1) Clearly $H = H^2$, hence $H$ is (symmetric) idempotent. Its eigenvalues are therefore 0 or 1. Hence $r(H) = \text{tr}(H) = n - 1$, as $H$ has $n - 1$ unit eigenvalues and one zero-eigenvalue. As usual $r(H)$ and $\text{tr}(H)$ denote rank and trace of $H$, respectively.

2) $B = H \cdot H = H(1_n - 1_n 1_n' H)$, where $H 1_n = 0$, $1_n' H = 0$. Hence

$$B = -\frac{1}{2} H (2(1_n 1_n' - 1_n)) H$$

and the (Euclidean) distance matrix is

$$D = \sqrt{2}(1_n 1_n' - 1_n)$$

i.e., $d_{ij} = \sqrt{2}$, $i \neq j$, and $d_{ii} = 0$, $i, j = 1, \ldots, n$.

3) Clearly, all distances are equal to $\sqrt{2}$. A dendrogram for the $n$ objects, assuming $n = 6$, is:

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