



Final Project

**A resource management scheme for
multi-user GFDM with adaptive
modulation in frequency selective
fading channels**

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Task description for Student Thesis

For: **Mr. Bernat Tornes Molins (Student # / ERASMUS)**

Topic: Design and evaluation of a resource management scheme for multi-user GFDM with adaptive modulation in frequency selective fading channels

For the next generation cellular networks, i.e. 5G, challenging application requirements exist, including low latency and very high data rate, as well as a massive number of devices and low energy consumption. While OFDM is currently the dominant waveform for multi-carrier systems, for 5G systems alternative waveforms are considered that are flexible and can be optimized for the different scenarios, such as for low latency requirements. One of these waveforms is GFDM, a block-based multicarrier filtered modulation scheme.

With GFDM, frames can be flexibly assigned to the resource grid. For example, a user block can be configured to allocate a single bit and several sub-carriers; this is one extreme case that is considered for low latency. In order to exploit the GFDM capabilities to adapt the user blocks in the time and frequency resource grid, a resource allocation scheme is required. Typically, in OFDMA systems such schemes target at maximizing the data rate under power and Quality-of-Service (QoS) constraints, or, alternatively, they minimize the transmit power/BER under the data rate constraint. However, applying these schemes to GFDM would considerably restrict its flexibility and limit its capability to meet the QoS requirements.

The task for this master thesis is to design and evaluate a resource management scheme for multi-user GFDM with adaptive modulation in frequency selective fading channels, where resources are assigned based on channel knowledge. Expected result of the thesis is an iterative algorithm, which performs the multi-user sub-carrier, sub-symbol and power allocation to meet QoS constraints. The performance should be compared with existing allocation schemes.

Objectives:

The master thesis shall cover the following aspects:

- Explore background of 5G application scenario, non-orthogonal wave forms and resource allocation schemes with adaptive modulation,
- Develop a MATLAB-based simulation environment to assess the performance of GFDM in multi-user settings for different application scenarios (low latency, etc.),
- Develop an iterative, low complexity algorithm for multi-user resource allocation with GFDM and adaptive modulation,
- Compare the performance of the proposed algorithm with existing ones.

Supervisor: Dr. Xin Zhang, Dr. Andreas Festag

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DECLARATION

I hereby declare that this document has been composed by myself and is based on my own work. Material from the published or unpublished work of others is credited to the author in the text. All sources of information have been specifically acknowledged. This work contains no material published elsewhere or extracted in whole or in part and has not been submitted for any other degree.

Bernat Tornés Molins

Dresden, May 11th 2016

Summary

This final project focus on designing and evaluating a resource management scheme for a multi-user generalized frequency division multiplexing (GFDM) system, when a frequency selective fading channel and adaptive modulation is used. GFDM with adaptive subcarrier, sub-symbol and power allocation are considered. Assuming that the transmitter has a perfect knowledge of the instantaneous channel gains for all users, I propose a multi-user GFDM subcarrier, sub-symbol and power allocation algorithm to minimize the total transmit power. This work analyzes the performance of using a specific set of parameters for aligning GFDM with long term evolution (LTE) grid. The results show that the performance of the proposed algorithm using GFDM is closer to the performance of using OFDM and outperforms multiuser GFDM systems with static frequency division multiple access (FDMA) techniques which employ fixed subcarrier allocation schemes. The advantage between GFDM and OFDM is that the latency of the system can be reduced by a factor of 15 if independent demodulation is considered.

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Nomenclature

LTE	Long Term Evolution
OFDM	Orthogonal Frequency Division Multiplexing
SC-FDM	Single Carrier Frequency Division Multiplexing
GFDM	Generalized Frequency Division Multiplexing
CR	Cognitive Ratio
PHY	Physical Layer
FSC	Frequency Selective Channel
QoS	Quality of Service
MTC	Machine Type Communication
IoT	Internet of Things
M2M	Machine-to-Machine
D2D	Device-to-Device
V2V	Vehicle-to-Vehicle
WRAN	Wireless Regional Area Network
FFT	Fast Fourier Transform
IFFT	Inverse Fast Fourier Transform
CP	Cyclic Prefix
OOB	Out Of Band
QAM	Quadrature Amplitude Modulation
AWGN	Additive White Gaussian Noise
NEF	Noise Enhancement Factor
TDMA	Time Division Multiple Access
FDMA	Frequency Division Multiple Access
ISI	Inter-Symbol Interference
ICI	Inter-Carrier Interference
SER	Symbol Error Rate
SNR	Signal to Noise Ratio

ZFR	Zero Forcing Receiver
MFR	Matched Filter Receiver
SMS	Short Message Service

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1 Introduction

1.1 Overview

It is an evidence that mobile communications hold great importance for our modern society. The first generation (1G) of cellular systems started providing voice transmission. In the 1990s the second generation (2G) emerged in a from the first one by using digital transmission and allowing users to send text messages (SMS). At that point the use of mobile phones became very popular in people daily lives and it started to raise the demand for data. The third generation (3G) enabled our mobile Internet access by the use of packet switching instead of circuit switching for data transmission. Then, after the great impact of social networks, the growth of internet content and bandwidth-intensive applications - like streaming media, it was clear that 3G networks would not be sufficient to satisfy user requirements. Now, the fourth generation (4G – LTE) allows higher-speed data rates up to 100 Mbps and enables a better Quality of Experience (QoE) than any other previous generation.

At the moment, Orthogonal Frequency Division Multiplexing (OFDM) is the main multiplexing scheme adopted because of its robustness against multipath channels and the easy implementation based on Fast Fourier Transform (FFT). Nevertheless, OFDM has some drawbacks that affect its application.

New requirements for future networks are predicted to go beyond LTE networks capacities and data rates. New application scenarios that will require much more network flexibility are foreseen and some of those will not be satisfied with current multiplexing schemes.

In this work I contemplate a frequency selective fading channel which means that the channel is affected by multipath propagation during the wave transmission. To counter this effect, adaptive modulation is considered. Adaptive modulation takes into account the conditions of the radio link in order to improve the rate of transmission by choosing the order of the modulation according to the channel state information.

1.2 Aims and Objectives

Firstly, the objective is to explore the background of 5G application scenario, non-orthogonal waveforms and resource allocation schemes with adaptive modulation. Next, it is to study more in detail the particular scenario where GFDM is aligned with the LTE grid. This scenario is especially important because it would be advantageous, for manufacturers and telecommunication operators, if the 5G standard is also based on the same LTE clock rate.

Lastly, design a MATLAB-based simulation environment for multi-user resource allocation with GFDM and adaptive modulation in frequency selective fading channels. Being more specific, resources will be assigned based on instantaneous channel information. Afterwards explain and analyze how the algorithm performs the multi-user sub-carrier, sub-symbol and power allocation to meet QoS constraints. After completing the design of the environment and the algorithm, the performance of the proposed algorithm will be compared with other existing ones.

1.3 Organization

The paper is organized as follows:

In Chapter 2 (Background) a reference on the concepts used during the system design and simulation is given.

Chapter 3 (System Model) details the whole transceiver model on which the study is done, detailing the different parameters that have been chosen.

Chapter 4 (Algorithm) explains how the algorithm works step by step and its implementation. It shows the evaluation of it including simulation results and a discussion about them.

Chapter 5 (Conclusions) concludes the report looking at what our objectives were and how I have contributed to analyze them. All difficulties and challenges that have been presented throughout the project from its beginning and possible future work are listed.

2 Background

2.1 GFDM for 5th Generation Cellular Networks

When the fourth generation for mobile systems was developed, it was optimized to provide both higher data rates and reliable coverage to mobile users. However, cellular systems for the next generation network will need to challenge new demands with more diversity of application requirements such as, ultra-high data rate, ultra-low latency, battery-driven communication sensors which need ultra-low power consumption, and some control applications that require a very short round trip time (RTT). In table 1 there is a description for the main scenarios foreseen for 5G networks [MMG14].

Scenario	Requirements
Bitpipe Communication	Low-out-of band is crucial to allow fragmented spectrum allocation with Cognitive Radio (CR) technology [LCL11]. The opportunistic utilization of white spaces, where radio terminals are able to detect white spaces in the spectrum and establish the communication in that space until they detect a primary user and do change to another channel. Especially for this particular case OFDM cannot be efficiently used because of its high out-of-band emission (OOB) and the high PAPR (Peak-to-Average-Power-Ratio).
Machine Type Communication	This is about enabling direct communications among devices and connect these devices to the Internet, also called Internet of Things (IoT). Normally a battery supports these devices and the power consumption is a very important factor to take into account. So for this scenario it is required to achieve reliable communication in exchange of a loose of synchronization [VWS13].
Tactile Internet / Device-to-device communication	Real-time control applications with at most 1 ms round-trip latency, i.e. tactile, haptic, visual and auditory interfaces. The ultra-low latency is going to be determined by the physical layer because the time budget for the physical layer (PHY) will be of less than hundreds μ s [FET14]. Current LTE structure, based on 70 μ s OFDM symbols, has a latency too high for achieving the target for the Tactile Internet. With GFDM we can solve this by having a small product of MK, being M and K the number of sub-symbols and subcarriers respectively, which will lead to larger bandwidth per subcarrier. However, as GFDM has M samples per subcarrier, frequency domain equalization can be applied.
Wireless Regional Area Network – (WRAN)	Wireless networks have relatively small cell size and operate in licensed frequencies, which makes them economical unfeasible in low populated areas. When using OFDM as air interface, it is difficult to attend the emission mask imposed by spectrum regulations. So next generation network should cover large coverage areas using dynamic channel allocation based on CR with low OOB emissions and the multipath effects by reducing the impact of the cyclic prefix (CP) in the overall data rate.

Table 1: Main scenarios for 5G [MMG14]

From the previous table, we deduce that the main characteristic for the next generation of cellular networks is the flexibility. The goal is to satisfy a set of completely different types of scenarios by just adjusting GFDM parameters (number of subcarriers and sub symbols).

