EFFECT OF RANDOM ERRORS IN CYLINDRICAL NEAR FIELD MEASUREMENTS

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1 Introduction.

So far [1,2], expressions have been derived to predict the effect of random errors in planar near field measurements. Similar equations for cylindrical near field measurements are not available. Random errors are due to A/D quantization, receiver noise or random room scattering, their nature is additive and will limit the accuracy of the far field pattern. Since the near to far field transformation is a linear operator, the effect of an additive noise can be analysed by superposition.

The near field noise is assumed to be a complex gaussian noise, space stationary, with zero mean, and variance σ^2. In this analysis the statistical characteristics of the far field noise will be found.

2 Far field transformation of the near field noise.

The cylindrical near to far field transformation is based on obtaining the cylindrical modal coefficients of the fields. The first step is the Fourier transform of the near field. If $n(z_i, \phi_j)$ is the measured noise, the Discrete Fourier Transform (DFT) is

$$N(k_z, n) = \frac{\Delta z \Delta \phi}{2\pi} \sum_{i}^{N_z} \sum_{j}^{N_\phi} n(z_i, \phi_j) e^{jk_z z_i} e^{-jn\phi_j} \tag{1}$$

The autocorrelation of the DFT of the noise is [3]

$$R_N(p,q) = \left(\frac{\Delta\phi\Delta z}{2\pi}\right)^2 \sigma^2 \sum_{i}^{N_z} \sum_{j}^{N_\phi} e^{-jp\phi_j} e^{jqz_i} = N_z N_\phi \sigma^2 \left(\frac{\Delta\phi\Delta z}{2\pi}\right)^2 \delta(p,q) \tag{2}$$

that is a white gaussian noise, since the spectral power density is flat.

The cylindrical modal coefficients are found as

$$a_n(k_z) = \frac{N_h b_n^{(2)}(k_z) - N_v b_n^{(1)}(k_z)}{a_n^{(1)}(k_z) b_n^{(2)}(k_z) - a_n^{(2)}(k_z) b_n^{(1)}(k_z)} \tag{3}$$

$$b_n(k_z) = \frac{N_v a_n^{(1)}(k_z) - N_h b_n^{(2)}(k_z)}{a_n^{(1)}(k_z) b_n^{(2)}(k_z) - a_n^{(2)}(k_z) b_n^{(1)}(k_z)}$$

where $a_n^{(i)}(k_z)$ and $b_n^{(i)}(k_z)$ are the modal coefficients of the probe that can be computed as [4] or [5], and N_h and N_v are the DFT of the noise in the horizontal and vertical channel.

In this case the coefficients $a_n(k_z)$ and $b_n(k_z)$ are random variables with autocorrelation

$$R_{b_n}(p,q) = \frac{|a_n^1(k_z)|^2 + |a_n^2(k_z)|^2}{|a_n^1(k_z) b_n^2(k_z) - a_n^2(k_z) b_n^1(k_z)|^2} \sigma^2 N_z N_\phi \left(\frac{\Delta\phi\Delta z}{2\pi}\right)^2 \delta(p,q) \tag{4}$$

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and

$$R_{a_n}(p,q)=\frac{|b_n^{(2)}(k_z)|^2+|b_n^{(1)}(k_z)|^2}{|a_n^1(k_z)b_n^2(k_z)-a_n^2(k_z)b_n^1(k_z)|^2}\sigma^2 N_z N_\phi\left(\frac{\Delta\phi\Delta z}{2\pi}\right)^2\delta(p,q) \tag{5}$$

