Research Group Advanced Analytics

MASTER’S THESIS

THE CENTRALIZED CHILD CARE SLOT ALLOCATION PROBLEM

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Information Summary

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**Department:** Research Group Advanced Analytics

**Motivation:** Master’s Thesis

Abstract

The child care market has become a recent issue these last years in Germany, triggered by the KiFöG Law (any 1-year-old child or older has the right to have a slot in a day-care facility) going into effect in August 2013.

Nowadays, the way child care facilities in Germany perform its children allocation is decentralized, causing significant inefficiencies and discontent among, not only families but other stakeholders involved.

This project will contribute with methods for optimizing children’s day-care demand fulfilment, particularly with regard to capacity planning and allocation. This study will consist of the creation of a centralized model of the market for children’s day-care and its simulation in an optimization software called IBM ILOG CPLEX. This simulation, which covers more than one possible scenario, will be calibrated and validated on empirical data of the city of Aachen (NRW). Its analysis leads to an evaluation of the success of new planning approaches as well as the consequences of different assumptions on stakeholders’ choice behaviour.
Acknowledgements

First of all, I would be very pleased to thank all those people who have taken part and have helped me performing my master’s thesis.

I would like to give special thanks to Prof. Dr. Catherine Cleophas who has been my supervisor and has given me the possibility to work on this project. In addition, she has given me the guide and council needed to best perform my thesis.

Many thanks to Meike Grewing, Martin Comis and Christina Büsing who have taken part of project meetings and have contributed with creative and useful ideas to reach a more realistic model.

Last but not least, I would like to thank all my family and friends who have supported me performing my Thesis since the first until the last day.

Àlex Octavio Bayón,
September 2016
Summary

KiFöG law in Germany going into effect in August 2013, has directly impacted to the German childcare market. The demand has consequently risen leading to a capacity issue in the day-care facilities.

This project focuses on finding a solution to this problem in the city of Aachen but replicable to any of the German cities. By means of centrally allocating children to the facilities, this issue considerably appeases. Working with real data and with different scenarios enable the reader to better understand how this children allocation is performed and how could it be improved.

Germany is not the only country who faces this challenge, thus there is still a lot of research to undertake and municipalities will certainly have to cooperate in order to find synergies.
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1. Introduction

1.1 Project Background

In August 2013, the KiFöG law, which claims that all children between one and three years old have the right to be assigned in a child-care facility, was established in Germany. Apparently, the legal claim could be fulfilled through the offer of 144,000 day-care slots for under three-year-olds (NRW Taskforce U3 Ausbau, 2013). However, the absolute number of available slots says little about their distribution and the resulting stakeholder satisfaction. In fact, there have already been examples of KiFöG non-compliance, such as the city of Leipzig which was sentenced to pay damages to families (faz.net, 2015).

An easy solution would be to increase the number of day-care slots but at the same time it would be cost-intensive. According to Hermann (2014), this resulting dilemma strongly motivates the need for new fulfilment planning approaches given scarce and highly localized capacity.

As a matter of fact, it has been studied very little so far about the child care slot allocation problem. In addition, there are many stakeholders involved when talking about children in day-care facilities assignment. Not only family’s satisfaction, but also municipal administrations or providers’ contentment is paramount in this topic.

1.2 Objectives

The main objectives of this project are as follows:

- Contextualize and define the relevance of performing further research in the areas of capacity planning and slot allocation in the domain of children’s day-care.
- Study and analyse the current situation of childcare slot allocation.
- Analyse the empirical data in order to gather insight of actual applying and contracting patterns in the childcare discipline.
- Create a centralized model for the childcare slot allocation problem, pursuing the satisfaction of all stakeholders involved in this social issue.
- Simulate the model with empirical data and analyse the subsequent results by means of different scenarios and what if analysis.
- Contribute to solving a problem that is of high current societal relevance.
1.3 Scope

The model, simulation and analysis of the centralized model will be focused on the Aachen municipality (NRW). This means that the empirical input data used in the simulating model will be extracted from real data from the city of Aachen. However, the model, data preparation and its analysis can certainly be extrapolated to any municipality of Germany.

The first part of this Thesis focuses on introducing the state of the art of the child care market, not only in Germany but also around the world. This section insists on the need to perform further research in this field as social trends lead to this problem.

Then, we propose a centralized model which solves the child care slot allocation problem. This chapter focuses on explaining the different parameters, variables and constraints this model has, so that it will help understanding the subsequent section of the input data feeding the model.

The following part refers to the input data the model takes the parameters from. This input data will be paramount for the proper execution of the model. This section explains the transformation of the data sources (Contract Data Set and Application Data Set) provided by the municipality of Aachen into the desired input data.

The next chapter shows the results obtained from the simulations of the model and its subsequent analysis, leading to a conclusion of these.
2. State of the art

Over the last decades the German birth rate has ceaselessly dropped, presenting nowadays the lowest rate worldwide (BBC, 2015). As a consequence, great efforts are being made by the German government to improve the reconciliation of work and family life (Kraemer, 2015). Figure 1 confirms the German low birth rate, compared to the other top European countries GDP ranked.

![Birth rate](image)

Figure 1: Birth rate evolution in top European countries GDP ranked (Google Public Data, 2014)

Notwithstanding, the demand of childcare slots has been increasing due to the women’s introduction in the workforce (see figure 2 below) during the last decades (Quast, 2011) and, in the German case, owing to the KiFöG law gone into effect in 2013. This fact has resulted in bigger difficulty accessing quality child care as it is in short supply and expensive (Harper and Leicht, 2007, Carlsson and Thomsen, 2015). For insolvent families, the struggle can be even worse because childcare represents a terrifying proportion of the family budget, although quitting work to look after the children can lead to financial disaster (The Economist, 2009). Besides this, as the number of applications for child care is larger than the number of places offered, child care facilities find themselves in a situation where they can choose which child to accept. This leads to displeasure and even competitive pressure on the parents’ side.
CESinfo (2014) confirms what Harper & Leicht or Carlsson & Thomsen warned of the short supply of day-care slots. This study goes beyond and affirms that the problem sharpens with the younger children. Parents of older children generally face fewer difficulties in finding a publicly subsidised ECEC (Early Childhood Education and Care) place. This is linked to the fact that, in many countries, older children are either legally entitled to ECEC, or they are obliged to participate for 1 or 2 years before primary education. Table 1 shows for European countries whether the demand is higher than supply or not, distinguishing between younger and older children.

Table 1: Demand vs supply of childcare slots in Europe 2012/2013 (CESinfo, 2014)

<table>
<thead>
<tr>
<th></th>
<th>Younger Children</th>
<th></th>
<th>Older Children</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Demand is</td>
<td>Demand meets</td>
<td>Demand is</td>
<td>Demand meets</td>
</tr>
<tr>
<td></td>
<td>higher than</td>
<td>supply</td>
<td>higher than</td>
<td>supply</td>
</tr>
<tr>
<td>Austria</td>
<td>X</td>
<td>X</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Belgium</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Bulgaria</td>
<td>X</td>
<td>O</td>
<td>O</td>
<td></td>
</tr>
<tr>
<td>Croatia</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Cyprus</td>
<td>NM</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Czech Republic</td>
<td>X</td>
<td>O</td>
<td>O</td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Estonia</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>X</td>
<td>O</td>
<td>O</td>
<td></td>
</tr>
<tr>
<td>Greece</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>
Very few countries satisfy the demand in the whole age range, actually just Norway, Finland, Sweden and Denmark. All the other European countries, including Germany, suffer a shortage in childcare slots supply, but especially with younger children. These figures refer to season 2012/13, hence the KiFöG effect it is not counted yet (law going to effect in August 2013, season 2013/14).