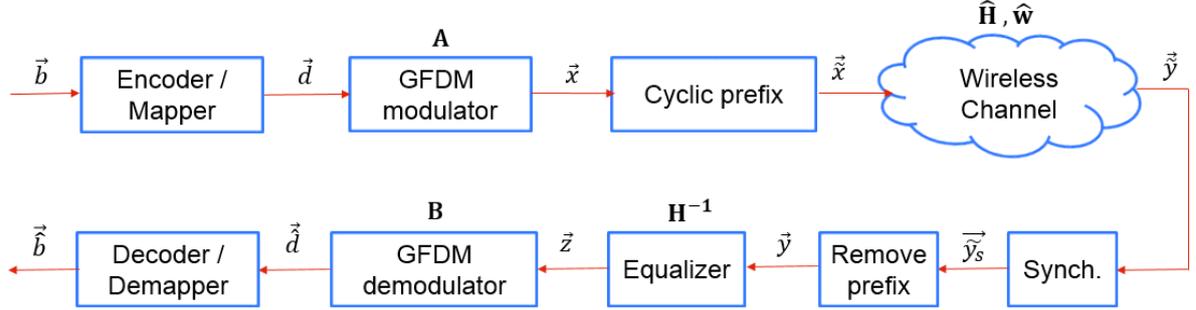


Fig. 1 Block diagram of the transceiver

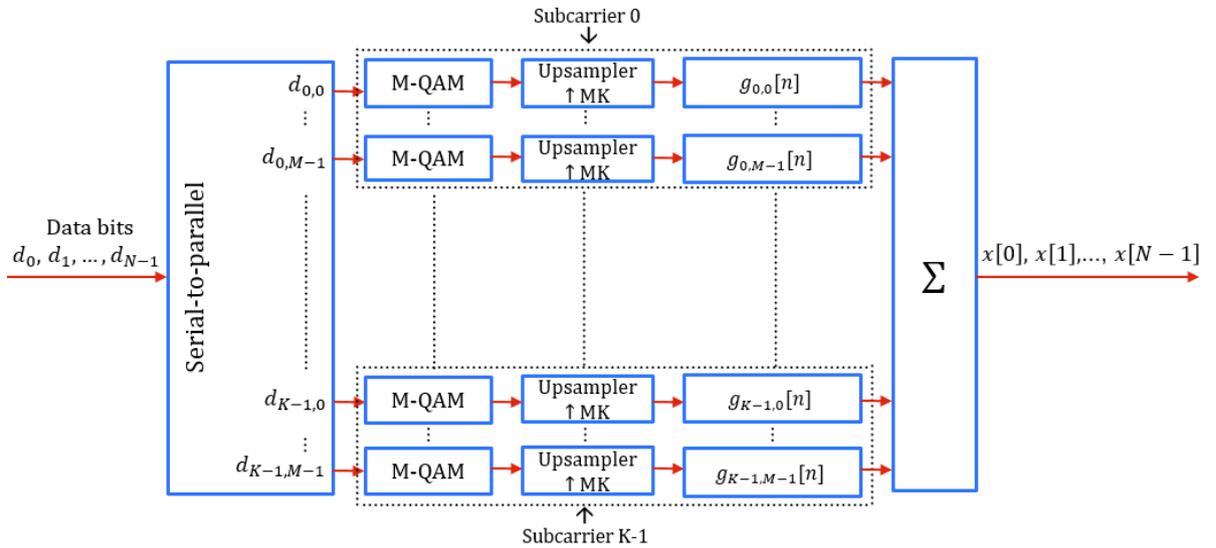


Fig. 2 GFDM modulator

Considering the block diagram depicted in figure 1, a data source provides bits of data to the vector \vec{b} . Then data is encoded and mapped, i.e. J-QAM is the modulation that will be used for this work, to symbols of 2^μ - valued complex constellation where μ is the modulation order. As a result of this, we get vector \vec{d} , which is the data block containing N elements. Those elements can be split into K subcarriers and M sub-symbols which cohere as $N=KM$ (M sub-symbols and K subcarriers compose each GFDM symbol), leading to $\vec{d} = (\vec{d}_0^T, \vec{d}_1^T, \dots, \vec{d}_{M-1}^T)^T$ where each vector is $\vec{d}_m = (d_{0,m}, d_{1,m}, \dots, d_{K-1,m})^T$. Every individual $d_{k,m}$ corresponds to the data transmitted on the kth subcarrier and in the mth sub-symbol of the block. Figure 3 shows how GFDM time-frequency grid is organized [MMG14].

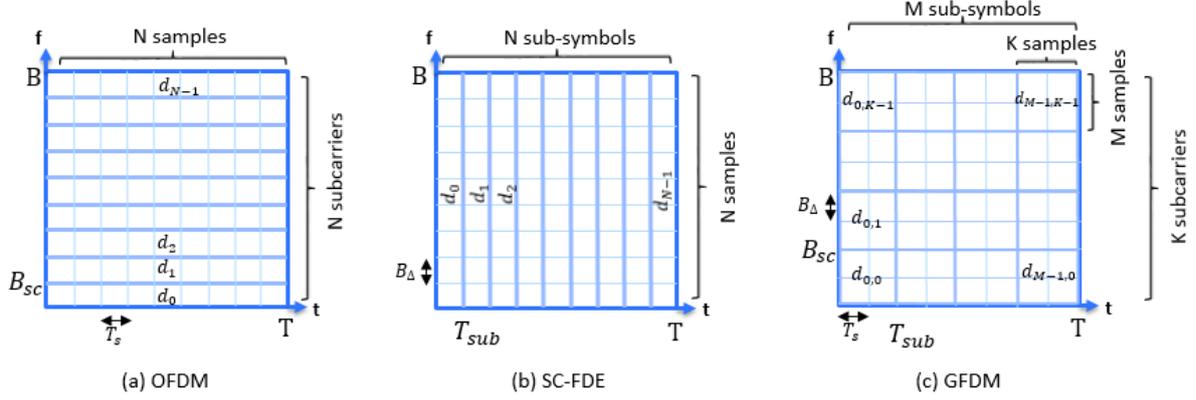


Fig. 3 Partitioning of time and frequency of different multiplexing schemes. (a) $K=N$ subcarriers and $M=1$ sub-symbol, (b) $K=1$ subcarrier and $M=N$ sub-symbols, (c) $K=4$, $M=4$ and $N=16$

All these data symbols are transmitted respectively with a corresponding pulse shape in time and frequency domain as follows,

$$g_{k,m}[n] = g[(n - mK) \bmod N] \cdot \exp[-j2\pi \frac{k}{K} n], \quad (1)$$

where n is denoting the sampling index. Each $g_{k,m}[n]$ is a time and frequency shifted version of the prototype filter $g[n]$, where mod N operation makes $g_{k,m}[n]$ a circularly shifted version of $g_{k,0}[n]$ and the exponential performs the shift in frequency domain. The transmit samples $\vec{x} = (x[n])^T$ are obtained by the following expression

$$x[n] = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} g_{k,m}[n] d_{k,m}, \quad n=0, \dots, N-1 \quad (2)$$

Equation (2) can be formulated in vector notation, if we put together filter samples in a vector

$$\overrightarrow{g_{k,m}} = (g_{k,m}[n])^T, \text{ as}$$

$$\vec{x} = \mathbf{A} \vec{d} \quad (3)$$

where \mathbf{A} is the transmitter matrix with $KM \times KM$ dimension, each element is

$$A = (\overrightarrow{g_{0,0}} \overrightarrow{g_{1,0}} \dots \overrightarrow{g_{K-1,0}} \overrightarrow{g_{0,1}} \overrightarrow{g_{1,1}} \dots \dots \overrightarrow{g_{K-1,M-1}}) \quad (4)$$

Now \vec{x} contains the transmit samples corresponding to the GFDM data block \vec{d} and only one more step is needed to add it to the cyclic prefix to send the signal. It is worth mentioning that GFDM only uses one CP for M sub-symbols. Figure 4 shows a comparison of how the CP is used between OFDM and GFDM.

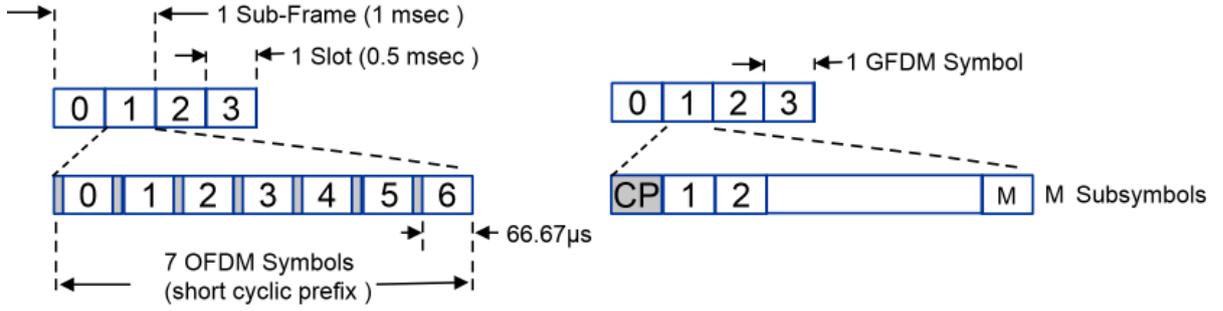


Fig. 4 CP in OFDM versus CP in GFDM

Once the cyclic prefix is added, signal \vec{x} is transmitted through the wireless channel and can be modeled by $\vec{y} = \tilde{\mathbf{H}}\vec{x} + \vec{w}$, where \vec{y} is the received signal. $\tilde{\mathbf{H}}$ denotes the channel matrix (which is a $N+N_{cp}+N_{ch}$ by $N+N_{cp}$ convolution matrix with band-diagonal structure based on the channel impulse response) and \vec{w} denotes additive white Gaussian noise (AWGN).

At the receiver, signal synchronization process is performed in time and frequency, obtaining \vec{y}_s . Afterwards, cyclic prefix is removed. Then, under the assumption of perfect synchronization $\vec{y}_s = \vec{y}$, the equation can be rewritten to $\vec{y} = \mathbf{H}\vec{x} + \vec{w}$. Notice that $\tilde{\mathbf{H}}$ has been replaced with the $N \times N$ matrix \mathbf{H} . Thanks to that zero-forcing channel equalization is applied, as used in OFDM [DJR13], but there are also other procedures that can be used.

After removing the CP, the received signal can be written as $\vec{y} = \mathbf{H}\vec{a} + \vec{w}$ and the signal after the channel equalization as $\vec{z} = \mathbf{H}^{-1}\mathbf{H}\vec{a} + \mathbf{H}^{-1}\vec{w} = \vec{a} + \vec{w}$. Finally, after the linear demodulation, signal can be written as

$$\vec{\hat{a}} = \mathbf{B}\vec{z} \quad (5)$$

where \mathbf{B} is the $KM \times KM$ receiver matrix.

