# SIGNALS AND REVISIONS IN ECONOMIC TIME SERIES: A CASE STUDY

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The paper estimates how much short-run monetary control may be affected by data noise and revisions, such as the ones implied by seasonal adjustment. The effects of the different types of data error are illustrated, and results on their empirical relevance and analytical properties are presented. The paper can be seen as an exercise that combines some elements of econometric, time series and economic analysis to answer a "real world" problem.

Keywords: MONETARY POLICY, ECONOMIC TIME SERIES, SEASONAL ADJUSTMENT, REVISIONS, SIGNAL EXTRACTION, ERROR IN THE VARIABLES.

# 1. INTRODUCTION.

In Maravall-Pierce /12/ revisions in the money supply series were analyzed and an attempt was made to measure the effect that revision errors could have on short-run monetary policy. This was done by estimating how often the preliminary measure of the rate of growth of the money supply Ml may give a wrong signal of whether Ml is growing as desired or not, the desired growth being the one lying inside the tolerance range set by the Federal Open Market Committee (FOMC) at each monthly meeting. Using actual data, we computed the number of times a preliminary figure was misleading over the period of the seventies. That frequency turned out to be surprisingly high (close to 40%), and most of the wrong signals could be attributed to seasonal revisions. In fact, the tolerance range used in policy could be interpreted as a relatively narrow confidence interval, under the hypothesis that the final rate of growth of Ml is equal to the preliminary measure.

Next, we estimated the probability of a wrong signal under the "ideal" situation in which there are no errors other than seasonal revisions and these revisions are associated with optimal and concurrent seasonal adjustment. This probability was about 20%. Thus, although the proportion of wrong signals could be considerably decreased through im-

proved seasonal adjustment methods, the existence of seasonal revision error sets a non-trivial lower bound to the precision of short run monetary policy.

However, it does not follow that in terms of setting policy the FOMC is necessarily misled by errors in preliminary data. In this paper it is found that noise in the data induces relatively little "noise" in actual policy. The results suggest that the incoming figures are not taken entirely at face value, rather than in effect a signal-plus-noise separation is made. In fact, we conclude that, on average, for a unit unexpected deviation in the rate of growth of Ml with respect to its target, measured with preliminary data, to a close approximation one-third of the deviation will represent transitory noise which should be ignored, one-third an undesired deviation which should be compensated, associated mainly with money supply shocks, and one-third an unexpected deviation which should be accomodated, primarily associated with shocks stemming from the demand side. seen how the different reaction towards demand and supply shocks, together with the signal extraction, explain why noise in the data have little effect on the setting of monetary targets.

The analysis includes some econometric results

- Agustín Maravall ; David A. Pierce Banco de España Federal Reserve Board Alcalá, 50 Madrid -4
- Article rebut el Maig de 1984
- Aquest treball es una ponencia invitada al "First Catalan International Symposium on Statistics" a Barcelona del 23 al 24 de Setembre de 1983.

on errors-in-variables models, which are presented in an Appendix. It should be noted that since 1979, some modifications in monthly operating procedures and in the definitions of the series have been made. As a consequence, the targeted series and the targeting procedures are not identical today to those during the period we consider. However, the present study is still of current interest. Experience with the redefined aggregates is limited and financial innovation is proceeding apace, so that it will be several more years before enough sufficiently wellbehaved preliminary data and revisions are available to enable a comparable study to be performed using contemporary aggregates. Of greater significance, the historical satatistical characteristics (sizes, variances and autocovariances) of the old and new definitions of the M1 series are broadly similar, the seasonal factor revision process is essentially unchanged, and tolerance ranges akin to those described continue to be used in monetary policy design. Thus, to a considerable degree inference from the 70's experience to the current outlook is warranted.

# 2. THE DATA SERIES AND THE TARGETS

Since the early 1970's, short run monetary policy has been characterized by the monthly setting of targets for the rate of growth of M1 (seasonally adjusted) over a two-month period, and for the level of the federal funds rate that should prevail until the next FOMC meeting.

The monthly value of M1 seasonally adjusted will be denoted  $\mathbf{M_t}$ . Let  $\mathbf{m_t}$  represent the monthly series of annualized rates of growth of M1, seasonally adjusted, calculated for the two-month beginning with the month t of the FOMC meeting. The series of montly averages of the Federal funds rate will be denoted  $\mathbf{r_t}$ .