But this issue does not just concern European states but also to all around the world. For instance, the city of Toronto suffers great difficulties to manage the endless waiting lists to access to a day-care facility. These waiting lists are currently decentralized, which does not ease to shorten them in a desired level. Implementing a centralized child care waiting list would certainly help solving the problem. Nevertheless, the capital resources for development, sustainment and administration for a system as large as Toronto’s are not currently included in the 10-year capital plan and would require debt funding to provide these resources.

Interesting is the particular German child care demand side. Prior to reunification, there were great differences between East and West Germany with the day care provision, likewise the consequent demand. At that moment in East Germany the day care supply was widely available to encourage mothers to return to work soon after giving birth.
Conversely, in West Germany, childcare facilities for under-threes were few and far between. Since 2005, post-reunification Germany has seen a significant expansion in day care services, primarily to help parents combine employment with family responsibilities. Despite these recent trends, however, 25 years after reunification, there are still major disparities between East and West Germany when it comes to childcare for the under-threes. Table 2 shows the evolution of the different attitudes between these areas towards childcare among different socio-economic groups (Schober & Stahl, 2014).

Table 2: Attitudes Towards childcare among Different Socio-Economic Groups in Germany (Schober & Stahl, 2014)

<table>
<thead>
<tr>
<th>Year</th>
<th>West Germany</th>
<th>East Germany</th>
<th>Level of Education</th>
<th>Non-german nationals</th>
<th>Single mothers</th>
<th>At risk of poverty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994</td>
<td>68.8</td>
<td>30.3</td>
<td>Lower than Abitur</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>68.1</td>
<td>55.6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Higher than Abitur</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>43.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>47.6</td>
<td>25.7</td>
<td></td>
<td></td>
<td>56.3</td>
<td>39.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>32.1</td>
<td>13.3</td>
<td></td>
<td></td>
<td>48.9</td>
<td>30.0</td>
</tr>
</tbody>
</table>

Childcare for pre-schoolers should be provided mainly by family members.

<table>
<thead>
<tr>
<th>Year</th>
<th>1994</th>
<th>2002</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1126</td>
<td>485</td>
<td>616</td>
</tr>
<tr>
<td></td>
<td>569</td>
<td>210</td>
<td>293</td>
</tr>
<tr>
<td></td>
<td>878</td>
<td>264</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>659</td>
<td>319</td>
<td>576</td>
</tr>
<tr>
<td></td>
<td>158</td>
<td>109</td>
<td>205</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>37</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>31</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>61</td>
<td>124</td>
</tr>
</tbody>
</table>

Sample Size*

*Results for cells entries under 30 are not shown

While methods of operations research and business analytics have been previously applied to great success in other application areas, day-care demand fulfilment has been considered only very rarely from this point of view. Young & Nelson (1973) and Young (1974) are one of the very few examples of the literature that appear to fit the research objective, being unfortunately out-dated. This stream of research, triggered by the
women’s entering the working place in the sixties and seventies of the 20th century, apparently vanished quickly.

To a certain extent, this may be owing to the fact that in most countries, the day-care is still contemplated as an individual problem, with particular providers struggling to gather the desired parents seen just as consumers (Cryer & Burchinal, 1997). Nevertheless, there is some evidence this is about to change in Germany. An ideal example is the Aachen’s unified web-based sign-up for day-care slots (Stadt Aachen, 2015), proving the influence KiFöG law already has.

Allocating children to day-care slots can be regarded as a so-called assignment problem. The assignment problem is one of the basic combinatorial optimization problems with many applications in scheduling, transportation and logistics (Burkard et al., 2009). As complexity increases with additional constraints on the assignment and the realistic problem size, several primal-dual, branch & bound, cutting plane and column generation methods have been developed to compute feasible and optimal solutions (Sørensen & Dahms (2014)). The particular challenges of assigning children to day-care facilities have not been addressed from this perspective yet.

As the responsibility for day-care provision in Germany lies at the community level (with no overall common guideline for regulations at a higher level), there is no uniform German day-care system. Instead, there is a broad spectrum with large differences on the supply side (but also on the demand side) (Carlsson and Thomsen 2015).

The German day-care system differs with respect to how the allocation of day-care spots to children is organized. Based on certain features, such as the existence of a central institution coordinating the allocation process and a consistent application deadline as well as the degree to which the application of children and the report of available spots are centrally handled, the different mechanisms in place can be characterized as “centralized” and “decentralized”. According to Carlsson and Thomsen, around 70% of the municipalities (among them Aachen) with more than 120,000 inhabitants in Germany use a form of a decentralized allocation. See Table 3 for further information for the cities of the Federal State of Nord Rhine-Westphalia (NRW).
Table 3: Daycare spot allocation systems in NRW (Carlsson and Thomsen 2015)

<table>
<thead>
<tr>
<th>City</th>
<th>Independent co-ordinating institution</th>
<th>Central application of children</th>
<th>Central registration of spots</th>
<th>Fixed application deadline</th>
<th>System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aachen</td>
<td>no</td>
<td>NM</td>
<td>NM</td>
<td>yes</td>
<td>decentralized</td>
</tr>
<tr>
<td>Bielefeld</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>decentralized</td>
</tr>
<tr>
<td>Bochum</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>decentralized</td>
</tr>
<tr>
<td>Bonn</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>centralized</td>
</tr>
<tr>
<td>Cologne</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>centralized</td>
</tr>
<tr>
<td>Dortmund</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>decentralized</td>
</tr>
<tr>
<td>Duisburg</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>decentralized</td>
</tr>
<tr>
<td>Düsseldorf</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>decentralized</td>
</tr>
<tr>
<td>Gelsenkirchen</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>decentralized</td>
</tr>
<tr>
<td>Hagen</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>centralized</td>
</tr>
<tr>
<td>Hamm</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>decentralized</td>
</tr>
<tr>
<td>Herne</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>decentralized</td>
</tr>
<tr>
<td>Krefeld</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>centralized</td>
</tr>
<tr>
<td>Leverkusen</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>decentralized</td>
</tr>
<tr>
<td>Möchengl.</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>centralized</td>
</tr>
<tr>
<td>Mülheim</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>decentralized</td>
</tr>
<tr>
<td>Münster</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>decentralized</td>
</tr>
<tr>
<td>Oberhausen</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>decentralized</td>
</tr>
<tr>
<td>Paderborn</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>decentralized</td>
</tr>
<tr>
<td>Solingen</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>decentralized</td>
</tr>
<tr>
<td>Wuppertal</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>centralized</td>
</tr>
</tbody>
</table>

NM = No monitoring data available at central level.

A study carried by Kennes, Monte & Tumennasan (2013) shows that a well-known German neighbour country, Denmark, currently performs a successful centralized model for childcare slot allocation. Taking a look back in table 1, Denmark was one of the few countries which meets the day care demand; certainly did not come about by chance.