Now $a_n(k_z)$ and $b_n(k_z)$ are gaussian random variables with zero mean, but are not stationary since their autocorrelation is a function of n and k_z. The far field is obained as

$$n_\theta(k_z,\phi)=\sum_n^{N_\phi} b_n(k_z)e^{jn\phi};\; n_\phi(k_z,\phi)=\sum_n^{N_\phi} a_n(k_z)e^{jn\phi} \tag{6}$$

the autocorrelation of the Θ far field noise component is

$$R_{n_\theta}(\tau,\xi)=E\{n_\theta(k_z+\tau,\phi+\xi)n_\theta^*(k_z,\phi)\}=\sum_m\sum_n R_{b_n}(n-m,\tau)e^{j(n-m)\phi}e^{jn\xi} \tag{7}$$

since $R_{b_n}(m-n,\tau)$ is zero if $n\neq m$

$$R_{n_\theta}(\tau,\xi)=\sigma^2 N_z N_\phi\left(\frac{\Delta\phi\Delta z}{2\pi}\right)^2\sum_n\frac{|a^{(1)}_n(k_z)|^2+|a_n^{(2)}(k_z)|^2}{|a_n^{(1)}(k_z)b_n^{(2)}(k_z)-a_n^{(2)}(k_z)b_n^{(1)}(k_z)|^2}e^{jn\xi}\delta(\tau) \tag{8}$$

The far field noise is white gaussian non stationary in kz, since the autocorrelation is a function of kz, and stationary and colored in ϕ since the autocorrelation is non zero for $\xi\neq 0$. The far field noise variance for each polarization is :

$$\sigma^2_{ff\theta}(k_z,\phi)=\sigma^2 N_z N_\phi\left(\frac{\Delta\phi\Delta z}{2\pi}\right)^2\sum_n\frac{|a_n^{(1)}(k_z)|^2+|a_n^{(2)}(k_z)|^2}{|a_n^{(1)}(k_z)b_n^{(2)}(k_z)-a_n^{(2)}(k_z)b_n^{(1)}(k_z)|^2}$$

$$\sigma^2_{ff\phi}(k_z,\phi)=\sigma^2 N_z N_\phi\left(\frac{\Delta\phi\Delta z}{2\pi}\right)^2\sum_n\frac{|b_n^{(1)}(k_z)|^2+|b_n^{(2)}(k_z)|^2}{|a_n^{(1)}(k_z)b_n^{(2)}(k_z)-a_n^{(2)}(k_z)b_n^{(1)}(k_z)|^2} \tag{9}$$

From the expressions above the effect of an additive gaussian white noise in the near field is an additive gaussian noise with variance proportional to the near field noise variance and function of Kz. Moreover the variance depends on the probe cylindrical coefficients (radiation pattern), so the effects of the noise will be dependent on the probe and in general different for each polarization.

3 Expected value of the radiation pattern.

The noise contaminated radiation pattern will be of the form

$$\tilde{E}(k_z,\phi)=\bar{E}(k_z,\phi)+\bar{n}(k_z,\phi) \tag{10}$$

where $\bar{n}(k_z,\phi)$ is a gaussian noise with variance $\sigma^2(k_z)$ given by equation [9]. The far field module is for a given polarization

$$|\tilde{E}|=|\Re(E+n)+j\Im(E+n)|=(\Re(E+n)^2+\Im(E+n)^2)^{\frac{1}{2}}=(X^2+Y^2)^{\frac{1}{2}} \tag{11}$$

where X and Y are the real and imaginary parts of $E(k_z,\phi)+n(k_z,\phi)$. X and Y are gaussian random variables with mean value $\Re(E)$ and $\Im(E)$ variance σ^2_{re} and σ^2_{im} respectively, and

$$\sigma^2_{re}=\sigma^2_{im}=\frac{1}{2}\sigma^2_{ff} \tag{12}$$

$|\tilde{E}|$ is a random variable with a Rice probalility density function (PDF) given by [6]

$$f(|\tilde{E}|)=\frac{|\tilde{E}|}{\sigma_{re}^2}e^{-\frac{|\tilde{E}|^2+|E|^2}{2\sigma_{re}^2}}I_0\left(\frac{|\tilde{E}||E|}{\sigma_{re}^2}\right) \tag{13}$$

where I_0 is the modified Bessel function of order zero.When $\frac{|E|^2}{\sigma_{re}^2} \gg 1$ the Rice PDF can be approximated by a gaussian PDF with mean value $|E|$, on the other hand if $\frac{|E|^2}{\sigma_{re}^2} \ll 1$ we have a Rayleigh PDF with mean value $\sigma_{re}\sqrt{\left(\frac{\pi}{2}\right)}$