While the FOMC sets a range of tolerance for  $m_{t}$  without specifying a point estimate, the midpoint of this tolerance range, which we denote  $\bar{m}_{t}$ , can be reasonably interpreted as a point target. This interpretation is implied by the wording of the FOMC Record of Policy Action. It is also supported, as we shall see, by empirical evidence. For the

funds rate a point target is specified, together with a relatively narrow range, and only occasionally does the point target differ from the midpoint of the range.

To summarize, approximately midway through month t there is a meeting of the FOMC, at which a target  $\bar{\mathbf{m}}_{t}$  is set for the growth of  $\mathbf{M}_{1}$  over the months t and (t+1). Also a target for the funds rate is set, which we shall denote  $\bar{\mathbf{r}}_{t}$ . Thus, in terms of monthly series, when  $\bar{\mathbf{m}}_{t}$  and  $\bar{\mathbf{r}}_{t}$  are set, information is available up to (and including) month (t-1).

In addition to this short-run target, during most of this period a tolerance range for the long run or annual growth of Ml was usually given as well. The long run targets were set from quarter to quarter, though they were tipically maintained constant for periods longer than three months. We shall use as point target the midpoint of this range, which we shall denote  $\mathfrak{m}_{t}^{LR}$ . For the first months (when long run targets were not made explicit) the series was set equal to the first available target value.

Since the subscript t refers to the time of the meeting, the first measure of  $\mathbf{m}_{t}$  will be available at the meeting held at (t+2). This preliminary estimate, denoted  $\mathbf{m}_{t}^{O}$ , will be revised over a period of approximately three years. When all revisions have been completed, the estimate becomes final and shall be denoted  $\mathbf{m}_{t}^{f}$ . Because of the time needed to complete the revision process, our analysis covers the five year period 1974-78. The three series  $\mathbf{m}_{t}$ ,  $\mathbf{r}_{t}$ , and  $\mathbf{m}_{t}^{LR}$  are shown in Figure 1. The series  $\mathbf{m}_{t}^{O}$ , and  $\mathbf{m}_{t}^{f}$  are displayed in Figure 2, together with the tolerance range. Finally, Figure 3 shows  $\mathbf{r}_{t}$  and its tolerance range.

# 3. A MODEL FOR THE SHORT RUN TARGETS.

The long run target  $m_{t}^{LR}$  is primarily set in accordance with what is believed to be consistent with such macroeconomic targets as GNP growth, employment, and inflation, and is fairly constant within a year. Thus for now we shall assume that, when setting  $m_{t}$  and  $\bar{r}_{t}$ ,  $m_{t}^{LR}$  is exogenously given. Our model simply states that, when setting short run targets, the FOMC should aim towards the long run one, correcting for undesired deviations

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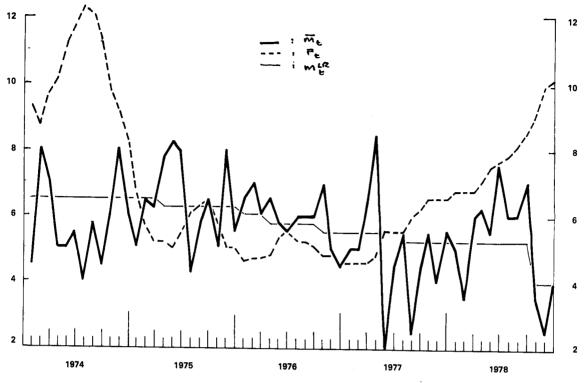


Figure 1: The series of targets

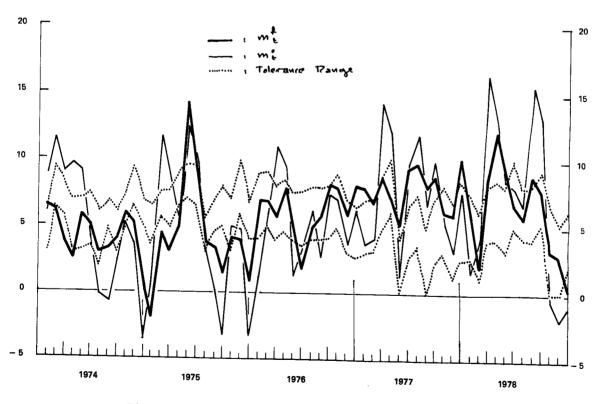


Figure 2: Preliminary and final measures of the rate of growth of  $\mathbf{M}_{\hat{\mathbf{l}}}$ 

