“Low-cost trajectories using Dynamical Systems”

[1] Josep M. Cors, [2] Esther Barrabés

[1] Dept. de Matemàtica Aplicada III, UPC (cors@epsem.upc.edu)

[2] Dept. d’Informàtica i Matemàtica Aplicada, UdG (barrabes@ima.udg.edu)

Escola Politècnica Superior d’Enginyeria de Manresa.

1. Introduction.

The study of astrodynamics has gain the attention of the scientific community specially the last decades. Not only the Earth-Moon space is becoming more common for space travels, also the whole Solar System is on the focus for interplanetary missions. The use of satellites as transport vehicles for scientific purposes requires the understanding of the fundamentals of the dynamics that govern the space navigation.

The classical approach for an interplanetary spacecraft trajectory has been based on patched-conics transfers (a sequence of pieces of joined ellipses) and planetary swing-bys (passages close to a planet in order to change the velocity), followed by numerical optimization procedures. Such kinds of trajectories were used for the Apollo lunar missions and for the Voyagers Grand Tour (visiting Jupiter and Saturn).

The space missions are gaining more complexity, willing to perform approaches to different objects in their journeys (planets, meteorites ...), staying long time travelling or, on the contrary, maintaining the position around a fixed point. One important restriction in designing a space mission is the low fuel requirements for the total journey.

Dynamical Systems tools allow the analysis of the natural dynamics of the problem in a systematic way. The *space manifold dynamics* is used to design "low-energy" orbits, cheap station keeping procedures and other strategies, fitting the required mission constraints. One example is the SOHO mission. The SOHO spacecraft was inserted in an inclined orbit (known as *halo* orbit) around an equilibrium point between the Earth and the Sun in order to perform studies of the Earth-Sun interactions. MAP, Genesis, and Herschel-Plank missions are other examples. Genesis was the first mission whose trajectory was completely designed using only Dynamical Systems tools.

Considering the Sun-Earth system, the equilibrium points of the dynamical model (a restricted three-body problem, where two main bodies and a spacecraft are taken into account) are the more important objects to start with. Around the equilibrium points, there exist different families of periodic and quasi-periodic orbits that are unstable, so associated to them there exists stable and unstable manifolds responsible of the transport between different regions. The orbits ‘inside’ the manifolds (that can be viewed as tubes) can travel from one region to a different one, depending on their relative position with respect the other manifolds. In this way, the intersections of the stable and unstable manifolds play an important role in the existence of the so-called transit or non-transit orbits.

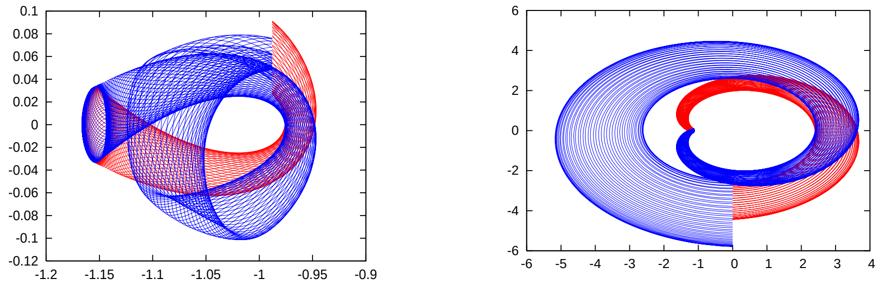

Figure: Invariant manifolds in configuration space (x,y) associated to a periodic orbit of the Restricted Three Body Problem Sun-Earth-Spacecraft. The trajectories inside the tubes can reach different regions depending on the branch and the intersections between them. An orbit inside a "red" branch first escape from a neighborhood of the equilibrium point, but can return to it if it goes inside the "blue" branch at its intersection with the red one.

2. A limit case of a planetary ring.

We create and study a model of a thin ring about a large planet, unaffected by any other bodies, with motion governed by Newtonian gravitation only. Surprisingly, our model has interesting internal dynamics and points out natural relationships between the stability of the motion of ring particles, the width of the ring, and the ratio of the masses of the ring and the planet.

The ring can be considered formed by a large number of bodies, which in a proper reference system can be located in a vertical axis distributed uniformly. Around each body, there exists an equilibrium point from which a family of unstable periodic orbits is born. The unstable and stable manifolds of the whole set of bodies on the ring are responsible of the dynamics of the ring. On one side, the particles (or a spacecraft) can travel from a neighborhood of a body to another one of a different particle (heteroclinic connections), or move from one side of the ring to the other one just following a specific branch of an invariant manifold. On another side, the study of the invariant manifolds gives an idea of the dynamics inside the ring, as the motion of the particles being trapped by the bodies of the ring is governed by the dynamics of the manifolds. In this way, the study of the behavior of the invariant manifolds associated to the periodic orbits around the equilibrium points gives an idea of the width of the ring.

3. Conclusions.

The space manifold dynamics is a tool that uses the existence and behavior of unstable objects of a dynamical system to construct specific trajectories of "low-cost" on energy (or fuel requirements): just following the dynamics, these kinds of "highways" allow to visit different regions of the space at no energy cost. Unfortunately, there is always a price: in this case, the total time of the trajectories are usually too big.

Using our model on a planetary ring, and the space manifold dynamics, a "low-cost" mission through the ring bodies can be performed.

4. Bibliografia.

- Barrabés, E; Cors, J.M; Hall, G.R. (2010). SIAM J. Appl. Dyn. Syst. 9(2), pp. 634-658
- http://www.scholarpedia.org/article/Space_Manifold_dynamics