Given the current legislation and habitual processes, it cannot expected a centralized planning to be enforceable as a "top-down" standard. Instead, it is necessary to rely on the proposed system to convince stakeholders to participate by offering unique benefits not available from alternative, decentralized planning approaches. This is illustrated by Figure 3: In the status quo, all slots are allocated through decentralized waiting lists, with
acceptance decisions being the result of iterative, bilateral communication between various pairs of families and day-care providers. In the alternative system to be explored by this project, families can choose to seek slots via the centralized market or via the shadow market or both. Providers have to decide how much capacity to offer on the centralized market and how much capacity to reserve for decentralized allocations.

The effectiveness of centralized planning will be visible through by the stakeholders’ choices over time: If they are satisfied with the central allocation and see that it reduces transactional costs and risks, providers will tend to allocate more capacity centrally. If they expect a high chance of success from the centralized process, families will save themselves the time and effort of applying for day-care slots via the decentralized process.

Figure 3: Alternative Children’s Day-Care Allocation Processes
3. Centralized Model

The objective of the centralized model is to grant every child a day-care slot at one of the available facilities. Within the design of the model, an important aspect to be considered has been the current childcare legislation. According to KiBiZ (NRW-law for early education and support of children), there are three different sorts of groups where the children can be enrolled. These groups, from now on called group constellations, are as follows (Kita, 2016):

- **Group constellation I**: Up to 20 children from the age of 2 until the end of the enrolment, but the number of 2-year-old children has to be between 4 and 6.
- **Group constellation II**: Up to 10 children younger until the age of 2.
- **Group constellation III**: Up to 25 children from the age of 3.

In order to evaluate the satisfaction of the families, which is an important stakeholder of the system, a ranking stated by parents when applying for their children is taken into account. It is also remarkable that the model focuses on different properties of every child. By this means, the age of the children, for instance, will be useful in order to guarantee the law compliance. As facilities are not composed of group constellations (previously aforementioned) but rooms, these group constellations will form different room constellations. That is to say, a room constellation will represent a common room of a child care facility. As a consequence, the upper and lower bounds of the room constellations will be in accordance with KiBiZ; these will depend on which group constellations shape the different room constellations. What is more, every room will have some possible room constellations that could fit in, but not all of them. This will depend on the children distribution in the different facilities. Furthermore, it will be important to meet the expectations of the childcare facilities when performing this slot allocation. So as to fulfil them, it has been considered for every facility a lower and upper bound on the desired minimum and maximum percentage of children at the day-care facility for every property. In figure 4 we can observe a diagram representing how nowadays the children allocation it is performed in the city of Aachen (decentralized) and how it would be with this centralized model.
The Centralized Child Care Slot Allocation Problem

Figure 4: Status quo and proposal of Children’s Day-Care Allocation Processes

Below are shown all the parameters considered in the centralized model.

**Parameters**

- \( C \) (which represent the set of children that have applied for a childcare slot)
- \( T \) (which represent the set of children that are currently in a childcare slot)
- \( K \) (which represent the set of facilities placed in Aachen)
- \( J \) (set of room constellations \( \{1, \ldots, J\} \))
- \( P \) (properties the children have \( \{1, \ldots, J\} \))
- \( W \) (set of rooms \( \{1, \ldots, W\} \))
- \( a^P_c \in \{0,1\} \) (1 only if applying child \( c \) satisfies property \( P \))
- \( e^P_e \in \{0,1\} \) (1 only if existing child \( e \) satisfies property \( P \))
- \( G^j_w \) (each room constl \( j \) is characterized by a total number of slots \( g^j_w \) with property \( P \))
- \( H^j_w \) (each room constl \( j \) has a minimum number of slots \( h^j_w \) with property \( P \))
- \( r_{ck} \in \{1, \ldots, 10\} \) (represents the preference a child has to be allocated in the facility \( k \))
- \( L^P_k, U^P_k \) lower and upper bounds of children with property \( P \) to each facility \( k \)
- \( b_{rwk} \in \{0,1\} \) (1 only if room \( r \) belongs to facility \( k \))
The Centralized Child Care Slot Allocation Problem

\[ n_{tw} \in \{0,1\} \ (1 \text{ only if existing child } t \text{ (when August 2014) belongs to room } w) \]

Variables

This centralized allocation problem can be modelled as mixed integer program. To that end, two binary variables are introduced: \( x_{cw} \) and \( y_{jw} \).

\[ x_{cw} \in \{0,1\} \ (1 \text{ if applying child } c \text{ is allocated to room } w) \]

\[ y_{jw} \in \{0,1\} \ (1 \text{ if room constellation } j \text{ is linked with room } w) \]

Objective function

As a result, the following objective function is obtained:

\[
[\text{MAX}] Z = \sum_{c \in C} \sum_{k \in K} \sum_{w \in W} r_{ck} \cdot b_{wk} \cdot x_{cw}
\]

Taking a look to the function, it is clear that it is of great importance assigning children to facilities where they had indicated a distinctive preference in the application form.

Constraints

This model will be subjected to the following constraints:

\[ \sum_{w \in W} x_{cw} = 1 \quad \forall c \in C \quad (1) \]

\[ \sum_{j \in J} y_{jw} = 1 \quad \forall w \in W \quad (2) \]

\[ \sum_{c \in C} a_{cp} \cdot x_{cw} + \sum_{t \in T} e_{pt} \cdot n_{tw} \leq \sum_{j=1}^{j} g_{wp}^{jp} \cdot y_{jw} \quad \forall p \in P, \forall w \in W \quad (3) \]

\[ \sum_{c \in C} a_{cp} \cdot x_{cw} + \sum_{t \in T} e_{pt} \cdot n_{tw} \geq \sum_{j=1}^{j} h_{wp}^{jp} \cdot y_{jw} \quad \forall p \in P, \forall w \in W \quad (4) \]

\[ \sum_{c \in C} \sum_{w \in W} a_{cp} \cdot b_{wk} \cdot x_{cw} + \sum_{t \in T} \sum_{w \in W} e_{pt} \cdot b_{wk} \cdot n_{tw} \]

\[ \geq L_{kp} \left( \sum_{c \in C} \sum_{w \in W} b_{wk} \cdot x_{cw} + \sum_{t \in T} \sum_{w \in W} b_{wk} \cdot n_{tw} \right) \quad \forall k \in K, \forall p \in P \quad (5) \]
The Centralized Child Care Slot Allocation Problem

\[ \sum_{c \in C} \sum_{w \in W} a^p_c \cdot b_{wk} \cdot x_{cw} + \sum_{t \in T} \sum_{w \in W} e^p_t \cdot b_{wk} \cdot n_{tw} \leq U^p_k \left( \sum_{c \in C} \sum_{w \in W} b_{wk} \cdot x_{cw} + \sum_{t \in T} \sum_{w \in W} b_{wk} \cdot n_{tw} \right) \quad \forall k \in K, \forall p \in P \]  

(6)

\[ \sum_{c \in C} x_{cw} + \sum_{t \in T} n_{tw} \leq 25 \quad \forall w \in W \]  

(7)

\[ x_{cw} \in \{0,1\} \quad \forall w \in W \quad \forall c \in C \]  

(8)

\[ y_{jw} \in \{0,1\} \quad \forall w \in W \quad \forall j \in J \]  

(9)

The first constraint guarantees that every children is allocated to exactly one child care facility. Constraint number two models that every room will be assigned to exactly one room constellation. Constraint three and four guarantees that the set of children assigned to a room respects the lower and upper bounds and the corresponding characteristics of the room constellation. The next two constraints guarantee that the group composition in the day-care facility is satisfied. The last constraint models that the maximum number of children for every room is 25 children.

