A Mathematical Autobiography

by Michael Goldberg

Résumé

Extrait de son autobiographie mathématique.

"Mes recherches mathématiques proviennent en partie de l'intérêt que je porte aux problèmes de dissection. J'ai montré, par exemple, comment découper un pentagone régulier en six parties qui peuvent être restructurées pour former un triangle équilatéral.

Le 18ème problème d'Hilbert a sensibilisé mon intérêt pour la juxtaposition des polyèdres, un sujet qui continue à me captiver.

Coxeter me fit connaître les chaînons mécaniques qui sont des chaînes fermées de tétraèdres, et ceci me conduisit à chercher d'autres chaînons mécaniques à trois dimensions.

Dans les mécanismes, j'inclus certaines généralisations d'ovales de largeur constante, que je nomme rotors.

Poursuivant les travaux de L. Fejes-Toth et de ses étudiants, j'ai étudié le diamètre maximal de n cercles égaux qui peuvent être placés sur la surface d'une sphère sans qu'il y ait recouvrement."

Much of my mathematical research was initiated by my interest in dissection problems. The earliest study involved dissection of a given polygon into parts which could be assembled to form another polygon, and to accomplish this transformation with the fewest parts. The best case is Dudeny's dissection of the equilateral triangle into four parts which can be arranged to form a square. I wrote a survey of such problems for H. S. M. Coxeter when he prepared the eleventh edition of Mathematical Recreations and Essays by W. W. R. Ball. My principal addition was the dissection of the regular pentagon into six parts which could be arranged to form an equilateral triangle. I proposed this problem in the American Mathematical Monthly and submitted the only solution, which appeared in 1952 (56).

B. M. Stewart, of Michigan State University, considered the problem of dividing each of n squares into k parts in the same way, and then arranging the kn parts into a square. With my collaboration, we prepared a paper in which we showed that k need not exceed four (30).

The tiling of the infinite plane into congruent polygons excited my interest. Work done by P. A. Macmahon and others considered only periodic arrangements of the parts. My contribution was the paper Central Tesselations which did not use this periodicity (19).

Hilbert's 18th problem asked for the derivation of convex polyhedral shapes which could fill three-space with congruent copies when varied juxtaposition of the parts was permitted. This initiated my series of papers on space-fillers on which I am still engaged (48, 50, 53, 60, 61, 62, 64, 65, 66, 67). (Figure 1)

Figure 1. Three copies of Somerville N° 1 make a triangular prism. Since each triangular prism is a space-filler, the tetrahedron Somerville N° 1 is also a space-filler.
A related problem is Hilbert's third problem. This problem asks for a pair of polyhedra of the same volume, neither of which could be dissected to form the other. It was shown by M. Dehn in 1902 that the regular tetrahedron and the cube was such a pair. There is still the open question of determining the other tetrahedra which could be dissected to form a cube. J.-P. Sydler described four such tetrahedra. H.-C. Lenhard found five more. Then I found sixteen more (23, 42, 54), (Figure 2). There may be still more of these rectifiable tetrahedra. All of these are tabulated in the book Hilbert's Third Problem by V.G. Boltianskii, translated from the Russian, John Wiley, 1978.

In 1927, E. Steinitz published some results on the problem of finding the polyhedral shapes of the minimal surface for a given volume and a given number of faces. This paper inspired me to extend these results. My findings are given in two papers (2, 4). The second paper has been cited in a number of papers on the structure of viruses since it describes a family of shapes that many viruses take (33).

Coxeter introduced me to linkage mechanisms which are closed chains of tetrahedra. This led me to investigate other linkage mechanisms in three-space. Those mechanisms which operate in spite of an excess number of constraints I consider as singular mechanisms. My findings are reported in several papers (7, 8, 10, 12, 63), (Figure 4). A novel application of three-dimensional linkages is a ten-piece linkage which duplicates the cube. It transforms a $1 \times 1 \times 2$ block into a cube (34). Another novel linkage is a six-plate linkage (57). A recent study of this linkage was published by J. Eddie Baker in the Journal of Mechanical Design, Transactions of the ASME, vol. 101 (1979), 509-513.

In the field of singular mechanisms, I include generalizations of ovals of constant width. I call these rotors. In the plane, such a non-circular oval is one which can rotate within a regular polygon while keeping contact with all the sides of the polygon. A series of papers on several families of these rotors and related mechanisms has been published (13, 14, 17, 18, 20, 21, 26, 29, 33, 40). These results have been generalized to three-space. All the known results have been published in my paper Rotors in Polygons and Polyhedra (25), (Figure 5).

L. Fejes-Tóth and his students have published a number of papers on the problem of determining the largest diameter of $n$ equal circles that could be packed on the surface of a sphere without overlapping. I was able to improve some of the results and to add to them. These and other results on packing and covering problems are revealed in a series of papers (15, 28, 31, 37, 38, 44, 46, 47, 59). A summary of some of these and other results is given in two papers (36, 45), (Figure 6). These results were incorporated in the survey paper Extremal arrangements of points and unit charges on a sphere by T. W. Melnyk, O. Knop and W. R. Smith, Canadian Journal of Chemistry 55 (1977), 1745-1761.
Figure 5. Circular-arc and regular trammel rotors in regular polygons.

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