it in the above action to derive the Lagrangian in
 30 equation (28):

$$31 \quad E^\gamma F_{\mu\nu} = E^\gamma \partial_\mu E_\nu - E^\gamma \partial_\nu E_\mu$$

$$32 \quad = \Gamma^\gamma_{\mu\nu} - \Gamma^\gamma_{\nu\mu} = \mathcal{T}^\gamma_{\mu\nu}$$

$$33 \quad E^\gamma E_\gamma = 1 \Rightarrow F_{\mu\nu} F^{\mu\nu} = F_{\mu\nu} E^\gamma E_\gamma F^{\mu\nu} = \mathcal{T}_{\mu\nu}^\gamma \mathcal{T}_\gamma^{\mu\nu}$$

$$34 \quad F_{\mu\nu} F^{\theta\nu} = F_{\mu\nu} E^\gamma E_\gamma E^\theta E_\theta F^{\theta\nu} = \mathcal{T}_{\mu\nu}^\gamma \delta_\gamma^\theta \mathcal{T}_\theta^{\theta\nu}$$

$$35 \quad F_{\mu\nu} \left(\frac{3}{2} F^{\mu\nu} - F^{\theta\nu} - F^{\theta\nu} \right)$$

$$36 \quad = F_{\mu\nu} E^\gamma E_\gamma \left(\frac{3}{2} F^{\mu\nu} - F^{\theta\nu} - F^{\theta\nu} \right)$$

$$37 \quad = F_{\mu\nu} E^\gamma \left(\frac{1}{2} [E_\gamma F^{\mu\nu} - E_\gamma F^{\nu\mu} + E_\gamma F^{\mu\nu}] - E_\gamma E^\theta E_\theta F^{\theta\nu} \right.$$

$$38 \quad \left. - E_\gamma E^\theta E_\theta F^{\theta\nu} \right)$$

$$39 \quad = \mathcal{T}_{\mu\nu}^\gamma \left(\frac{1}{2} [\mathcal{T}_\gamma^{\mu\nu} - \mathcal{T}_\gamma^{\nu\mu} - \mathcal{T}^{\mu\nu}_\gamma] - \delta_\gamma^\theta \mathcal{T}_\theta^{\theta\mu} - \delta_\gamma^\theta \mathcal{T}_\theta^{\theta\nu} \right)$$

$$40 \quad = \mathcal{T}_{\mu\nu}^\gamma (K^{\mu\nu}_\gamma - \delta_\gamma^\mu \mathcal{T}^{\theta\nu}_\theta - \delta_\gamma^\nu \mathcal{T}^{\theta\mu}_\theta) = 2S_\gamma^{\mu\nu} \mathcal{T}^\gamma_{\mu\nu} = 2\mathcal{T}$$

$$41 \quad X^0 = t \Rightarrow g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu = 1 + g_{LK} \partial_a X^L \partial_b X^K, \quad L, K = 1, 2, 3$$

$$42 \quad \Rightarrow D_{ab} = 1 - g_{LK} \partial_a X^L \partial_b X^K + \frac{1}{2} F_{\mu\nu} \left(\frac{3}{2} F^{\mu\nu} - F^{\theta\nu} - F^{\theta\nu} \right)$$

$$43 \quad \Rightarrow S_{D3} = -\frac{2T_3}{g_s} \int d^4x \left[1 - \frac{1}{2} g_{LK} \partial_a X^L \partial_b X^K \right.$$

$$44 \quad \left. + \frac{1}{4} F_{\mu\nu} \left(\frac{3}{2} F^{\mu\nu} - F^{\theta\nu} - F^{\theta\nu} \right) \right]$$

$$45 \quad \Rightarrow S_{D3} = -\frac{2T_3}{g_s} \int d^4x \left[1 - \frac{1}{2} g_{LK} \partial_a X^L \partial_b X^K + \frac{1}{2} \mathcal{T} \right]$$

$$66 \quad \rho = \frac{1}{2} g_{LK} \partial_a X^L \partial_b X^K \Rightarrow S_{D3} = -\frac{2T_3}{g_s} \int d^4x \left[1 - \rho + \frac{1}{2} \mathcal{T} \right]$$

$$67 \quad \Rightarrow S_{D3} = -\frac{2T_3}{g_s} \int dt L \Rightarrow L = -\rho V + \frac{1}{2} \mathcal{T}V \quad (31)$$