Standard receiver methods for GFDM such as zero forcing receiver (ZF) $\mathbf{B}_{ZF} = \mathbf{A}^{-1}$ are already available. The advantage is that it removes the self-interference at the expense of enhancing the noise. On the other hand there is the matched filter receiver (MF) which maximizes the signal-to-noise ratio (SNR) $\mathbf{B}_{MF} = \mathbf{A}^H$, but at the cost of introducing self interference when a non-orthogonal pulse is applied. In a middle term there is the linear minimum mean square error (MMSE) receiver $\mathbf{B}_{MMSE} = (\mathbf{R}_w^2 + \mathbf{A}^H\mathbf{H}^H\mathbf{H}\mathbf{A})^{-1}\mathbf{A}^H\mathbf{H}^H$ that takes into account both self-interference and noise enhancement problems. At the end, estimated symbols $\vec{\hat{a}}$ are demapped and decoded to obtain \vec{b} .

2.2 LTE-compatible 5G PHY based on GFDM

One of the most important features for a transition between generations is how the future generation can take advantage of the already existing infrastructure in the previous one. Mobile operators and device manufacturers are interested in strategies and solutions that will enhance their existing 3G or 4G networks while addressing their 5G deployment requirements without needing a complete equipment upgrade. Only solutions that are able to support multiple functions in a single node [BIN90] through a software upgrade, will protect today's investment for tomorrow's network and avoid a costly replacement of the existing systems. In this section it is shown, based on [GMM14], how 5G can be efficiently aligned with 4G time-frequency grid using the same clock rate of LTE (30.72 MHz).

GFDM has the very big flexibility propriety of covering single carrier frequency division multiple access (SC-FDM) and OFDM as corner cases, or in other words, it makes it possible to operate both modulation schemes used in LTE. More than one approach are already available in the literature, but for this study, just one of these approaches is used.

In this approach, GFDM is configured to address low latency, which is a very interesting feature for future scenarios. GFDM is used to reduce the sub-symbols duration and the use of only one CP for each M sub-symbols. Consequently, system latency is reduced. It is also important to point out that the shorter the symbol is the wider the system has subcarriers and they might be affected from frequency selective effects of multipath channel. Nevertheless, each GFDM subcarrier has M times more samples in frequency domain than in LTE and it is not a big problem to achieve an acceptable symbol error rate.

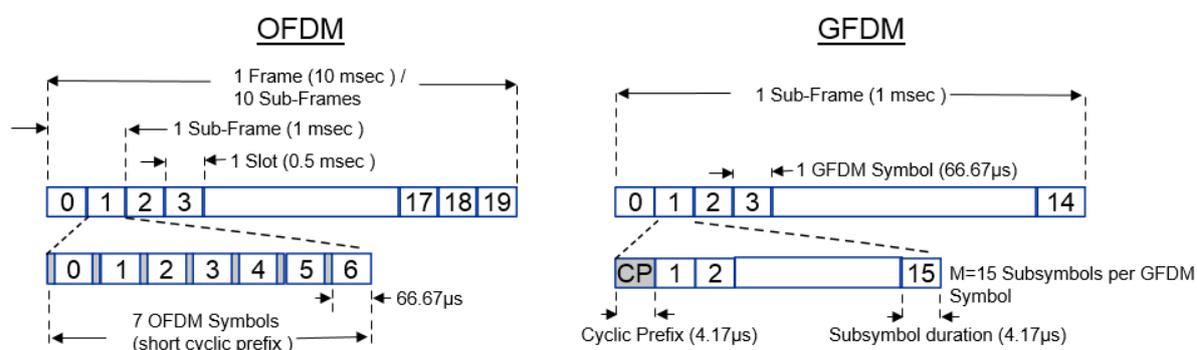


Fig. 5 GFDM frame aligned with LTE.

In figure 5 differences between LTE frame structure and the proposed frame structure in the approach can be seen.

To make the alienation possible a set of GFDM subcarriers must fit into an integer multiple of 180 KHz, which is the bandwidth that occupy on one LTE resource block. Taking parameter $K = 128$ subcarriers, we have a $30.72 \text{ MHz}/128 = 240 \text{ kHz}$ subcarrier bandwidth. So, three GFDM subcarriers $240 * 3 = 720 \text{ kHz}$, which is the band that occupy for four LTE resource blocks.

An other aspect that can be seen likewise in figure 5 and in table 2 is that GFDM symbol duration which is 7.5 times smaller than the LTE slot duration. In addition, if we consider independent demodulation for each symbol, latency is reduced by a factor of 15 (compared with the LTE frame structure that requires two slots of 0.5 ms to demodulate the data). Also the number of data symbols transmitted are much more than in LTE, in the same amount of time (while in LTE there are 14 OFDM symbols per sub-frame or 1 ms using the normal mode, in GFDM there is space for 225 GFDM sub-symbols per sub-frame).

To sum up, OFDM carries less data in comparison to GFDM for the LTE grid retro compatibility. This comes from the fact that data symbols are being transmitted more often, i.e. in this work 15 times more. Moreover, it is more efficient due to the reduction of the cyclic prefix.

Parameter	Normal mode
Subframe duration	1ms or 30.720 samples
GFDM symbol duration	66.67 μ s or 2048 samples
Subsymbol duration	4.17 μ s or 128 samples
Subcarrier spacing	240 kHz
Subcarrier bandwidth	240 kHz
Sampling freq. (clock)	30.72 MHz
Subcarrier spacing factor K	128
Subsymbol spacing M	128
Active subcarriers K_{on}	128
Subsymbols per GFDM symbol M	15
GFDM symbols per subframe	15
CP length	4.17 μ s or 128 samples
Prototype filter	Dirichlet

Table 2. GFDM configuration with the LTE grid.

2.3 Existing allocation schemes literature

2.3.1 Water-filling power allocation

In information theory the capacity of AWGN channel is widely known. The capacity C of one channel is by definition the maximum data rate at which reliable communication is possible. The capacity of the AWGN (complex) channel with noise variance σ^2 and power constraint P is:

$$C_{awgn} = \log\left(1 + \frac{P}{\sigma^2}\right) = \log(1 + SNR) \text{ bits/s/Hz} \quad (6)$$

And if we consider a continuous-time AWGN channel with bandwidth W Hz, power constraint P watts, and additive white Gaussian noise with power spectral density $N_0/2$:

$$C_{awgn} = \log\left(1 + \frac{P}{N_0W}\right) \quad (7)$$

It can be seen that the channel capacity depends on the received power P and bandwidth W . It's important to notice the corner cases: low SNR and high SNR. If we vary the SNR to 0 or $\gg 1$ we can see:

$$C_{awgn} = \log(1 + SNR) \approx SNR \log_2(e) \quad \text{when } x \approx 0 \quad (8)$$

$$C_{awgn} = \log(1 + SNR) \approx \log_2(SNR) \quad \text{when } x \gg 1$$

It follows that the capacity increases linearly for low SNR but logarithmically for high SNR.

Now that the capacity for the AWGN channel has been already shown (detailed explanation in [LUE69]), it is time to focus on the channel type that will be used later for this work which is the frequency-selective channel.

We consider a time-invariant L -tap frequency-selective AWGN channel:

$$y[m] = \sum_{l=0}^{L-1} h_l x[m-l] + w[m] \quad (9)$$

Where y is the received signal, h the channel, x the transmitted signal and w the noise with an average power constraint P on each symbol. The frequency-selective channel can be transformed into N_c independent sub-carriers by adding a CP of length $L-1$ to a data vector of length N_c . So if we repeat this over more blocks, the expression we obtain for the i th OFDM block is:

$$y_n[i] = h_n d_n[i] + w_n[i] \quad n = 0, 1, \dots, N_c \quad (10)$$

This can be seen as a group of sub-channels, one for each sub-carrier n . By definition a set of non-interfering sub-channels, each of which corrupted by independent noise, is called a parallel channel:

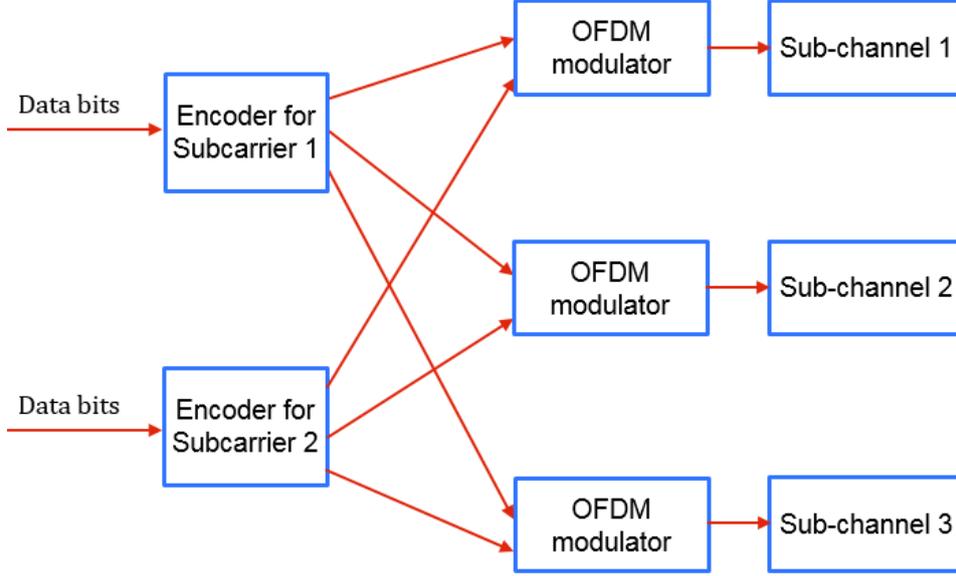


Fig.6 Coding independently over each of the sub-carriers.

