The optimized kinematic dynamo in a sphere

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The Earth's magnetic field is generated and sustained by the complex motion of a conducting fluid in the liquid outer core. This phenomenon can be understood in the framework of dynamo theory, which mathematically describes the interaction between the flow and the magnetic field. An outstanding question is which kind of flow can amplify a seed magnetic field. The growth rate of the magnetic field is determined by the competition between magnetic advection and magnetic diffusion. The ratio between the two effects is given by a dimensionless parameter called the magnetic Reynolds number (Rm). A seed magnetic field may grow at a sufficiently high Rm, but the precise threshold for a dynamo driven by a general type of flow is unknown. Given a conducting fluid confined in a domain, what is the lowest Rm to generate a dynamo? We base our Rm on the unit enstrophy norm for the flow, since Rm based on unit kinetic energy is known to have no lower bound from Proctor (2015) . We use an optimization method inspired by Willis (2012) to search for the most efficient dynamo solution. This method allows us to maximize the growth rate of the magnetic field over a time window T while imposing other constraints using Lagrange multipliers. We simultaneously look for the optimal steady flow field U and the optimal seed magnetic field B0. We reported the optimization results for flows confined in a cube in Chen et al. (2015). In this talk, I will present the new results in a sphere with electrically insulating boundary condition (BC). The flow satisfies no-slip BC. Compared with previously known dynamo models with the same BC, e.g., Livermore & Jackson (2004), our optimal flow has a lower critical Rm where the magnetic field becomes self-sustaining. We also compare it with other known dynamo models with low critical Rm which may have different flow BCs, e.g., Dudley & James (1989), again our optimal flow exhibits superior efficiency in driving a dynamo, yet still respect the lower bounds found by Backus (1958), Childress (1969) and Proctor (1977). The profile of this flow will be discussed supplemented with visualization.
