

Results show that the method can be used to classify two materials with similar responses, as is the case with steel and aluminum. In the case of the small bearing-balls, it has been found that for the large cylinders there is no significant difference among diameters, due all responses present similar characteristics. In the case of the small cylinders, it has been found that the large ball (b1) impacts are better, in order to observe difference among materials.

Table I. Validation test and ANN output. Size=0, 1 means short, and long. Type=0, 1 means aluminum, and steel.

Input Sequence (Validation)	ANN Output Output code		
	Size	Type	Detected Material
A1-S	0	0	Aluminum-Short
A1-S	0	0	Aluminum-Short
St-L	1	1	Steel-Large
A1-L	1	0	Aluminum-Large
St-S	0	1	Steel-Short
St-L	1	1	Steel-Large
A1-L	1	0	Aluminum-Large
St-S	0	1	Steel-Short

Table I. Validation test and ANN output. Size=0, 1 means short, and long. Type=0, 1 means aluminum, and steel.

### 3. Conclusions

We have presented a method which allows classifying materials using impacts and neural networks. The main problem on material's classification arises when responses are similar, in this case the method has been tested with steel and aluminum, and the ANN has proven to be a robust solution to detect differences. Further analysis will involve other materials as well.

### 4. Acknowledgement

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### 5. References

- [1] P. Cawley and R. D. Adams, "The mechanics of the coin-tap method of non-destructive testing," *Journal of Sound and Vibration*, 122(2), pp. 299-316, 1988.
- [2] A. Migliori and T. W. Darling, "Resonant ultrasound spectroscopy for materials studies and non-destructive testing," *Ultrasonics*, Vol 34, pp. 473-476, 1996.
- [3] S. Baglio and N. Savalli, "Fuzzy tap-testing sensors for material health-state characterization," *IEEE Trans. Instrum. Meas.*, vol. 55, no. 3, pp. 761-770, Jun. 2006.
- [4] H. Wu and M. Siegel, "Correlation of Accelerometer and Microphone Data in the "Coin-Tap Test"" *IEEE Trans. Instrum. Meas.*, Vol 49, No. 3. pp. 493-497, June 2000.
- [5] C. M. Harris and A. G. Piersol, "Harris' shock and vibration handbook", McGraw-Hill, 5th ed., 2002.

## APPLICATION OF A KALMAN FILTER ON MECHANICAL SYSTEMS TO ANALYZE IMPACTS

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### 1. Introduction

The purpose of this paper is to test the feasibility of using a Kalman filter and a simple mechanical model to analyze the velocity and the acceleration of an impact generated from the collision between two rigid bodies. To achieve this, the Kalman filter and a mechanical model are compared with experimental signals that have been obtained from real impacts, between a sensorized hammer and a steel cylinder.

The cylinder has been modeled as a first order dynamic system, as shown in Figure 1, with the impact signal,  $f(t)$ , applied on its surface. The mechanical model is shown on equation (1).

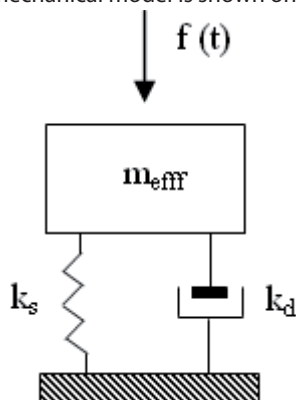


Figure 1. Metallic cylinder model.

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k_s}{m_{eff}} & -\frac{k_d}{m_{eff}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \quad (1)$$

Where  $m_{eff}$  : effective mass [kg]  
 $k_d$  : damping constant [N·s/m]  
 $k_s$  : spring constant [N/m]  
 $f(t)=u(t)$  : input force [N]  
 $x_1, x_2$  : displacement and speed.

According to [1], the effective mass is the joint masses of the hammer and the cylinder, and this is given by equation (2).

where  $m_h$  : hammer mass  
 $m_c$  : cylinder mass  
 And constants  $k_d$  and  $k_s$  are for steel.

### 1.1 Kalman Algorithm

Figure 2, shows the block diagram of the Kalman estimator.

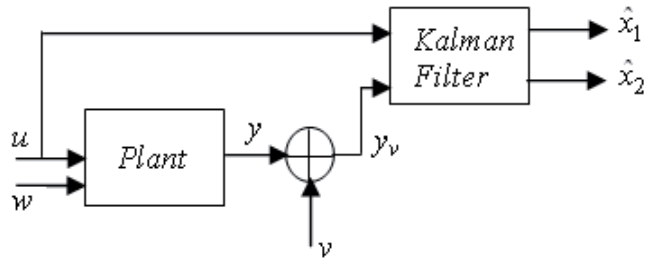


Figure 2. Kalman estimator.

To describe the Kalman algorithm some equations need to be defined. The discrete Kalman filter state equations are specify by (3) and (4).

$$y_k = H \cdot x_k + v_k \quad (3)$$

$$x_{k+1} = F \cdot x_k + B \cdot u_k + G \cdot w_k \quad (4)$$

On equation (3),  $y_k$  is the observable at time  $k$ , and  $H$  is the measurement matrix. The measurement noise,  $v_k$ , and the process noise,  $w_k$ , are assumed to be additive, white, and Gaussian, with the covariance matrix defined by (5) and (6):

$$R = E\{v_k v_k^T\} \quad (5)$$

$$Q = E\{w_k w_k^T\}. \quad (6)$$

Terms in (4) are:  $x_{k+1}$  which is the state vector in  $k+1$ ;  $F$  is the transmission matrix;  $x_k$  is the state vector in  $k$ ;  $B$  is the input signal vector; and  $u_k$  is the input signal. The complete Kalman algorithm is described on Figure 3.