So if we allocate P_n power in each sub-carrier, we obtain the expression for the maximum rate of reliable communication over all sub-carriers:

$$C_{N_c} := \max_{P_0, \dots, P_{N_c-1}} \sum_{n=0}^{N_c-1} \log \left(1 + \frac{P_n |h_n|^2}{N_0} \right) \text{ bits/OFDM symbol} \quad (11)$$

Subject to:

$$\sum_{n=0}^{N_c-1} P_n = N_c P, \quad P_n \geq 0, \quad n = 0, \dots, N_c - 1 \quad (12)$$

At this point the system has been defined and water-filling power allocation can be applied now. There is the objective function and the constraint so the Lagrangian can be defined as:

$$\mathcal{L}(\lambda, P_0, \dots, P_{N_c-1}) := \sum_{n=0}^{N_c-1} \log \left(1 + \frac{P_n |h_n|^2}{N_0} \right) - \lambda \sum_{n=0}^{N_c-1} P_n \quad (13)$$

where λ is the Lagrange multiplier. The Kuhn-Tucker condition for the optimality of a power allocation is

$$\frac{\partial \mathcal{L}}{\partial P_n} \begin{cases} = 0 & \text{if } P_n > 0 \\ \leq 0 & \text{if } P_n = 0 \end{cases} \quad (14)$$

So defining $x^+ := \max(x, 0)$, the power allocation it results in to

$$P_n^* = \left(\frac{1}{\lambda} - \frac{N_0}{|h_n|^2} \right)^+ \rightarrow \frac{1}{N_c} \sum_{n=0}^{N_c-1} \left(\frac{1}{\lambda} - \frac{N_0}{|h_n|^2} \right)^+ = P \quad (15)$$

Where (15) satisfies conditions in (14) and is therefore optimal, with the Lagrange multiplier λ chosen such that the power constraint is met.

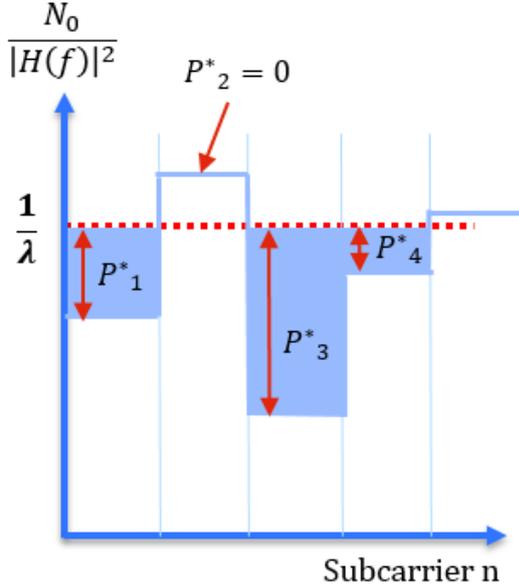


Fig.7 Water-filling power allocation

Figure 7 shows an example of how this method can be viewed from the perspective of the frequency domain. As we can see either in the figure or from the formula, this power allocation schemes allocates P_n^* units of water per subcarrier depending how good the channel conditions are. If channel conditions are good ($H(f)$ high) the subtract $\frac{1}{\lambda} - \frac{N_0}{|h_n|^2}$ is positive and then that subcarrier can be filled with the resulting power of that subtract. Otherwise, as can be seen in figure 7 parameter P_2^* , if the channel conditions are not good or not worth for carrying information then no power is given to those subcarriers.

2.3.2 Multiuser OFDM with adaptive subcarrier, bit, and power allocation

In this section some of the already existing allocation schemes for OFDM are shown. Adaptive modulation is a very strong technique to achieve high spectral efficiency. This technique chooses the order of the modulation based on the channel conditions. Nevertheless, when OFDM with adaptive allocation is used in a frequency selective channel each user is given a predetermined frequency band or time slot. Because of that, some of the subcarriers may not be used and these unused subcarriers are wasted and not used by other users. However, if one subcarrier is in deep fade for one user it does not mean to be in deep fade for all users as the fading parameters are mutually independent. The same happens with GFDM. For this reason, a resource management scheme is needed in combination with the adaptive modulation technique.

The system consists of Q users and K subcarriers, and the bits transmitted for each OFDM symbol for the q th user is R_q . A perfect channel state information is assumed in both the transmitter and the receiver. For this system each subcarrier can only be used by one user, so allocating bits to one subcarrier prevents other users to transmit on that subcarrier.

In the transmitter, the data from these Q users are put into the subcarrier and bit allocation block. Depending on the number of bits assigned to each subcarrier the adaptive modulator will use the corresponding modulation scheme. We define $c_{q,k}$ the number of bits assigned to the q th user on the k th subcarrier and $D = \{0,1,2,\dots,S\}$ where S is the maximum number of bits/OFDM symbol allowed for each subcarrier. The magnitude of the channel gain is defined as $\alpha_{q,k}$ (considering coherent reception) of the q th user seen by the k th subcarrier. Finally, the function $f_c(c)$:

$$f_c(c) = \frac{N_0}{3} [Q^{-1} \left(\frac{P_e}{4}\right)]^2 (2^c - 1) \quad (6)$$

Denoting the required received power (in energy per symbol) for a MQAM symbol in a subcarrier for reliable reception of c bits per symbol when channel gain is equal to unity. The noise power spectral density is denoted by N_0 and the probability error by P_e . Hence, the optimization problem becomes

$$\begin{aligned}
P_T = \min_{\substack{c_{q,k} \in [0,S] \\ \rho_{q,k} \in [0,1]}} & \sum_{q=1}^Q \sum_{k=1}^K \frac{f_q(c_{q,k})}{\alpha_{q,k}^2} \rho_{q,k} & (7) \\
\text{s.t.} & \\
R_q = & \sum_{k=1}^K \rho_{q,k} c_{q,k} & (7.i) \\
1 = & \sum_{q=1}^Q \rho_{q,k} & (7.ii)
\end{aligned} \quad \left. \vphantom{\begin{aligned} P_T = \min \\ \text{s.t.} \\ R_q = \\ 1 = \end{aligned}} \right\} \begin{aligned}
P_T = \min_{\substack{c_{q,k} \in [0,S, \rho_{q,k}] \\ \rho_{q,k} \in [0,1]}} & \sum_{q=1}^Q \sum_{k=1}^K \frac{\rho_{q,k}}{\alpha_{q,k}^2} f_q\left(\frac{r_{q,k}}{\rho_{q,k}}\right) & (8) \\
\text{s.t.} & \\
R_q = & \sum_{k=1}^K r_{q,k} \text{ for all } q \in \{1, \dots, Q\} & (8.i) \\
1 = & \sum_{q=1}^Q \rho_{q,k} \text{ for all } k \in \{1, \dots, K\} & (8.ii)
\end{aligned} \quad (8)$$

The $\rho_{q,k}$ is a time sharing factor (in this system it will take values of 0 or 1 only). So the purpose of the system (7a) is to minimize the transmit power over all users and subcarriers, constrained to:

- i) Total rate of qth user is equal to the sum of the data over all subcarriers being used by that user
- ii) Ensures that only one user can use each subcarrier.

For making the problem more tractable, variable $c_{q,k}$ has to be rewritten as $\frac{r_{q,k}}{\rho_{q,k}}$, otherwise the function $\rho f_q(c_{q,k})$ is not convex (7b). Following the procedure in [WCL99], using standard optimization techniques [7] we obtain the Lagrangian

$$L = \sum_{k=1}^K \sum_{q=1}^Q \frac{\rho_{q,k}}{\alpha_{q,k}^2} f_q\left(\frac{r_{q,k}}{\rho_{q,k}}\right) - \sum_{q=1}^Q \gamma_q \left(\sum_{k=1}^K r_{q,k} - R_q \right) - \sum_{k=1}^K \beta_k \left(\sum_{q=1}^Q \rho_{q,k} - 1 \right) \quad (9)$$

Where γ_q and β_n are the Lagrangian multipliers for the constraints (i)(ii). After differentiating L with respect $r_{q,k}$ and $\rho_{q,k}$, we obtain proportionately:

$$r_{q,k}^* = \rho_{q,k}^* f_q'^{-1}(\gamma_{q,k} \alpha_{q,k}^2) \quad (10)$$

where

$$\gamma_{q,k} = \begin{cases} f_q'(0)/\alpha_{q,k}^2 & \text{if } f_q'^{-1}(\gamma_q \alpha_{q,k}^2) < 0 \\ \gamma_q & \text{if } 0 \leq f_q'^{-1}(\gamma_q \alpha_{q,k}^2) \leq S \\ f_q'(M)/\alpha_{q,k}^2 & \text{if } f_q'^{-1}(\gamma_q \alpha_{q,k}^2) > M \end{cases} \quad (11)$$

and

$$\rho_{q,k}^* = \begin{cases} 0 & \text{if } \beta_k < H_{q,k}(\gamma_{q,k}) \\ 1 & \text{if } \beta_k > H_{q,k}(\gamma_{q,k}) \end{cases} \quad (12)$$

$$H_{q,k}(\gamma) = \frac{1}{\alpha_{q,k}^2} \left[f_q \left(f_q'^{-1}(\gamma \alpha_{q,k}^2) \right) - \gamma \alpha_{q,k}^2 f_q'^{-1}(\gamma \alpha_{q,k}^2) \right] \quad (13)$$

Equation (12) can be seen in another way. Since constraint (7.ii) must be satisfied, we deduce that for each subcarrier if $H_{q,k}(\gamma)$ for $q=1,\dots,Q$ are all different, only the user with the smallest $H_{q,k}(\gamma)$ can use that subcarrier. This can be formulated as follows:

$$\rho_{q',k}^* = \mathbf{1}, \quad \rho_{q,k}^* = \mathbf{0}, \quad \text{for all } q \neq q' \quad (14)$$

where

$$q' = \arg \min_q H_{q,k}(\gamma_{q,k}) \quad (15)$$

The way this algorithm works is by iterative searching. It starts giving small values for all lambdas and then increases them individually until the individual rate constrain is reached. Then it switches to another user. This procedure is repeated until the total data rate constraint is satisfied. More detailed information and figures about this scheme are shown in [WCL99].

3 System Model

The configuration of our multi-user GFDM with adaptive allocation system can be seen in fig.6. Firstly, it is assumed the system has $Q=5$ users, but in next section the performance with different number of users will be shown. It is supposed that each subcarrier has a bandwidth much smaller than the coherence bandwidth of the channel and that the instantaneous channel state information of all subcarriers are known at the transmitter.