Knowing the PDF of the module of the far field, an upper and lower bound M and m for $|\tilde{E}|$ can be calculated by

$$P(|\tilde{E}|\le M)=\int_0^M f|\tilde{E}|d|\tilde{E}|=p \tag{14}$$

$$P(|\tilde{E}|>m)=1-\int_0^m f|\tilde{E}|d|\tilde{E}|=q \tag{15}$$

where p and q are the probabilities to exceed the bound M and not to exceed m respectively. The probability that thte far field is between M and m is

$$P(m<|\tilde{E}|\le M)=\int_m^M f|\tilde{E}|d|\tilde{E}|=p+q-1 \tag{16}$$

4 Results.

Numerical simulations have been carried out to validate expressions 9. The far field variance has been computed by averaging the square of the real and imaginary part of the result of performing the near to far field transform, when the input is a gaussian noise. Figures 1 and 2 show the result compared with equations (9) for both polarizations. The probe is a magnetic probe, the frequency is 3 GHz and the measurement radius is 50.9 cm.

The following figures show the radiation pattern of a low sidelobe array. Figures 3 and 4 without noise, and figures 5 and 6 with a 40 dB S/N ratio in the near field measurement. The dotted lines show the 80 % probability strip.

5 References.

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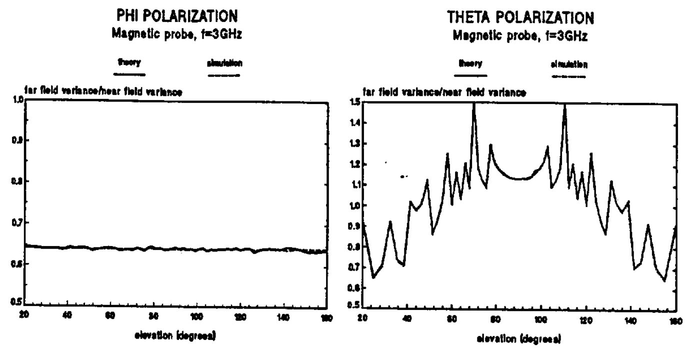

Figure 1,2 . Far field variance for a magnetic probe.

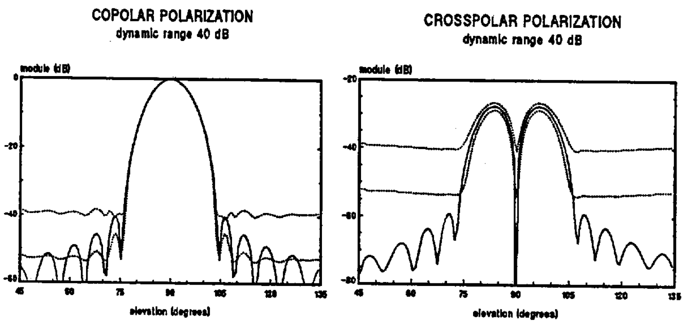

Figure 3,4. Radiation pattern and 80 % probability strip for 40 dB S/N.

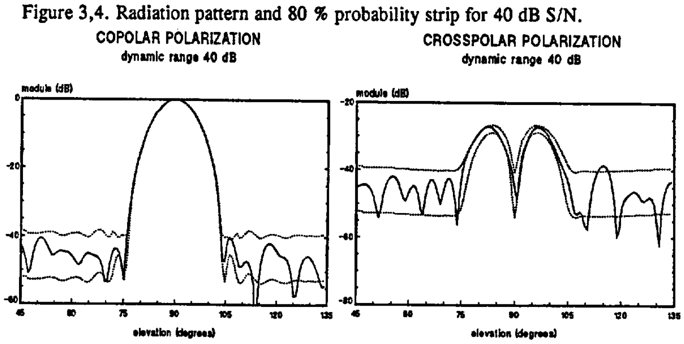

Figure 5,6. Radiation pattern with 40 dB S/N and 80 % probability strip.