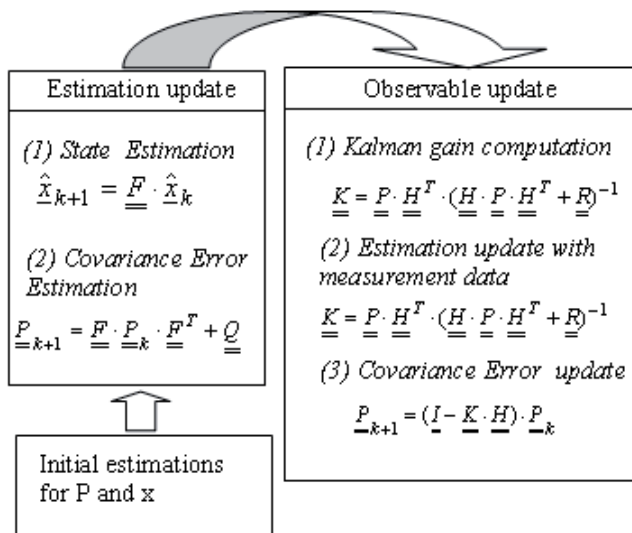


Figure 3. Kalman algorithm.

2. Results and Discussion

Figure 4 shows the simulations results from the mechanical model, stated on (1), and from the Kalman algorithm, described on Figure 3. On Figure 4a, we can observe the results of the Kalman estimations of the system, Figure 4a top, and the system simulation results, Figure 4a bottom. From these graphics a good approximation between both can be observed, even if there is an error at the beginning of the estimation, shown in Figure 5, due to the initial conditions defined on matrix  $P$  (covariance of error) and (state variable). After a few cycles, around 20, the error reaches a minimum stable value. Also, from Figure 4a, can be observed another interesting feature of the Kalman estimator, which is its capacity to reduce the noise on the measured signal.

On Figure 4b, we present the velocity and acceleration of the impact obtained by numerical derivation of the displacement and velocity shown on Figure 4a (top), from the Kalman estimator. These signals are compared with the experimental data, shown on Figure 4c, obtained from a sensorized hammer tapping a steel cylinder. These graphics indicate that there is a good approximation between them, and that Kalman algorithm can be considered as a suitable solution to analyze the model of an impact.

3. Conclusions

On this work a method to study impacts has been presented, using a Kalman filter and a mechanical model. Results show that with a correct model, the Kalman filter achieves a good estimation.

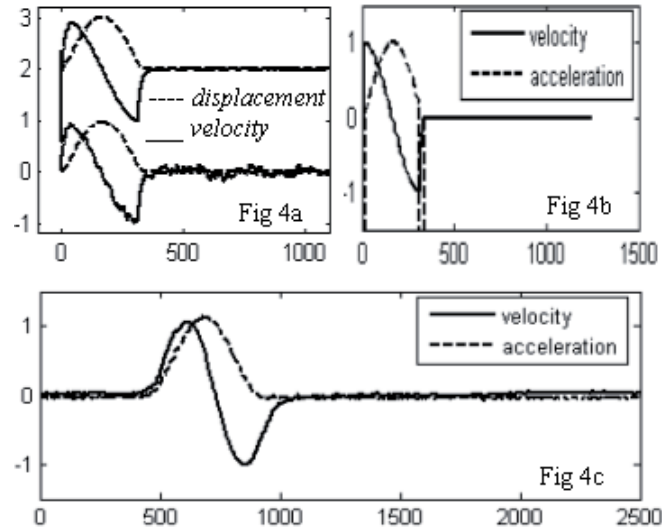


Figure 4. a) Simulation results. b) Acceleration from numerical derivation of velocity. c) Experimental signal from a sensorized hammer.

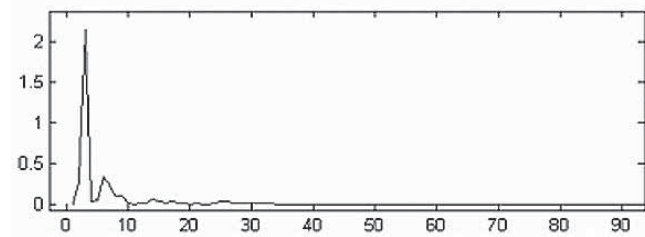


Figure 5. Estimation error

The Kalman filter is a well explored solution to many complex problems; however it requires a very good understanding of the plant. On Figures 4b and 4c, we can see that the simulations results are similar to the experimental impact. In future works, we will study the sensor's influence by modeling the plant as a second order dynamic system.

4. References

[1] A. Tornambe, Modelling and controlling one-degree-of-freedom impacts, IEEE proceedings, Roma, 1996.  
 [2] Jae-Yoo Yoo, Tae-Sik Park, Speed Estimation of an IM using Kalman Filter Algorithm at Ultra-low Speed, IEEE, Korea, 1997.  
 [3] S. Grewal, P Andrews, Kalman filtering: Theory and Practice Using Matlab., John Wiley & Sons inc, 2001