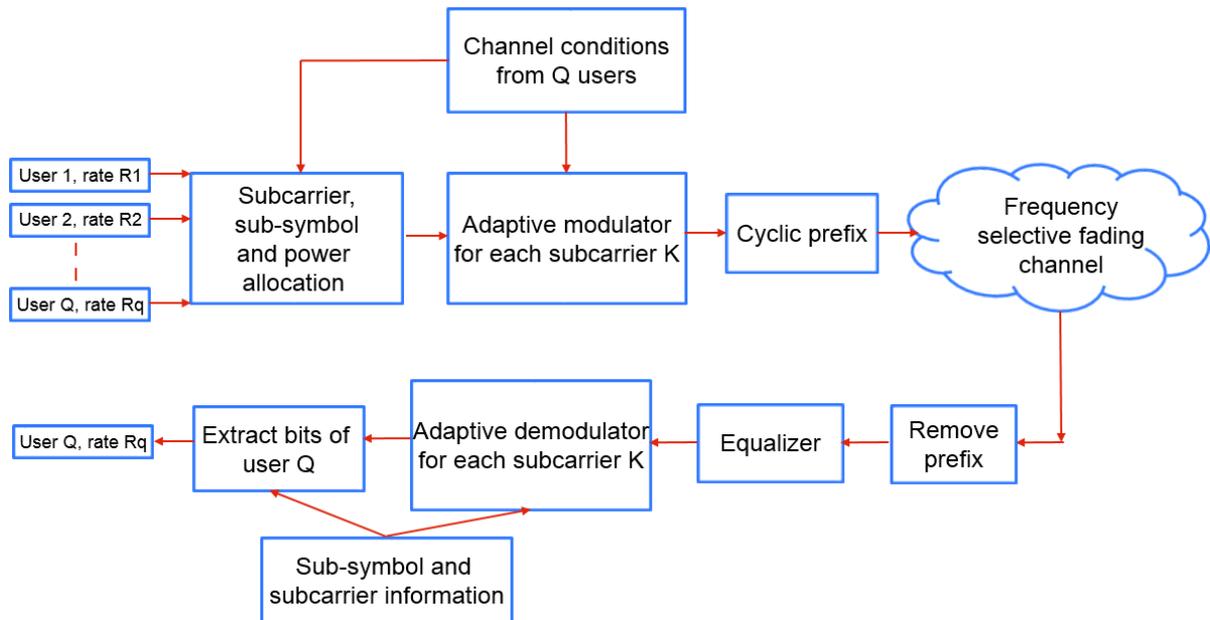


Fig.7 Multi-user GFDM with adaptive allocation

Each of these Q users requests a data rate equal to R_q . Using channel state conditions from all users, the subcarrier, sub-symbol and power allocation algorithm assign the resources to the users. Afterwards adaptive modulation depending on the channel gain, impute more bits to those subcarriers that present a higher channel gain. Then the Inverse Fast Fourier Transform (IFFT) is applied to the signal and the cyclic prefix will be added.

At this step, the signal is ready to be transmitted through the frequency selective fading channel. At the receiver side perfect synchronization is assumed and first of all we remove cyclic prefix. Next, the signal goes through the channel equalizer. The equalizing method employed in this work is the zero-forcing channel equalizer which can be efficiently used in the same way as OFDM.

Once the equalization is done, the demodulation procedure starts. At this point, the adaptive demodulator demodulates the signal depending on the modulation order of each subcarrier using the sub-symbol and subcarrier information and extracts the corresponding bits for each concrete user.

4 Proposed Resource Allocation Scheme

4.1 Description

The main goal of this work was to design a low complexity algorithm (resource management scheme) based on a MATLAB code and evaluate it. One of those schemes is detailed in [3] and that algorithm will be compared with this proposed allocation scheme for their similarities. Nevertheless the scheme in [3] is evaluated in OFDM and the flow chart of the algorithm is quite complex, and the proposed scheme in this work is evaluated for the GFDM case and with a lower complexity.

In this proposed scheme, the main objectives are:

1. Minimize the total transmit power by using the best channel conditions for each user. For this reason, perfect channel knowledge at the transmitter side is required and assumed.
2. Guarantee that all users will obtain their desired data requested. With this constraint even a user with poor channel conditions will be able to transmit data but in exchange of increasing transmit power.
3. Reward with a higher modulation order (more bits per symbol) those subcarriers that have better conditions. That is by definition what adaptive modulation does in this system.

To make this possible some trade-offs between users that have very good channel conditions versus other ones that never have good channel conditions have to be made. Also the combination of using higher or lower modulation order will have an impact to the necessity of using more or less subcarriers.

4.2 Modeling and implementation

In this section, the algorithm mechanics are explained step by step as and the parameters chosen in each simulation.

First of all a structure with system parameters is initiated, which is common for all simulations. Parameters are the following:

Parameter	Number
Number of users	5
Number of subcarriers	128
# Sub-symbols OFDM	1
# Sub-symbols GFDM	15
Total requested data rate (OFDM)(All users) R_{Tot}	512 bits/block
Total requested data rate (GFDM)(All users) R_{Tot}	7680 bits/block (512*15 sub-symbols)
Individual data rate R_q	#Sub-symbols*[104,100,104,100,104] bits
Pulse OFDM	Raised cosine with roll-off 0 (time domain)
Pulse GFDM	Raised cosine with roll-off 0.3 (freq. domain)

Table 3: System parameters

Secondly, five random Rayleigh fading channels with 5-taps each are created and their channel gain information are saved into a channel gains matrix.

Once all channel gains are saved in to matrix \mathbf{H} (with dimensions 5x128) the bit and subcarrier allocation process starts assigning 2 bits (lowest modulation scheme 4-QAM) for each subcarrier to the user that has a higher channel gain. If all subcarriers are already assigned it sums up the total data rate from each user. Normally, in this step some users may not have any subcarrier ascribed to, i.e. a user, whose channel conditions are bad over all the available bands, so it could not transmit data.

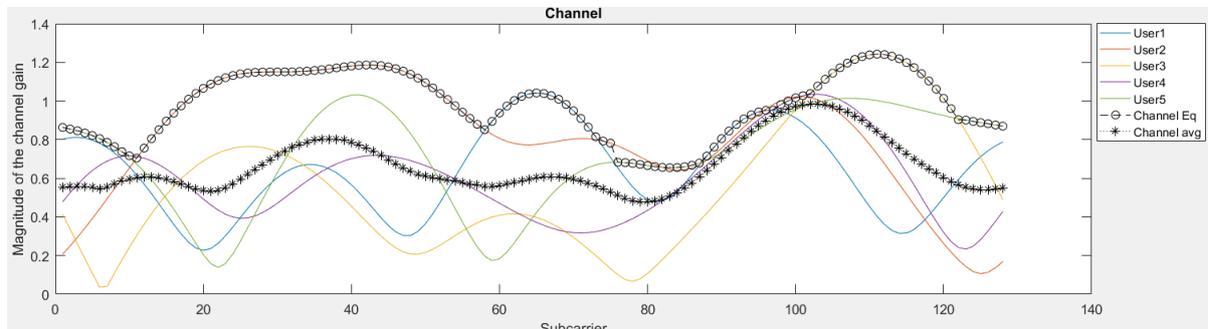


Fig.8 Example of subcarrier allocation after initialization

Afterwards, it has to be checked if each user can transmit his desired data rate with that allocation. In case one user can't reach the individual data rate, it means that he do not

have enough subcarriers or that the subcarriers assigned to that user are already carrying the maximum bits permitted per subcarrier, so that user needs more subcarriers. Hence those users that needs more subcarriers are given another one (at this point subcarrier which should be changed from one user to another is not important). This process is repeated until all users are able to transmit their desired data with that allocation. At the same time, for each extra subcarrier that is accessed to this user another one has to be taken away from another user. For this reason, all users are checked and the ones with more subcarriers associated with are marked with leftover subcarriers.

The next step is to choose carefully which subcarrier can be taken from the user with leftover subcarriers. An iterative procedure will be done here, which searches for the subcarrier with minimum channel gain of the user with leftover subcarriers. Then we modify the parameter $\rho_{q,k}^*$ that indicates for which user q that subcarrier is activated. Also that subcarrier will be initialized with two bits. This procedure is repeated until all leftover subcarriers are assigned.

Now it can be guaranteed that with that subcarrier allocation it is will be possible to satisfy all users. Following the same example as in figure 7 and in figure 8, it is shown that some subcarriers have changed from one user to another. In circles there is the equivalent channel. It just shows which user is using which each subcarrier. Furthermore, channel gain average at each subcarrier is shown by stars. Figure 9 shows a flow chart providing the detailed description of the multiuser subcarrier, sub-symbol and power allocation.

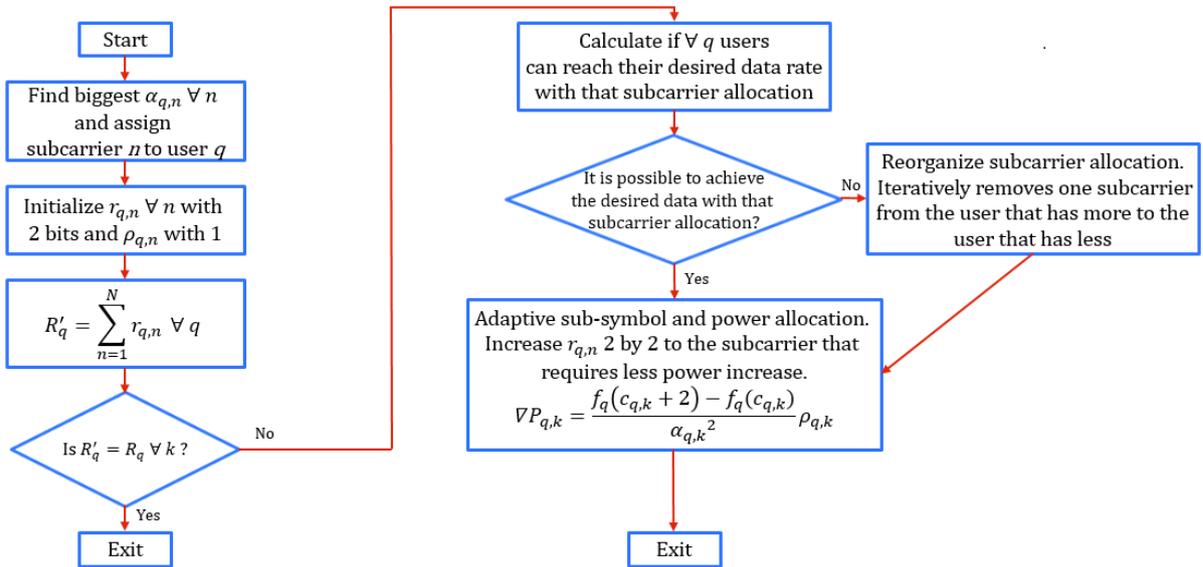


Fig.9 Flow chart of the multiuser subcarrier, sub-symbol and power allocation algorithm.