This model will be implemented and simulated in IBM ILOG CPLEX Optimizer which enables modelling and solving business issues mathematically through powerful algorithms to produce precise and logical decisions (IBM, 2016).

In appendix I it can be observed the whole code written in this software, both the model and the parameters pointing to the input data, which will be thoroughly explained in the next chapter.
4. Data mining

4.1 Data Sets

In order to solve the centralized model previously presented, an input data will be required. The municipality of Aachen has provided to the department of Research Group of Advanced Analytics of RWTH Aachen all relevant information about the childcare business of the city. This data consists of two different Excel sheets: Application Data Set and Contract Data Set.

4.1.1 Contract Data Set

This Excel sheet includes information related to the children who has been sometime enrolled to one of the childcare facilities. Actually, it holds data since 2006. Although there is plenty of relevant information, it has been extracted just some of the fields provided, such as the facility where the children are enrolled, sex and birthday. In the next chapters it will be explained in detail how this data has been manipulated to get an appropriate input data for the model.

4.1.2 Application Data Set

This document contains all applications that have been submitted since 2012. Despite the thorough information, as in the case of the Contract Data Set, only some of the fields will be of our interest. In our particular case study, it has been taken the applications from September 2013 until August 2014. The needed information for the execution of the model are the list of applications during this period, with its respective facility interested, birthday, sex and preference. The calculated age is considered when the simulation is going to be carried out, that is to say August 2014. In the subsequent sections it will be clarified how this data has been handled to obtain a suitable input data for the model.

4.2 Input Data

Once the data sources (contract and application data set) have been introduced, the next step is to transform all this information, so that the model can properly understand the
data in them. Below (see table 4) is explained in detail where all parameters come from and how these Excel sheets have been filled.

Table 4: Input data preparation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Excel sheet</th>
<th>Setting Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>ApplyingChild</td>
<td>From the application data set, the data is filtered so that it is only considered children who applied from September 2013 to August 2014. The age of this children, as it has been abovementioned, it is calculated when August 2014. This has been possible adding a column of the actual date and by means of Excel formulation the age is obtained.</td>
</tr>
</tbody>
</table>
| $E$       | ExistingChild | From the contract data set, we previously analyse the children who were in October 2013 at one day by filtering the appropriate column. Afterwards, the age of these children is calculated when August 2014. This has been possible adding a column of the actual date and by means of Excel formulation the age is obtained. At this point, there will be 2 scenarios:
- Only children younger than 7 will stay enrolled (scenario1)
- Only children younger than 6 will stay enrolled (scenario2)
It has been assumed that children who were in October 2013 and were older than 7 years old, would not be considered in the input data. |
| $K$       | Facility    | From the contract data set it can be extracted the whole list of childcare facilities. By means of a pivot table, it is been observed the amount of children every facility has. It is been assumed that facilities with less than 10 children will not be considered in the input data. |
| $J$       | RoomConstellation | In this case, this sheet has not been fed by neither the application nor the contract data set. In this case it has been decided to constitute a total number of 11 room constellations, being composed of the different type of group constellations (previously explained). |
| $P$       | i.e ApplyingChild | This properties can be found in more than one Excel sheet. In the end, it has been considered 6 properties, so that it will enable the accomplishment of some lower and upper bounds extracted from KiFöG. |
| $R$       | RoomLocation _VBA | From RoomLocation_VBA it can be observed the total amount of existing rooms in the city of Aachen. |
| $a_c^p$   | ApplyingChild | In order to fill this sheet, it has been used Vlookup and If conditions from data of the Application Data Set. |
| $e_t^p$   | ExistingChild | As in the previous case, by means of Vlookup and If conditions the data has been completed, but in this case the data source is the Contract Data Set. This sheet will vary on the scenario considered (remember there is 2 scenarios depending on when we consider the leaving age of children). |
### The Centralized Child Care Slot Allocation Problem

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{lw}^{lp}$</td>
<td>RoomConstellation</td>
<td>This parameter represents the upper bounds of the different room constellations. In this case it is been respected the upper bounds of the group constellations that form every room constellation.</td>
</tr>
<tr>
<td>$h_{lw}^{lp}$</td>
<td>RoomConstellation</td>
<td>In this case this parameter represents the lower bounds of the different room constellations. As in the previous case, it respects the lower bounds of the group constellations that form every room constellation. i.e room constellations that are formed with group constellation type 1 will have a lower bound for the group of children aged 2.</td>
</tr>
<tr>
<td>$r_{ck}$</td>
<td>Preference</td>
<td>For this parameter, data has been extracted from the application data set. By means of vlookups and concatenation it has been filled all cells. It is important to point out that the preference column of the application data set does not follow the logic of this ranking parameter. This parameter ranges from 1 to 10 (being 10 the top preference, and 1 the lowest). However in the application data set the values range from 1 to 6 (being 1 the top preference and 6 the lowest). Therefore, this data has been adapted to our particular case. When a child has not applied to one facility its corresponding ranking will be -1000.</td>
</tr>
<tr>
<td>$U_k^p$</td>
<td>Facility</td>
<td>This parameter represents the upper bound of children with property p every facility permits. In order to establish this bound as more realistic as possible, it has been used the real assignment information of the contract data set (through vlookups) and an additional 10% margin has been given.</td>
</tr>
<tr>
<td>$L_k^p$</td>
<td>Facility</td>
<td>In this case, this parameter represents the lower bound of children with property p every facility permits. The logic is the same of the previous case but this 10% margin is lowered and not added like before.</td>
</tr>
<tr>
<td>$b_{wk}$</td>
<td>RoomLocation_VBA</td>
<td>This parameter indicates how many rooms every facility has. In order to fill appropriately the cells, it is been analysed through a pivot table the amount of children every facility has in maximum capacity (a day in October 2013). It has been necessary to program a Macro in Visual Basic in order to perform this task. This subroutine has taken into account that the maximum capacity of rooms is 25 children. This Macro called RoomLocation can be found in Appendix II.</td>
</tr>
<tr>
<td>$n_{tw}$</td>
<td>ExistingChildrenPlacement</td>
<td>This parameter shows in which room the existing children are assigned. It is important that this allocation is aligned with the RoomLocation information and contract data set. That is to say, if child A is assigned to Facility K, then this child has to be allocated in a room belonging to Facility K. Besides this, it will also important to consider that a child can only be assigned in one room and there are no more than 25 children in any room. To perform this, it has been programed through Visual Basic a subroutine called “room_assignation” that can be found in Appendix II.</td>
</tr>
</tbody>
</table>
Once all parameters have been introduced, it is interesting to focus on the different Excel sheets which will feed the proper execution of the model in CPLEX. These Excel sheets will be as follows:

- **Parameters**

This sheet specifies the range of the different parameters, so that the sums in the code are defined. Table 5 shows the format of this sheet.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

The column *Parameters* includes all parameters from the model and the column *Amount* indicates the number of the different parameters specified on the left column, i.e Parameter: properties, Amount: 6.

