SOLUCIÓ AL PROBLEMA PROPOSAT AL VOLUM 26 N. 1 i 2

PROBLEMA N. 92

1) Clearly $\mathbf{H} = \mathbf{H}^2$, hence $\mathbf{H}$ is (symmetric) idemptotent. Its eigenvalues are therefore 0 or 1. Hence $r(\mathbf{H}) = \mathrm{tr}(\mathbf{H}) = n - 1$, as $\mathbf{H}$ has $n - 1$ unit eigenvalues and one zero-eigenvalue. As usual $r(\mathbf{H})$ and $\mathrm{tr}(\mathbf{H})$ denote rank and trace of $\mathbf{H}$, respectively.

2) $\mathbf{B} = \mathbf{H} = \mathbf{H} \cdot \mathbf{H} = \mathbf{H}(\mathbf{I}_n - \mathbf{1}_n \mathbf{1}_n')\,\mathbf{H}$, where $\mathbf{H}\mathbf{1}_n = 0$, $\mathbf{1}_n'\mathbf{H} = 0$. Hence

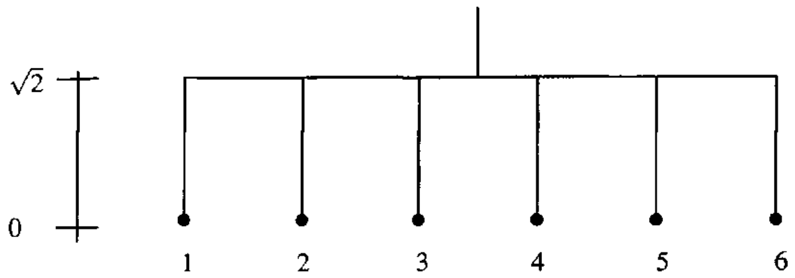
$$\mathbf{B} = -\frac{1}{2}\mathbf{H}\left(2(\mathbf{1}_n \mathbf{1}_n' - \mathbf{I}_n)\right)\mathbf{H}$$

and the (Euclidean) distance matrix is

$$\mathbf{D} = \sqrt{2}(\mathbf{1}_n \mathbf{1}_n' - \mathbf{I}_n)$$

i.e., $d_{ij} = \sqrt{2}$, $i \neq j$, and $d_{ii} = 0$, $i, j = 1, \ldots, n$.

3) Clearly, all distances are equal to $\sqrt{2}$. A dendrogram for the n objects, assumming $n = 6$, is:



Heinz Neudecker
Cesaro
The Netherlands