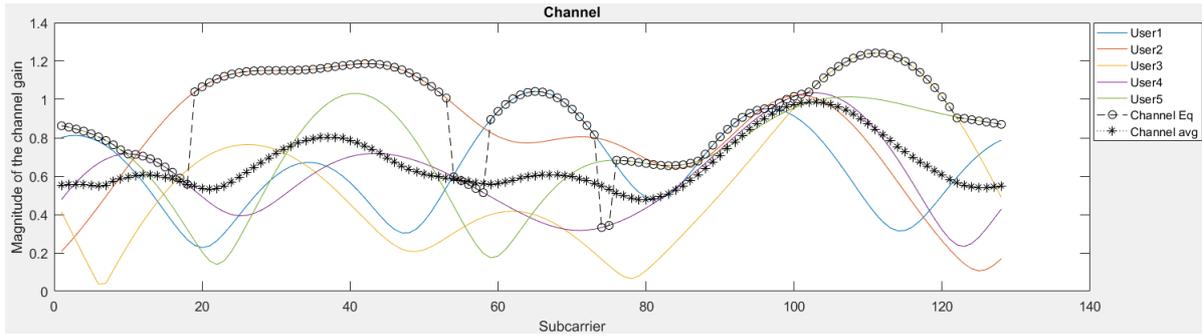


Fig.10 Subcarrier allocation after leftover subcarriers process

Lastly, the adaptive bit allocation is applied. In this step those subcarriers with better channel gains will be given a higher modulation order to carry more bits. To do that, the increment of power which is needed to transmit 2 bits more in the subcarriers/sub-symbols with $\rho_{q,k}^*=1$ has to be calculated:

$$\Delta P_{q,k} = \left(\frac{f_q(c_{q,k+2}) - f_q(c_{q,k})}{\alpha_{q,k}^2} \right) \quad (16)$$

Then there is an iterative function that increases 2 bits by 2 bits of those subcarriers that require less extra power to transmit those 2 bits. This is done until the individual data rate constraint of all users is fulfilled.

Finally the information of which user is using which subcarrier and the modulation order of each subcarrier is saved into a vector. It will be used at the receiver side, in order to demodulate each subcarrier according to the modulation order and the user.

4.3 Evaluation

4.3.1 Simulation results

To evaluate the performance of my proposed scheme, I have simulated 1000 sets of five-path frequency selective Rayleigh fading channels with exponential power delay profile. In almost all simulations the number of users is five, except one – where the purpose of that simulation is precisely to show what happens for different number of users. I use an OFDM system with extended mode and a GFDM system (for comparison purposes) both of them with 128 subcarriers. The data rate transmitted is 512 bits/OFDM symbol or 7680 bits/GFDM symbol (both within the 128 subcarriers), where the total number of data symbols transmitted are 12 OFDM data symbols and 225 GFDM data symbols (15 sub-symbols each GFDM symbol) over one sub-frame/subcarrier respectively.

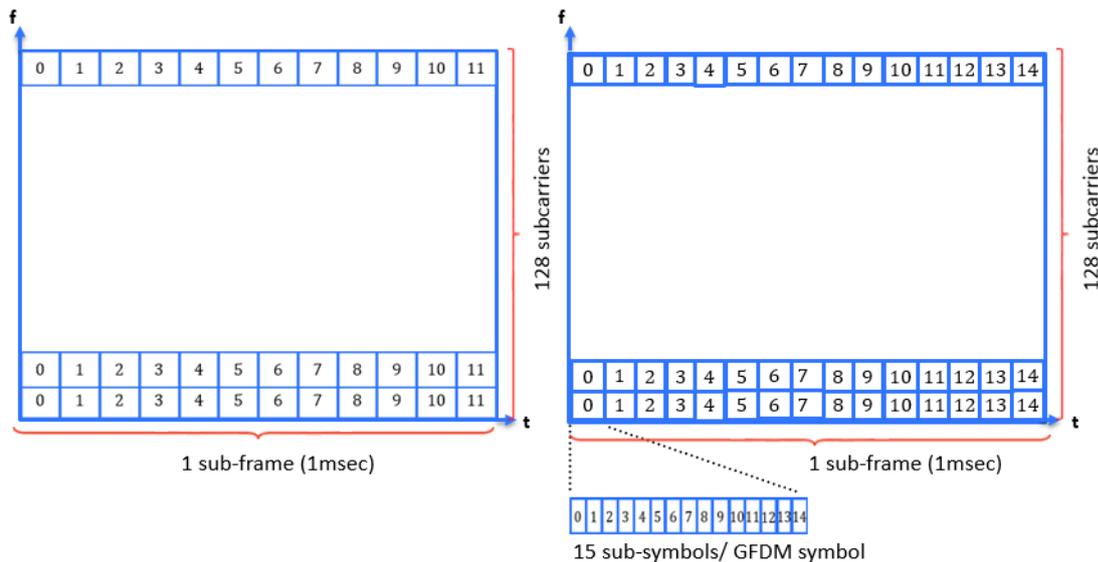


Fig. 11 Transmitted data in simulations

For comparison purposes, I also considered two other static multiuser subcarrier allocation methods. Both of them are multiple access methods and the first one is described in [CIS13] and the second one in [3]. These two methods are the following:

- GFDM-FDMA: each user is assigned to a predetermined group of subcarriers and can only use those subcarriers exclusively for data transmission.

The main drawback of this method is that in frequency selective fading channels, there is a high correlation between the channel gains of adjacent subcarriers. So, by using this method, one user may have all subcarriers in deep fade and therefore it may not satisfy QoS requirements. To avoid this situation, in [3] was proposed an enhanced version of OFDM-FDMA, named OFDM Interleaved-OFDM and I will use here the same technic but applied to GFDM.

- GFDM Interleaved-FDMA: this method is the same as GFDM-FDMA but instead of giving users a set of consecutive subcarriers, all users are interlaced with other user subcarriers in frequency domain.

The subcarrier assignment for these schemes are illustrated in figure 12. It is worth mentioning that in both static schemes the subcarrier allocations are independent from the channel gains of the users. The difference between these schemes and my proposed algorithm is that mine assigns the subcarriers adaptively based on the channel gains of the users.

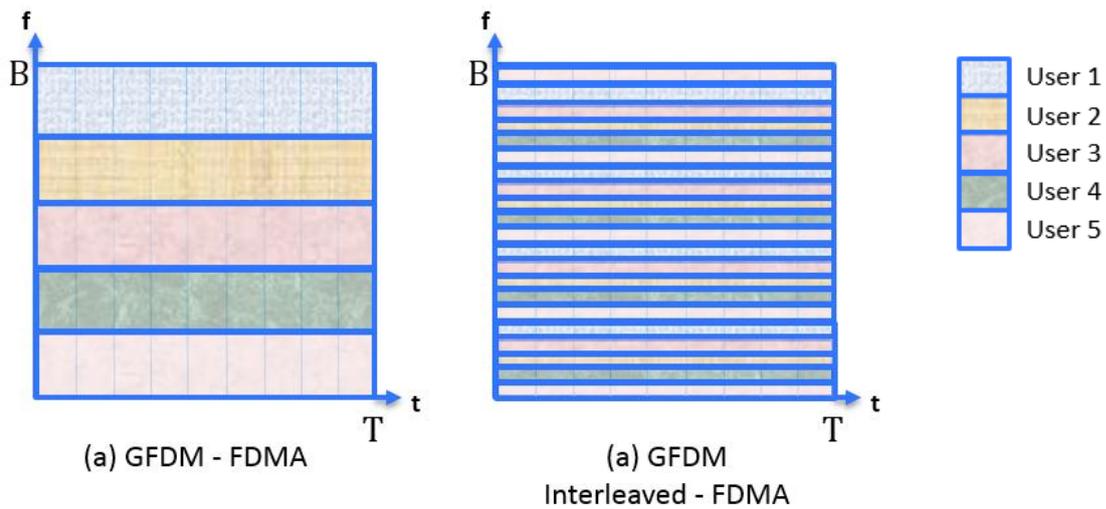


Fig. 12 Subcarrier allocation of GFDM-FDMA and GFDM Interleaved-FDMA scheme.

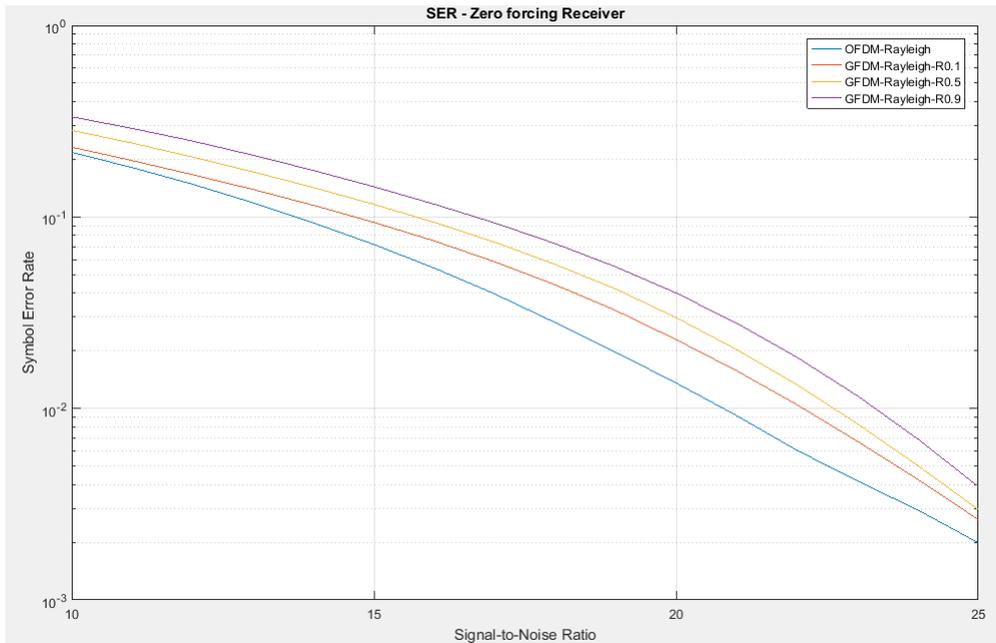


Fig. 13 OFDM and GFDM SER performance in frequency selective channels.

Figure 13 compares the SER performance of OFDM and GFDM when a different roll-off factor is used for the transmitter pulse of the GFDM signal under a frequency Rayleigh fading channel. The figure shows that the pulse shape is an important factor to take into account in the GFDM performance. From the theory we know that when using a root cosine (RC) filter with roll-off=0 the OFDM and GFDM curves should match. For this reason, GFDM plots starts with a roll-off factor equal to 0.1 / 0.5 / 0.9. On the other side, it is shown that when roll-off is equal to 0.9 the performance is much worst and that's because of the noise enhancement when an RC filter with roll-off=0.9 is used. The higher the roll-off factor is the higher noise enhancement the signal has.