- **RoomLocation_VBA**

This table indicates which rooms every facility has. As it has been abovementioned, this table is filled through a subroutine programed in Visual Basic. In table 6 can be observed the format of the table.

<table>
<thead>
<tr>
<th>Room/Facility</th>
<th>Facility_k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room_w</td>
<td>...</td>
</tr>
</tbody>
</table>

- **ExistingChildrenPlacement**

This table shows where the existing children in August 2014 are allocated. It is important to remember this table will vary with the scenario studied. In Table 7 appears the format of this table.

<table>
<thead>
<tr>
<th>Existing Children/Room</th>
<th>Room_w</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child_t</td>
<td>...</td>
</tr>
</tbody>
</table>
- **RoomConstellation**

In this Excel sheet it is presented a table with information related to the upper and lower bounds every room constellation has. The disposition of this table is shown in table 8.

<table>
<thead>
<tr>
<th>Room Constellation/Bound_p</th>
<th>Lower bound_p</th>
<th>Upper bound_p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room Constellation_j</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

- **ApplyingChild**

This table indicates the properties from all children who have applied from September 2013 to August 2014. In table 9 can be observed the disposition of the table.

<table>
<thead>
<tr>
<th>Applying Children/Property</th>
<th>Property_p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child_c</td>
<td>...</td>
</tr>
</tbody>
</table>

- **ExistingChild**

In this case, table 10 shows the properties from the existing children in August 2014. As for the ExistingChildrenPlacement sheet, it will vary with the scenario analysed.

<table>
<thead>
<tr>
<th>Existing Children/Property</th>
<th>Property_p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child_t</td>
<td>...</td>
</tr>
</tbody>
</table>

- **Facility**

In this Excel sheet appears the list of facilities with the lower and upper bounds for all properties analysed (see format in table 11).

<table>
<thead>
<tr>
<th>Facility/Bound_p</th>
<th>Lower bound_p</th>
<th>Upper bound_p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facility_k</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
• **Preference**

In this last sheet, there appear the list of applicants from September 2013 until August 2014 with the preference stated for all the facilities. In case the child has not applied for a facility, the value will be -1000 as explained in the previous section. Table 12 shows the format of this Excel sheet.

**Table 12: Format Preference Excel sheet**

<table>
<thead>
<tr>
<th>Applying Children/Facility</th>
<th>Facility_k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child_c</td>
<td>...</td>
</tr>
</tbody>
</table>

Before moving to the next chapter, it is important to explain a consideration it is been taken into account to solve the model. In order to satisfy the demand, we have created an additional facility with a subsequent room with unlimited capacity. This room and facility are called “dummy room” and “dummy facility” from now on because they are not a real room nor a real facility. It is created in order to visualize the possible dissatisfaction of the demand and its magnitude and to make the model feasible. Different assumptions of these room and facility is that it has no upper or lower bounds, neither capacity limit. The assumed ranking for every children has been considered 0 (lower than any of the applications, but higher than the case where the child has not applied to any of the facilities).

**4.3 Further data analysis**

Besides this, from the data sets can be extracted other interesting information that may provide a better insight of the child care market in the city of Aachen.

In order to establish the lower and upper bounds of the different facilities for the different properties (age and sex), in the Contract Data Set have been created some Pivot Tables which permit the visualization of these figures in an intuitive and dynamic way. Sheets of this book are “%Age distribution” and “%Gender distribution”. Hence, these sheets have been used when building the input data (cells formulation), specifically sheet “facility” of the input data.

On the other hand, it is interesting to take a close look to the application data set. In this sheet one of the fields are the priority stated by every child and facility. This priority
represents the ranking parameter placed in the objective function. As it has been explained this ranking ranges from 1 to 10. However, in the application data set it does not follow the same logic, so that it will be necessary to adapt this data by means of “Replace” functionality. Table 13 specifies this required adaptation.

<table>
<thead>
<tr>
<th>Priority Application Data Set</th>
<th>Equivalent ranking for CPLEX Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

As in the case of the Contract Data Set, in the Application Set sheet it has been created a pivot table which enables an intuitive and dynamic visualization of the child care applications (sheet “Pivot table” of Application Data Set Excel book). This pivot table shows the number of applications for every facility, having the possibility to filter this data by priority.

The facility which received more amount of applications within the period from September 2013 to August 2014 was Stštdt. Tageseinrichtung fšr Kinder   Boxgraben. Surprisingly, this facility possess one of the lowest capacities (can be seen in Contract Data Set). Filtering by the number of applications with top priority, this same facility appears at the top, which makes full sense.

For further information these pivot tables can be easily modified and can provide a thorough insight in all data presented so far. In appendix III there are two Excel workbooks attached (Contract and Application Data Set) in order to manipulate and better visualize these tables.
5. Results and analysis

5.1 Results

The next step is to implement the model previously detailed in IBM ILOG CPLEX Optimizer. It is important to remember that in this study there are going to be two scenarios: scenario I where children leave the childcare facilities before 7 years old and scenario II where children leave the facilities before they are 6 years old. Depending on the scenario the results will presumably differ as in the case of the 2nd scenario there will appear more free slots, hence the demand will be more easily fulfilled. In appendix I can be seen the code implemented in the IBM software, both the model and the data connection to Excel (it is shown scenario II).

Observations:

After executing the model in IBM ILOG CPLEX, the model has not been able to find any optimal solution. Actually it is been running for hours without finding the desired solution. The reason of this is because of the luck of capacity of the computer used. In order to make sure of this, we have tried with a smaller Input Data and the model has given a feasible and optimal solution with the same model. Therefore, we have considered to work with not the whole input data built with all information of the city of Aachen, but with a representative sample of this with real data. For the two scenarios, this is the magnitude of the input data of the final study (see table 14):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number (scenario I)</th>
<th>Number (scenario II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>958</td>
<td>958</td>
</tr>
<tr>
<td>E</td>
<td>1031</td>
<td>721</td>
</tr>
<tr>
<td>K</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>J</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>P</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>W</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

We have taken the biggest facilities of Aachen, so that the sample is as more realistic as possible.
Scenario I

Once the model is executed in IBM ILOG CPLEX, after one hour of looking for an optimal solution, the software indicates the model has no solution. In order to solve this problem, we have added for all constraints inequalities relaxation variables. These variables would certainly make the model feasible and they would show which constraints made the model unfeasible at first. The process of relaxation works as follows (example):

*Taking into account “feasible” is a relaxation variable:*

\[
\text{Objective function: [MAX] } Z = \text{expression} - \text{feasible}^2
\]

\[
\text{Constraint: expression } \geq \text{expression} - \text{feasible}
\]

After performing this modification, the model gives a solution after 1 minute and 27 seconds. The complete results can be found in the Excel file “Input Data Simulation” in “Xcw1” and “Yjw1” sheets. Figure 5 shows the assignment of all the applying children. These results show that the offer is much lower than demand, so that it would be very convenient that children would have to leave sooner their child care slot.