71 As can be seen from this equation, the Lagrangian of torsion can
 72 be obtained from the relevant action of D3-branes in string theory.
 73 Now, the question arises as to what is the origin of this torsion in
 74 Einstein cosmology? In the previous section, we have shown that
 75 the action of the D3 brane can be constructed by summing over
 76 the actions of M0-branes. Ignoring fields of other branes, we can
 77 re-obtain the Lagrangian in (28). We use mappings in (13) and the
 78 method in [7,21–25] and write:

$$79 \quad X^\gamma = E^\gamma \quad X_\mu = E_\mu \quad [X^\gamma, X_\mu, X_\nu] = E^\gamma \partial_\mu E_\nu = \Gamma^\gamma_{\mu\nu}$$

$$80 \quad [X^\gamma, X_\nu, X_\mu] = -E^\gamma \partial_\nu E_\mu = -\Gamma^\gamma_{\nu\mu}$$

$$81 \quad [X^\gamma, X_\nu, X_\mu] = -[X^\gamma, X_\mu, X_\nu]$$

$$82 \quad \Rightarrow \Sigma_{m,n=\mu,\nu} [X^\gamma, X_m, X_n]$$

$$83 \quad = [X^\gamma, X_\mu, X_\nu] + [X^\gamma, X_\nu, X_\mu]$$

$$84 \quad = E^\gamma \partial_\mu E_\nu - E^\gamma \partial_\nu E_\mu = \Gamma^\gamma_{\mu\nu} - \Gamma^\gamma_{\nu\mu} = \mathcal{T}^\gamma_{\mu\nu}$$

$$85 \quad \Rightarrow \Sigma_{m,n=\mu,\theta} [X^\theta, X_m, X_n]$$

$$86 \quad = [X^\theta, X_\mu, X_\theta] + [X^\theta, X_\theta, X_\mu]$$

$$87 \quad = E^\theta \partial_\mu E_\theta - E^\theta \partial_\theta E_\mu = \Gamma^\theta_{\mu\theta} - \Gamma^\theta_{\theta\mu} = \mathcal{T}^\theta_{\mu\theta}$$

$$88 \quad \Rightarrow \Sigma_{m,n,l=\mu,\nu,\gamma} [X^m, X^n, X_l]$$

$$89 \quad = \frac{1}{2} \left[\Sigma_{m,n=\mu,\nu} ([X^m, X^n, X_\gamma] + [X^m, X^n, X_\gamma]) \right]$$

$$90 \quad = \frac{1}{2} \left[\Sigma_{m,n=\mu,\nu} ([X^m, X^n, X_\gamma] - [X_\gamma, X^m, X^n]) \right]$$

$$91 \quad = \frac{1}{2} \left[\Sigma_{m,n=\mu,\nu} ([X^m, X^n, X_\gamma] - [X^n, X^m, X_\gamma]) \right.$$

$$92 \quad \left. - \Sigma_{m,n=\mu,\nu} [X_\gamma, X^m, X^n] \right]$$

$$93 \quad = \frac{1}{2} [\mathcal{T}^{\mu\nu}_\gamma - \mathcal{T}^{\nu\mu}_\gamma - \mathcal{T}_\gamma^{\mu\nu}] = K^{\mu\nu}_\gamma$$

$$94 \quad \Rightarrow \Sigma_{m,n,l=\mu,\nu,\gamma,\theta} [X^m, X^n, X_l]$$

$$95 \quad = \Sigma_{m,n,l=\mu,\nu,\gamma} [X^m, X^n, X_l]$$

$$96 \quad + \Sigma_{m,l=\mu,\theta} [X^m, X^\theta, X_l] + \Sigma_{n,l=\nu,\theta} [X^m, X^\theta, X_l]$$

$$97 \quad = K^{\mu\nu}_\gamma - \Sigma_{m,l=\mu,\theta} [X^\theta, X^m, X_l] - \Sigma_{n,l=\nu,\theta} [X^\theta, X^n, X_l]$$

$$98 \quad = K^{\mu\nu}_\gamma - \delta_\gamma^\nu \mathcal{T}^{\theta\mu}_\theta - \delta_\gamma^\mu \mathcal{T}^{\theta\nu}_\theta = S^{\mu\nu}_\gamma$$

$$99 \quad \Rightarrow S_{torsion}$$

$$100 \quad = \Sigma_{a=0}^3 S_{M0} = -\Sigma_{a=0}^3 T_{M0} \int dt Tr$$

$$101 \quad \times \left(\Sigma_{l=\gamma,\theta} (\Sigma_{m,n=\mu,\nu,\theta} [X^m, X^n, X_l], \Sigma_{m,n=\mu,\nu} [X_m, X_n, X^l]) \right)$$

$$102 \quad = -\frac{2T_3}{g_s} \int d^4x \frac{1}{2} (S^{\mu\nu}_\gamma \mathcal{T}^\gamma_{\mu\nu}) = -\frac{2T_3}{g_s} \int d^4x \frac{\mathcal{T}}{2}$$

$$103 \quad = \frac{-2T_3}{g_s} \int dt \frac{1}{2} \mathcal{T}V = \frac{-2T_3}{g_s} \int dt L_{torsion} \Rightarrow$$

$$104 \quad L_{torsion} = \frac{1}{2} \mathcal{T}V \quad (32)$$

This equation shows that by ignoring the interaction of the $D3$ brane with intersecting anti- $D8$ and $D4$ branes, the Lagrangian of torsion in Einstein cosmology can be produced. In fact, this equation implies that this torsion can be generated as due to the interaction of scalars in the transverse direction with our own brane. However, loop quantum cosmology may appear in a system of oscillating branes in which the angle between intersecting branes changes with time.