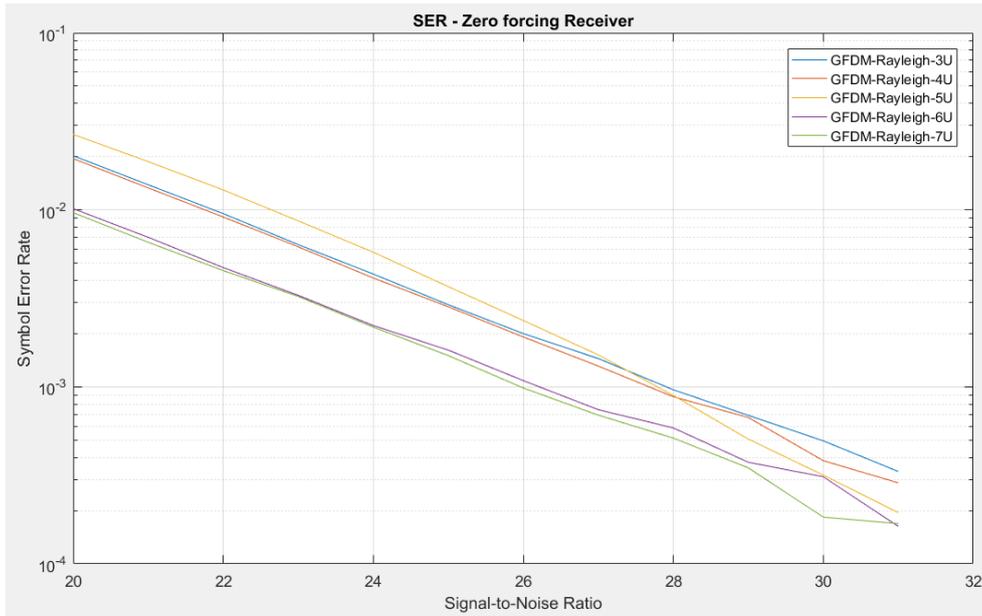


Fig. 14 GDFM SER performance for different number of users

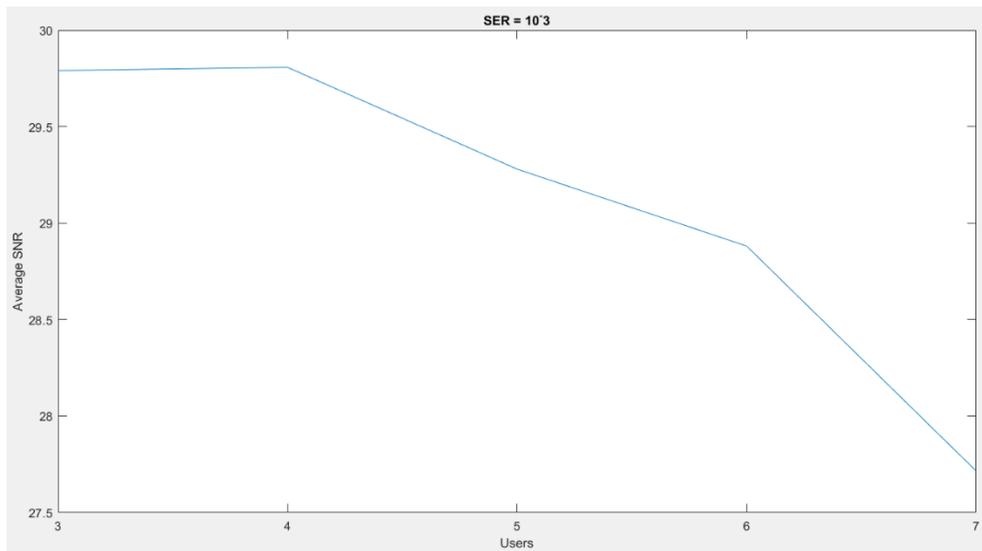


Fig. 15 Average SNR required to achieve $SER=10^{-3}$ versus the number of users

Figure 14 shows the average SNR needed to achieve a certain SER when having five systems with different number of users. We see that when the number of users change (that means more channels, one for each user) the average SNR to achieve a certain SER is lower as more users the system has. The reason of this is that when the number of channels is bigger, the resource allocation algorithm has more chances to satisfy users with better channel conditions. Notice that for this simulation the total requested data rate is the same for all systems, 512 bits, hence when the system has 7 users the individual average requested data is less than when the system has 3 users. On the other hand, when the system has 3 users the band is given to those 3, that means that if one user has poor channel conditions 1/3 of 512bits will be transmitted by this channel, whilst if the system has 7 users and one has bad conditions the proportion will be around 1/7.

From figure 15 it can be seen from another perspective that the average bit SNR to achieve a certain SER (in this case SER is equal to 10^{-3}) decreases when the number of users increase.

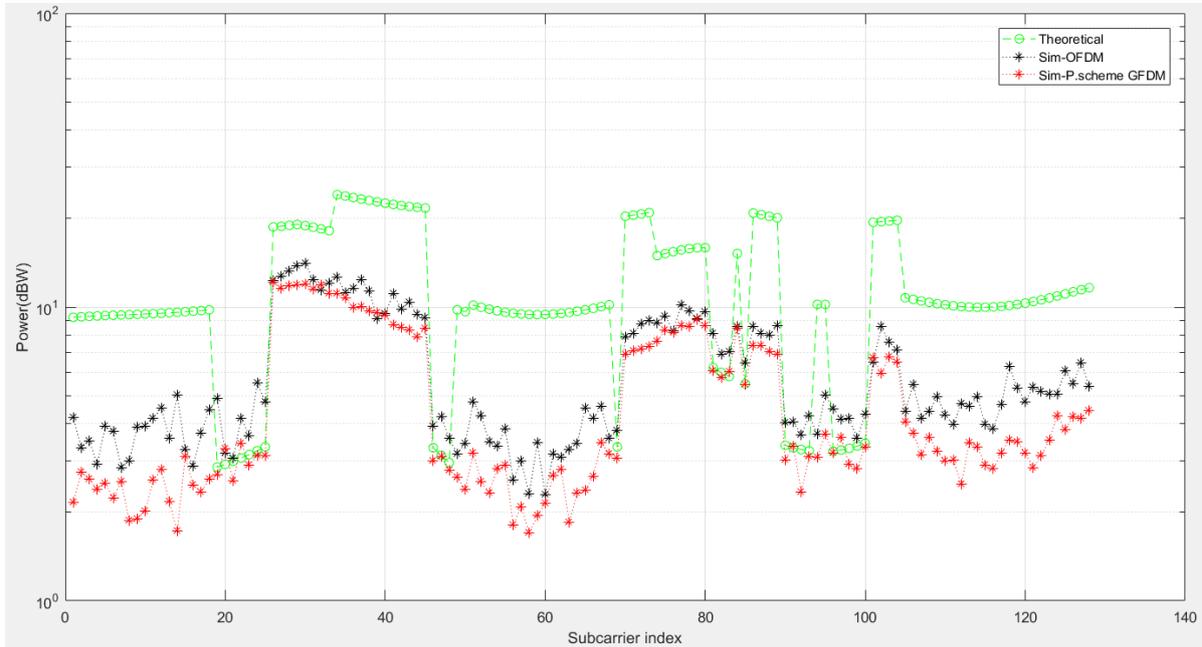


Fig. 16 Received power per subcarrier in OFDM and GFDM versus theoretical required power for supporting c bits/symbol when $P_e = 10^{-3}$

Figure 16 is just the illustration of the system when one iteration is done. It is shown within two figures to make it look clearer than showing it all together, but both are from the same iteration.

Theoretical curve is obtained from equation (6) (notice that for this equation N_0 is equal to 1 and the P_e is equal to 10^{-3}). On the other hand, the power of the simulated signal is obtained from multiplying the received samples versus their conjugate and then divided by the channel gain. Another point that is worth mentioning, is that for plotting the power per subcarrier of the GFDM signal it only takes into account one sub-symbol for each GFDM symbol (first calculates the power including all sub-symbols then is divided by the number of sub-symbols to have an average).

Looking into the plot it is clear that the results obtained when using the theoretical formula versus the simulated results differs a bit from each other. I attribute this differences due to the noise power and the probability error. In the theoretical values N_0 is fixed to 1 and P_e to 10^{-4} , while in the simulated results the power is obtained directly from the received samples that have convoluted with the channel.

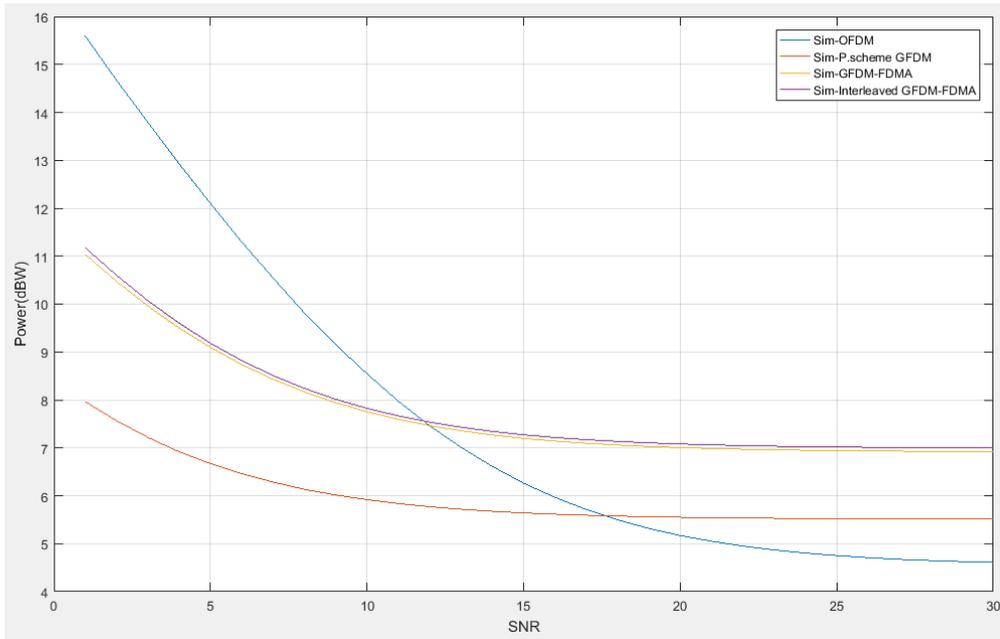


Fig. 17 Total received power of one sub-frame in OFDM and GFDM for SNR=1:30

Figure 17 shows that for low SNR the received power of OFDM sub-frame is higher than GFDM sub-frame in the three schemes. As the SNR increases the power reduction for OFDM frame is bigger than in GFDM. From SNR=12 dB OFDM performance starts to be better than the static allocation schemes GFDM-FDMA and GFDM Interleaved-FDMA. It is not until when the SNR is equal to 17.5dB that the OFDM performance overtake the GFDM one.