![Figure 5: Children assignment scenario 1](image)

Besides this, figure 6 shows the distribution of room constellations assigned to all rooms. Surprisingly, type 6 of room constellation is the one which abound in the different rooms.
Finally, table 15 shows the relaxation variables abovementioned. These results indicate the lower and upper bounds of the facilities and the upper bound of the room constellations cannot be satisfied with these input data. A solution for this fact would be making the range of these bounds larger.

Table 15: Relaxation variables scenario 1

<table>
<thead>
<tr>
<th>Relaxation variables</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>aaa</td>
<td>25</td>
</tr>
<tr>
<td>bbb</td>
<td>0</td>
</tr>
<tr>
<td>ccc</td>
<td>12,1116279</td>
</tr>
<tr>
<td>ddd</td>
<td>12,1116279</td>
</tr>
<tr>
<td>eee</td>
<td>0</td>
</tr>
</tbody>
</table>

Scenario II

As in the case of scenario I, we perform a relaxation in the model so that we can properly compare these two scenarios.

In this case, the model has found a solution after 2 minutes 31 seconds. In this scenario, children have to leave their child care slot at the age of 6, that is to say, one year earlier compared to scenario I. This fact will make the offer of slots bigger, so that the applying children will be easily assigned. Figure 7 shows that there is a higher number of assigned children but still there is a big quantity who would be allocated to the dummy facility.
On the other hand, figure 8 indicates the distribution of room constellations assigned to all rooms. If we take a look back to scenario I we will see there has been a big change in this result. Actually room constellation type 1 is the one with more assigned rooms.

As in the case of scenario 1, some of the relaxation variables take a value as we can see in table X. In the following “analysis” section we will explain and analyse the logic of these results.
Table 16: Relaxation variables scenario 2

<table>
<thead>
<tr>
<th>Relaxation variables</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>aaa</td>
<td>25</td>
</tr>
<tr>
<td>bbb</td>
<td>0</td>
</tr>
<tr>
<td>ccc</td>
<td>9,22790698</td>
</tr>
<tr>
<td>ddd</td>
<td>9,22790698</td>
</tr>
<tr>
<td>eee</td>
<td>0</td>
</tr>
</tbody>
</table>

5.2 Analysis

Owing to the fact that the input data does not correspond to the totality of the market of the city of Aachen, the conclusions extracted from the results will not be as reliable as we wish. However, there is always useful information we can obtain from the simulation of the model.

Apparently, observing figures 5 and 7 (corresponding to scenario I and II respectively), it looks like the city of Aachen has a dramatic problem with the supply of childcare slots. Nevertheless, it is important to take into account that the sample analysed only includes the largest childcare facilities of Aachen, being the majority of them, the most desired centres. If we take a look at the supply and demand numbers it is easy to understand these results. Table 17 shows a comparison between the two scenarios in terms of % of satisfied demand but at the same time the ratio supply slots compared to demand slots.

Table 17: Demand fulfilment for the simulation

<table>
<thead>
<tr>
<th>% satisfied demand</th>
<th>Supply slots/demand slots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario I</td>
<td>20,25%</td>
</tr>
<tr>
<td>Scenario II</td>
<td>52,6%</td>
</tr>
</tbody>
</table>

The fact that many applying children are assigned to a dummy facility is because the demand is much higher than the slots available. True is that we assumed a number of rooms and a capacity of 25 for each one obtaining a total distribution of facility-rooms. Although there may be a slight deviation, it is still reliable this reasoning.

However, in this case it would be even more reliable to analyse these demand fulfilment with the whole data of the city of Aachen. Table 18 shows these figures.
Table 18: Demand fulfilment for the sample

<table>
<thead>
<tr>
<th>Scenario</th>
<th>% satisfied demand</th>
<th>Supply slots/demand slots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario I</td>
<td>Not simulated</td>
<td>0.81</td>
</tr>
<tr>
<td>Scenario II</td>
<td>Not simulated</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Although we do not possess the demand fulfilment, we can observe the number offer slots is much closer to the demand slots. This means that it is true that the city of Aachen has slightly a higher demand than offer, but it is not as dramatic as the results may show.

Regarding the results from the relaxation variables introduced, we can extract two conclusions:

- The first one is that it is been necessary to add some relaxation in the model to get a solution. Otherwise, the model would keep running indefinitely seeking an optimal solution without success. That is to say, these relaxation variables have enabled the simulation finding much faster a solution.

- The other one is that the relaxation variables that have taken a value different from 0 are the ones that relax the constraints from the lower and upper bounds. The fact that the range of these bounds is relatively narrow, has tremendously complicated the simulation of the model. At the same time these relaxation variables have possibly impacted the solution of the decision variable of room constellations assignment, that is why the results from this variable may differ from reality.
6. Conclusions

After having performed the study, it is paramount to draw some conclusions. Therefore, it is convenient to analyse the grade of fulfilment of the objectives set at the beginning of the Thesis.

The first part of the project has been focused on contextualising and defining the relevance of performing further research in the areas of capacity planning and slot allocation in the domain of childcare slot allocation. We have seen it is an arising social problem derived, among others, from women’s in the workforce introduction of the last decades. We have seen that KiFöG law have directly impacted the relevance of slot allocation in the German childcare market, but still there is evidence that not only Germany faces this challenge of capacity planning.

By means of creating Pivot Tables in Excel, it has been possible to get a dynamic image of the information of the contract and application data sets, which enabled us to gather insight of the demand side of this domain, such as the preferable childcare depending on children age or sex.

An important objective of the Thesis was to create a centralized model for the childcare slot allocation problem, pursuing the satisfaction of all stakeholders involved in this social issue. We can say it is been achieved although the model could be even more compliant with reality. The model seeks the satisfaction of not just the families (including a ranking of the applications) but also of the facilities as the lower and upper bounds have been adjusted to the real values.

Another aim proposed at the beginning of the study was to perform a simulation of the model and analyse its results. This objective has been partially accomplished, due to the fact that the input data has not been the whole information of the city of Aachen but a sample of it. This fact diminishes reliability of the results obtained. Nevertheless, an important conclusion of this is that the city of Aachen possess an offer slot shortage. A contingency solution while the city increases its childcare supply would be implementing scenario II, that is to say, setting up the childcare leaving age at 6 years old.

Last but not least, although there is still a long way to go, this project has definitely contributed to solving a problem that is of high current societal relevance.
7. References

BBC (2015) *Germany passes Japan to have world’s lowest birth rate – study.*


Google Public Data (2014)
https://www.google.es/publicdata/explore?ds=d5bncppjof8f9&m=met_y=sp_dyn_tfrt_in&idim=country:DEU:GBR:FRAl=es&dl=es