4. Summary and discussion

In this research, we have considered the origin of teleparallel LQC in a system of intersecting branes in which the angle between branes change with time. This system has been constructed by two intersecting $D8$ -branes, one compacted $D4$ -brane and another $D3$ -brane. The $D4$ brane is compacted on a circle and is located between anti- $D8$ -branes. The $D3$ brane wraps around the $D4$ one from one end and attaches to one of the anti- $D8$ -branes from another end. On each brane, one type of field lives, which interacts with other fields on another brane and causes the oscillation of the angle between the intersecting branes. Decreasing this angle and approaching the anti- $D8$ branes, the $D3$ brane wraps around the $D4$ one and the contraction branch begins. Increasing the angle between branes and moving away from each other, the $D3$ brane separates from the $D4$ brane and the expansion branch starts. This wrapping of the $D3$ brane around the compacted $D4$ one and the opening of the $D3$ from the $D4$ brane leads to the emergence of teleparallel LQC. Ignoring the interaction of the $D3$ brane with the intersecting anti- $D8$ -branes and $D4$ branes, and considering the interaction of scalars in the transverse direction with the $D3$ brane, the Lagrangian of teleparallel LQC is reduced to the Lagrangian of torsion in Einstein cosmology.

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References

- [1] Jaume Haro, Emili Elizalde, *Europhys. Lett.* 89 (2010) 69001.
- [2] A. Ashtekar, P. Singh, *Class. Quantum Gravity* 28 (2011) 213001, arXiv:1108.0893 [gr-qc].
- [3] J. Amorós, J. de Haro, S.D. Odintsov, *Phys. Rev. D* 87 (2013) 104037.
- [4] Jaume de Haro, Jaume Amorós, *Phys. Rev. Lett.* 110 (2013) 071104.
- [5] Jaume de Haro, Jaume Amorós, *J. Cosmol. Astropart. Phys.* 08 (2014) 025.
- [6] Jaume Haro, *J. Cosmol. Astropart. Phys.* 11 (2013) 068.
- [7] A. Sepehri, *Phys. Lett. B* 748 (2015) 328335, arXiv:1508.01407 [gr-qc].
- [8] A. Sepehri, F. Rahaman, M.R. Setare, A. Pradhan, S. Capozziello, I.H. Sardar, *Phys. Lett. B* 747 (2015) 1, arXiv:1505.05105 [gr-qc].
- [9] A. Sepehri, F. Rahaman, A. Pradhan, I.H. Sardar, *Phys. Lett. B* 741 (2015) 92, arXiv:1501.00428 [gr-qc].
- [10] M.R. Setare, A. Sepehri, arXiv:1410.2552 [physics.gen-ph].
- [11] Gianluca Grignani, Troels Harkmark, Andrea Marini, Niels A. Obers, Marta Orselli, *Nucl. Phys. B* 851 (2011) 462.
- [12] M.R. Setare, A. Sepehri, *J. High Energy Phys.* 1503 (2015) 079.
- [13] M.R. Setare, A. Sepehri, *Phys. Rev. D* 91 (2015) 063523.
- [14] A. Sepehri, accepted in *Mod. Phys. Lett. A* (2016).
- [15] Shigenori Seki, *J. High Energy Phys.* 1007 (2010) 091; A. Sen, *Phys. Rev. D* 68 (2003) 066008, arXiv:hep-th/0303057; Avinash Dhar, Partha Nag, *J. High Energy Phys.* 0801 (2008) 055; M.R. Garousi, *J. High Energy Phys.* 0501 (2005) 029; Avinash Dhar, Partha Nag, *Phys. Rev. D* 78 (2008) 066021.
- [16] R.C. Myers, *J. High Energy Phys.* 9912 (1999) 022, arXiv:hep-th/9910053.
- [17] Neil R. Constable, Robert C. Myers, Oyvind Tafjord, *J. High Energy Phys.* 0106 (2001) 023.
- [18] Chong-Sun Chu, Douglas J. Smith, *J. High Energy Phys.* 0904 (2009) 097.
- [19] B. Sathiapalan, Nilanjan Sircar, *J. High Energy Phys.* 0808 (2008) 019.
- [20] J. Kluson, *J. High Energy Phys.* 0011 (2000) 016.
- [21] J. Kluson, *Phys. Rev. D* 64 (2001) 126006.
- [22] J. Bagger, N. Lambert, *Phys. Rev. D* 77 (2008) 065008, arXiv:0711.0955 [hep-th].
- [23] A. Gustavsson, *Nucl. Phys. B* 811 (2009) 66, arXiv:0709.1260 [hep-th].
- [24] Pei-Ming Ho, Yutaka Matsuo, *J. High Energy Phys.* 0806 (2008) 105.
- [25] Sunil Mukhi, Constantinos Papageorgakis, *J. High Energy Phys.* 0805 (2008) 085.

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