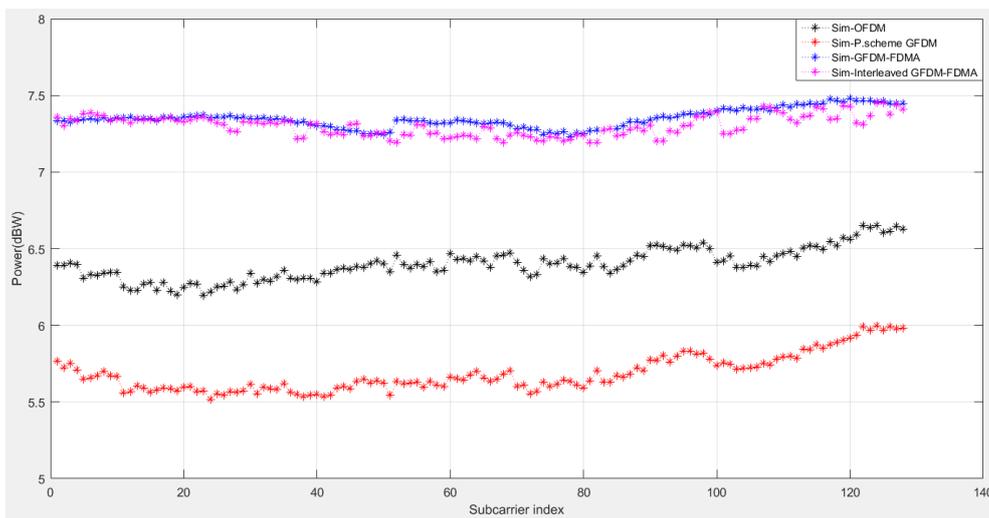


Fig. 18 Received power per subcarrier of one sub-frame in OFDM and GFDM for SNR=15

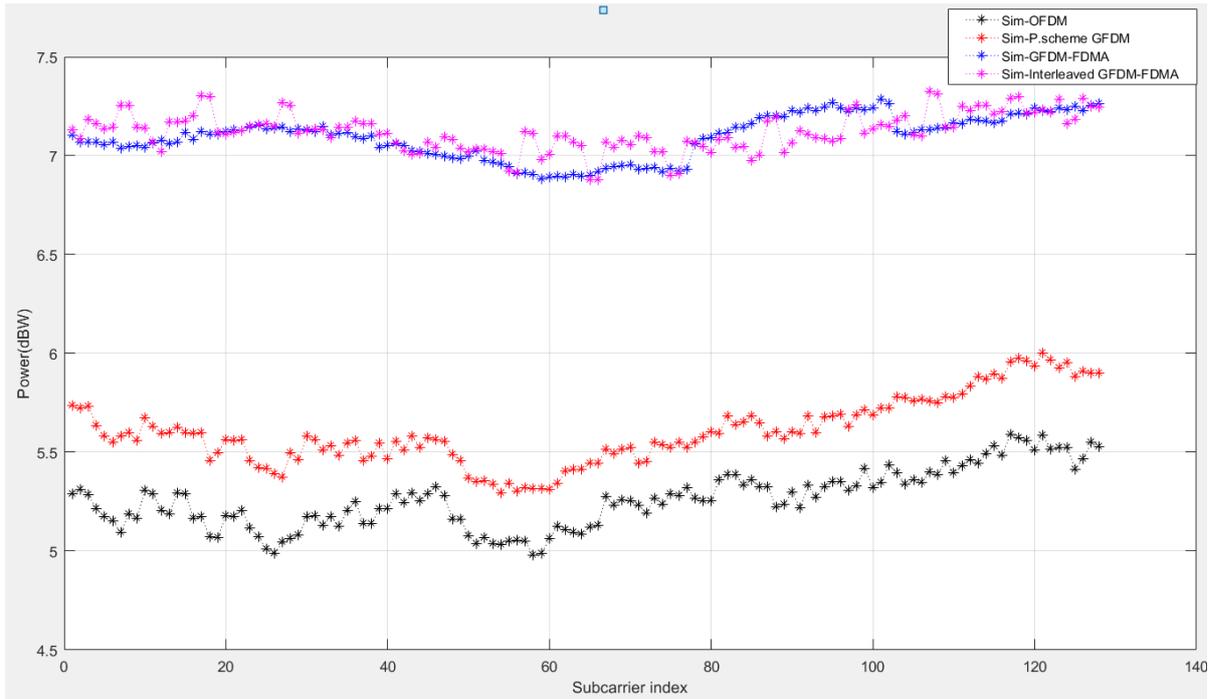


Fig. 19 Received power per subcarrier of one sub-frame in OFDM and GFDM for SNR=20

Figure 18 confirms that the average received power for OFDM is above the average received power for GFDM when is normalized by the number of sub-symbols and the SNR=15. The average difference between the proposed scheme GFDM and OFDM is about 0.8dB average.

On the other hand, in figure 19 it is shown that the average received power for OFDM is below the average received power for GFDM and therefore has a better performance for high SNR. Nevertheless, it is worth to point that the difference between using GFDM with the proposed scheme and OFDM is in average no more than 0.5dB when the SNR=20.

It is important to mention that all curves have a minimum power level. It follows that for GFDM after SNR=21dB this minimum is reached and there is no any performance improvement when the SNR increase more. Besides, the OFDM scheme reaches its limit after 28 dB.

To sum up, the extra power needed for GFDM at high SNR with the proposed algorithm is not critical in terms of power. This effect is produced due to the non-orthogonality of GFDM subcarriers and therefore the introduction of the ISI. This interference is not considered in the model and is one of the future lines of work to include it. Furthermore, it is important to notice that GFDM sub-symbol duration is 15 smaller than OFDM symbol, so by using this proposed scheme not only achieves almost the same power performance per subcarrier/data symbol it also allows to reduce the latency by a factor of 15.

5 Conclusion

In this report, the performance of using a multi-user GFDM system, aligned with LTE resource grid, with adaptive subcarrier, sub-symbol and power allocation is analyzed. After testing and simulating the performance of the proposed scheme, it is clear that knowing the channel gains helps to reduce the average transmitting power.

To this end, it can be stated that when using the proposed scheme for GFDM it is possible to achieve a very close behavior compared with OFDM in terms of SER and received power when the same amount of data is shown. Furthermore, if each GFDM symbol is demodulated independently we can achieve very low latency maintaining a more than acceptable SER with the average transmitting power being slightly above than it is in OFDM for high SNR, and a better performance than OFDM for low SNR, when the same amount of data is compared. Low latency is an important feature for future 5G networks and here we have seen that when GFDM is configured in this way it reduces the impact of the PHY layer overall system latency. This is possible because of the use of one CP for each M sub-symbols and the reduction of the GFDM symbol duration.

As a future line of work, two main points arise that could potentially lead to interesting results:

- Since the theoretical formula used in this work does not include any reference to the inter symbol interference, it would be great to include this interference to be more accurate and therefore obtain a better approximation of how much power is required to support c bits at a given probability error.
- Analyzing the same scheme described in this report but not only focusing with the zero forcing receiver. Also it would be interesting to test the performance with more different pulses or changing the system parameters such as the number of sub-symbols and subcarriers, but then the system will no longer be aligned with the LTE grid. Nevertheless, not only the scenario where GFDM is aligned with LTE grid is interesting, other new 5G scenarios may be of interest to test.

6 References

- [BIN90] J. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Commun. Mag.*, vol. 28, no. 5, pp. 5–14, May 1990.
- [CIS13] Cisco, Long-Term Evolution: Simplify the Migration to 4G Networks, available http://www.cisco.com/c/en/us/products/collateral/wireless/asr-5000-series/white_paper_c11-602886.pdf, Updated Feb 2013
- [DJR13] Y. Ding, Y. Jin, L. Ren, and K. Hao, "An intelligent self-organization scheme for the Internet of things," *IEEE Comput. Intell. Mag.*, vol. 8, no. 3, pp. 41–53, Aug. 2013.
- [FA14] G. Fettweis and S. Alamouti, "5G: Personal mobile Internet beyond what cellular did to telephony," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 140–145, Feb. 2014.
- [FET14] G. P. Fettweis, "The tactile Internet: Applications and challenges," *IEEE Veh. Technol. Mag.*, vol. 9, no. 1, pp. 64–70, Mar. 2014.
- [GMM14] Gaspar et al. - 2014 - LTE-compatible 5G PHY based on Generalized Frequency Division Multiplexing
- [LCL11] Y.-C. Liang, K.-C. Chen, G. Y. Li, and P. Mahonen, "Cognitive radio networking and communications: An overview," *IEEE Trans. Veh. Technol.*, vol. 60, no. 7, pp. 3386–3407, Sep. 2011.
- [LUE69] D. G. Luenberger, *Optimization by Vector Space Methods*. New York: Wiley, 1969.
- [MMG14] Michailow et al. - 2014 – "Generalized Frequency Division Multiplexing for 5th Generation Cellular Networks", *IEEE transactions on communications*, vol.62, no.9, September 2014.
- [RG97] H. Rohling and R. Grunheid, "Performance comparison of different multiple access schemes for the downlink of an OFDM communication system," in *Proc. IEEE Vehicular Technology Conf. (VTC'97)*, Phoenix, AZ, pp. 1365–1369.
- [VWS13] V. Vakilian, T. Wild, F. Schaich, S. ten Brink, and J.-F. Frigon, "Universal filtered multi-carrier technique for wireless systems beyond LTE," in *Proc. IEEE Globecom Workshop*, Atlanta, GA, USA, Dec. 2013, pp. 223–228

- [WCL99] Cheong, Y. W. Cheng, R.S. Lataief, K.B. Murch - 1999 - Multiuser OFDM with Adaptive Subcarrier, Bit, and Power Allocation
- [WUN14] G. Wunder et al., "5GNOW: Non-orthogonal, asynchronous waveforms for future mobile applications," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 97–105, Feb. 2014.