8. Appendix

8.1 Appendix I

In this appendix we include the code of the whole model in IBM ILOG CPLEX. In this case, it refers to scenario II. At the beginning there is the definition of the parameters and variables. Afterwards, we can see the objective function and constraints. At the end there is the data connection from the Input Data to the simulating software and there are also the commands to write to solution in the same document of the Input Data.

```plaintext
// parameters
int C=...; // number of applying children
int T=...; // number of existing children
int K=...; // number of facilities
int J=...; // number of room constellations
int P=...; // number of properties
int W=...; // number of rooms

range app_children=1..C;
range exist_children=1..T;
range facilities=1..K;
range room_const=1..J;
range properties=1..P;
range rooms=1..W;

float a[app_children][properties]=...; // if applying children c possess property p
float e[exist_children][properties]=...; // if existing children e possess property p
float g[room_const][properties]=...; // upper bound slots of every room constellation by property
float h[room_const][properties]=...; // lower bound slots of every room constellation by property
float r[app_children][facilities]=...; // ranking given by family to every facility
float L[facilities][properties]=...; // lower bound every facility has for every property
float U[facilities][properties]=...; // upper bound every facility has for every property
float b[rooms][facilities]=...; // if a room belongs to a facility
float n[exist_children][rooms]=...; // in which room stay existing children

//decision variables

dvar boolean y[room_const][rooms];
dvar boolean x[app_children][rooms];

//relax variables

dvar float aaa;
dvar float bbb;
dvar float ccc;
```
dvar float ddd;
dvar float eee;

// objective function

maximize sum(c in app_children, k in facilities, w in rooms) (r[c][k]*b[w][k]*x[c][w]) -1000*(aaa+bbb+ccc+ddd+eee);

// constraints

subject to {

forall(c in app_children)
    sum(w in rooms)x[c][w]=1; // a children will be only assigned to one room

forall(w in 1..49)
    sum(j in room_const)y[j][w]=1;

forall(w in 1..49, p in properties)
    sum(c in app_children)a[c][p]*x[c][w]+sum(t in exist_children)e[t][p]*n[t][w]<=sum(j in room_const)g[j][p]*y[j][w]+aaa;

forall(w in 1..49, p in properties)
    sum(c in app_children)a[c][p]*x[c][w]+sum(t in exist_children)e[t][p]*n[t][w]>=sum(j in room_const)h[j][p]*y[j][w]-bbb;

forall(k in 1..12, p in properties)
    sum(c in app_children, w in 1..49)a[c][p]*b[w][k]*x[c][w]+sum(t in exist_children, w in 1..49)e[t][p]*b[w][k]*n[t][w]<=L[k][p]*(sum(c in app_children, w in 1..49)b[w][k]*x[c][w]+sum(t in exist_children, w in 1..49)b[w][k]*n[t][w]))-ccc;

forall(k in 1..12, p in properties)
    sum(c in app_children, w in 1..49)a[c][p]*b[w][k]*x[c][w]+sum(t in exist_children, w in 1..49)e[t][p]*b[w][k]*n[t][w]<=U[k][p]*(sum(c in app_children, w in 1..49)b[w][k]*x[c][w]+sum(t in exist_children, w in 1..49)b[w][k]*n[t][w]))+ddd;

forall(w in 1..49)
    sum(c in app_children)x[c][w]+sum(t in exist_children)n[t][w]<=25+eee;

forall(w in 50..50)
    sum(c in app_children)x[c][w]+sum(t in exist_children)n[t][w]<=10000;
}

// connect book from Excel

SheetConnection HojaExcel("Input Data Simulation.xlsm");

// data connection
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C from SheetRead(HojaExcel, "Parameters!C1");
T from SheetRead(HojaExcel, "Parameters!C2");
K from SheetRead(HojaExcel, "Parameters!C3");
J from SheetRead(HojaExcel, "Parameters!C4");
P from SheetRead(HojaExcel, "Parameters!C5");
W from SheetRead(HojaExcel, "Parameters!C6");
a from SheetRead(HojaExcel, "ApplyingChild!B2:G959");
e from SheetRead(HojaExcel, "ExistingChildScen2!B2:G722");
g from SheetRead(HojaExcel, "RoomConstellation!I2:N12");
h from SheetRead(HojaExcel, "RoomConstellation!C2:H12");
r from SheetRead(HojaExcel, "PreferenceSim!B2:N959");
L from SheetRead(HojaExcel, "Facility!B2:G14");
U from SheetRead(HojaExcel, "Facility!H2:M14");
b from SheetRead(HojaExcel, "RoomLocation_VBA!B2:N51");
n from SheetRead(HojaExcel, "ExistChildPlaceScen2!B2:AY722");

// writing solution
x to SheetWrite(HojaExcel,"Xcw2!B2:AY959");
y to SheetWrite(HojaExcel,"Yjw2!B2:AY12");
aaa to SheetWrite(HojaExcel,"relaxresults2!B1");
bbb to SheetWrite(HojaExcel,"relaxresults2!B2");
ccc to SheetWrite(HojaExcel,"relaxresults2!B3");
ddd to SheetWrite(HojaExcel,"relaxresults2!B4");
eee to SheetWrite(HojaExcel,"relaxresults2!B5");
8.2 Appendix II

In Appendix II we will see the macros formulated in order to build the input data to an understandable format for the model. The first subroutine (RoomLocation) writes in which facility the different rooms belong to. The second subroutine (room_assignment) calculates in which room all existing children are assigned. And the last subroutine (Room(Constellations)) calculates which possible room constellations a room can have. However, this last parameter has not been used in the simulation of the model because this information together with the other parameters made directly the model unsolvable.

**Sub RoomLocation()**

    Dim facility As String
    Dim i As Integer
    Dim j As Integer
    Dim NumChildren As Integer

    j = 1
    While Worksheets("RoomLocation_VBA").Cells(1, j + 1).Value <> ""
        facility = Worksheets("RoomLocation_VBA").Cells(1, j + 1)
        NumChildren = Application.WorksheetFunction.VLookup(facility, Worksheets("AgeDistribution").Range("A2:B116"), 2, False)
        i = 1
        While Worksheets("RoomLocation_VBA").Cells(i + 1, 1).Value <> ""
            If (NumChildren > 0 And Application.WorksheetFunction.Sum(Worksheets("RoomLocation_VBA").Range(Cells(i + 1, 2), Cells(i + 1, j + 1))) = 0) Then
                Worksheets("RoomLocation_VBA").Cells(i + 1, j + 1).Value = 1
                NumChildren = NumChildren - 25
            Else
                Worksheets("RoomLocation_VBA").Cells(i + 1, j + 1).Value = 0
            End If
        i = i + 1
    Wend
    j = j + 1
Wend
End Sub
Sub room_assignation()
Application.ScreenUpdating = False
Dim nen As String
Dim guarderia As String
Dim edat As Integer
Dim sexe As String
Sheets("ExistingChildrenPlacement_VBA").Select
r_total = Columns(1).Find("TOTAL").Row
Cells(1, 1).End(xlToLeft).Select
last_column = Selection.Column
a = Cells(r_total - 1, last_column).Address
Range("B2:" & a).Select
Selection.Value = 0
'ActiveSheet.Paste
Sheets("InfoChildrenStillAssigned").Select
f = 2
last_row = Cells(1, 1).End(xlDown).Row
Do While f <= last_row
    Sheets("InfoChildrenStillAssigned").Select
    guarderia = Cells(f, 1).Value
    nen = Cells(f, 2).Value
    edat = Cells(f, 3).Value
    sexe = Cells(f, 4).Value
    c_room = 1
    f_room = 1
    assignat = False
    exist_room = True
    Do While Not assignat And exist_room
        Sheets("RoomLocation_VBA").Select
        Cells.Find(What:=guarderia, After:=Cells(1, 1), LookIn:=xlFormulas, LookAt:=}
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xlPart, SearchOrder:=xlByRows, SearchDirection:=xlNext, MatchCase:=False
_, SearchFormat:=False).Activate

c_room = Selection.Column

On Error GoTo ErrHandler:

Columns(c_room).Find(What:=1, After:=Cells(f_room, c_room), LookIn:=xlFormulas, _
  LookAt:=xlPart, SearchOrder:=xlByRows, SearchDirection:=xlNext, _
  MatchCase:=False, SearchFormat:=False).Activate

f_room = Selection.Row

'    If exist_room Then
'        Columns(c_room).Find(What:=1, After:=Cells(f_room, c_room), LookIn:=xlFormulas, _
'            LookAt:=xlPart, SearchOrder:=xlByRows, SearchDirection:=xlNext, _
'            MatchCase:=False, SearchFormat:=False).Activate
'    f_room = Selection_Row
'    If f_room > 100100 Then
'        assignat = True
'    End If
'
room = Cells(f_room, 1).Value

If exist_room Then
    Sheets("ExistingChildrenPlacement_VBA").Select
total_ = Cells(r_total, f_room).Value
    If total_ < 25 Then
        Cells(f, f_room).Value = 1
        assignat = True
    End If
End If

End If

Loop
    f = f + 1
Loop

Application.ScreenUpdating = True
Sub Room_Constellations()

    Application.ScreenUpdating = False
    Sheets("ExistingChildrenPlacement_VBA").Activate
    r = 2
    n_r = Cells(1, 1).End(xlToLeft).Column  
    ' n_r = 2
    n_total_nens = Cells(1, 1).End(xlDown).Row - 1
    'n_total_nens = 2
    Do While r <= n_r
        exist_nen = True
        nen = 1
        n_child = 0
        n_male = 0
        n_female = 0
        n_under_2y = 0
        n_2y = 0
        n_over_2y = 0
        Do While nen <= n_total_nens
            Sheets("ExistingChildrenPlacement_VBA").Activate
            Columns(r).Find(What:=1, After:=Cells(nen, r), LookIn:=xlFormulas, _
            LookAt:=xlPart, SearchOrder:=xlByRows, SearchDirection:=xlNext, _
            MatchCase:=False, SearchFormat:=False).Activate
            nen = Selection.Row

ErrHandler:
    exist_room = False
Resume Next
End Sub
If nen <= n_total_nens Then
    id_nen = Cells(nen, 1).Value
    Sheets("InfoChildrenStillAssigned").Activate
    Columns(2).Find(What:=id_nen, After:=Cells(1, 2), LookIn:=xlFormulas, _
    LookAt:=xlPart, SearchOrder:=xlByRows, SearchDirection:=xlNext, _
    MatchCase:=False, SearchFormat:=False).Activate
    row_nen = Selection.Row
    edat_nen = Cells(row_nen, 3).Value
    sexe_nen = Cells(row_nen, 4).Value
    n_child = n_child + 1
    If sexe_nen = "weiblich" Then
        n_female = n_female + 1
    Else
        n_male = n_male + 1
    End If
    If edat_nen < 2 Then
        n_under_2y = n_under_2y + 1
    ElseIf edat_nen = 2 Then
        n_2y = n_2y + 1
    Else
        n_over_2y = n_over_2y + 1
    End If
End If
Loop
'G1
Sheets("Room_VBA").Activate
If (n_child <> 0 And n_2y >= 4 And n_child <= 20 And n_male <= 15 And n_female <= 15 And n_under_2y <= 0 And n_2y <= 6 And n_over_2y <= 20) Then
    Cells(r, 2).Value = 1
Else
    Cells(r, 2).Value = 0
End If

'G2

If (n_child <> 0 And n_2y >= 8 And n_child <= 40 And n_male <= 30 And n_female <= 30 And n_under_2y <= 0 And n_2y <= 12 And n_over_2y <= 40) Then
    Cells(r, 3).Value = 1
Else
    Cells(r, 3).Value = 0
End If

'G3

If (n_child <> 0 And n_2y >= 4 And n_child <= 30 And n_male <= 23 And n_female <= 23 And n_under_2y <= 10 And n_2y <= 16 And n_over_2y <= 20) Then
    Cells(r, 4).Value = 1
Else
    Cells(r, 4).Value = 0
End If

'G4

If (n_child <> 0 And n_2y >= 4 And n_child <= 40 And n_male <= 31 And n_female <= 31 And n_under_2y <= 20 And n_2y <= 26 And n_over_2y <= 20) Then
    Cells(r, 5).Value = 1
Else
    Cells(r, 5).Value = 0
End If

'G5

If (n_child <> 0 And n_2y >= 4 And n_child <= 45 And n_male <= 36 And n_female <= 36 And n_under_2y <= 0 And n_2y <= 6 And n_over_2y <= 45) Then
    Cells(r, 6).Value = 1
Else
    Cells(r, 6).Value = 0
End If

'G6

If (n_child <> 0 And n_2y >= 0 And n_child <= 10 And n_male <= 8 And n_female <= 8 And n_under_2y <= 10 And n_2y <= 10 And n_over_2y <= 0) Then
Cells(r, 7).Value = 1
Else
    Cells(r, 7).Value = 0
End If
'G7
If (n_child <> 0 And n_2y >= 0 And n_child <= 20 And n_male <= 16 And n_female <= 16 And n_under_2y <= 20 And n_2y <= 20 And n_over_2y <= 0) Then
    Cells(r, 8).Value = 1
Else
    Cells(r, 8).Value = 0
End If
'G8
If (n_child <> 0 And n_2y >= 0 And n_child <= 30 And n_male <= 24 And n_female <= 24 And n_under_2y <= 30 And n_2y <= 30 And n_over_2y <= 0) Then
    Cells(r, 9).Value = 1
Else
    Cells(r, 9).Value = 0
End If
'G9
If (n_child <> 0 And n_2y >= 0 And n_child <= 35 And n_male <= 29 And n_female <= 29 And n_under_2y <= 10 And n_2y <= 10 And n_over_2y <= 25) Then
    Cells(r, 10).Value = 1
Else
    Cells(r, 10).Value = 0
End If
'G10
If (n_child <> 0 And n_2y >= 0 And n_child <= 25 And n_male <= 21 And n_female <= 21 And n_under_2y <= 0 And n_2y <= 0 And n_over_2y <= 25) Then
    Cells(r, 11).Value = 1
Else
    Cells(r, 11).Value = 0
End If
'G11

If (n_child <> 0 And n_2y >= 0 And n_child <= 50 And n_male <= 42 And n_female <= 42 And n_under_2y <= 0 And n_2y <= 0 And n_over_2y <= 50) Then

    Cells(r, 12).Value = 1

Else

    Cells(r, 12).Value = 0

End If

Cells(r, 13).Value = n_child
Cells(r, 14).Value = n_male
Cells(r, 15).Value = n_female
Cells(r, 16).Value = n_under_2y
Cells(r, 17).Value = n_2y
Cells(r, 18).Value = n_over_2y

r = r + 1

Loop

Application.ScreenUpdating = True

End Sub
8.3 Appendix III

In this appendix there are two Excel workbooks attached (Contract and Application Data Set) in order to manipulate and better visualize some pivot tables created in order to gather further information of the raw data and to establish, among others, more exact bounds.