as they occur. We shall assume that the full correction extends over a period of several months. Such a gradual response is in agreement with the wording of the FOMC Record of Policy Action. Likely, it reflects mistrust of the preliminary measure on one hand and FOMC concern with orderly markets on the other. This concern typically translates into avoiding unexpected short run fluctuations in the Federal funds rate-see De Rosa-Stern /5/ and Lombra-Torto /11/. In this sense, short run targeting should react both to recent deviations in the growth of Ml with respect to its target (so as to be able to meet the long run target) and to deviations in the funds rate with respect to its target (in order to avoid disorderly markets). Letting

$$dm_t = m_t - \bar{m}_t \tag{1a}$$

$$dr_t = r_t - \bar{r}_t \tag{1b}$$

represent both deviations, we shall assume that the targeted money growth rate and interest rate are given, respectively, by

$$\bar{m}_{t} = \omega(L) dm_{t-1} + \lambda(L) dr_{t-1} + \gamma m_{t}^{LR} + u_{t}$$
 (2a)

and

$$\bar{r}_{t} = \alpha(L) dm_{t-1} + \beta(L) dr_{t-1} + \pi m_{t}^{LR} + v_{t}$$
 (2b)

where  $\omega\left(L\right)$  ,  $\lambda\left(L\right)$  ,  $\alpha\left(L\right)$  , and  $\beta\left(L\right)$  are polynomial distributed lags (DLs) in the lag operator L.

# 3.1 SUPPLY AND DEMAND SHOCKS

To a significant extent the targets' variability is assumed to be explainable by whether Ml growth is as desired. We may distinguish three conceptually different reasons for the existence of the discrepancies  $\mathrm{dm}_{\mathtt{t}}$  between actual and targeted values:

- a) an unexpected shock in the money demand function,  $\mathbf{D}_{\mathsf{t}}$ ;
- b) an unexpected shock in the money supply function,  $S_{+};$
- c) an unexpected shock in the IS function.

Policy responses to these different shocks

should, in principle, be different. (See for example, Friedman /7/, Davis /4/, and Lindsey /10/, and Section 9 of this paper.) For our purposes, we may group (a) and (c) together, and refer to them as "Demand shocks".

The differential effects of supply and demand shocks are illustrated in Figure 4. The targets set are  $\overline{\mathbf{m}}_{\mathbf{t}}$  and  $\overline{\mathbf{r}}_{\mathbf{t}}$ , the equilibrium values associated with the demand and supply fuctions  $\mathbf{D}_{\mathbf{t}}$  and  $\mathbf{S}_{\mathbf{t}}$ . Assume there is an unexpected shock in demand, so that  $\mathbf{D}_{\mathbf{t}}$  moves to  $\mathbf{D}_{\mathbf{t}}'$ . The equilibrium point attained will then be B instead of A. Hence the deviation in money growth is dm\_t and that in interest rates is dr\_t. Alternatively, assume the shock affects supply, and  $\mathbf{S}_{\mathbf{t}}$  moves to S'. The deviations are now dm\_t and dr\_'.

If only  $\mathrm{dm}_{\mathsf{t}}$  is included in equation (2), the two different shifts in Figure 4 would lead to the same response. However, this will not be the case if  $\mathrm{dr}_{\mathsf{t}}$  is included: an unexpected increase in demand induces a positive  $\mathrm{dr}_{\mathsf{t}}$ , while an unexpected increase in supply induces a negative  $\mathrm{dr}_{\mathsf{t}}$ . Hence the two types of shifts can be differentiated.

Other variables are of course also relevant in explaining the setting of targets. For example, if monthly FOMC forecasts of income were available, together with monthly measurements, unexpected deviations in income could then be incorporated into equation (2). In principle this would allow us to identify the three different sources of shocks (i.e., to separate money demand from IS shocks). However some of those relevant variables (such as income) are not observed monthly. Even when monthly values are available, or when monthly estimates can be determined, monthly FOMC targets or forecasts are not available. Reasonably, the informational value of new data to the FOMC depends on the underlying target of forecast implicit in its behavior (Duesenberry, /6/). Hence for analyzing policy such monthly information is of little value.

Equations (2a, 2b) attempt to capture the dynamic reaction to deviations with respect to short run targets. They are in part implied by declared FOMC behavior, and they allow for different reactions to supply and demand shocks. Also, through  $\bar{r}_t$ ,  $\bar{m}_t$ , and  $m_t^{LR}$ , policy changes due to shifts in other variables

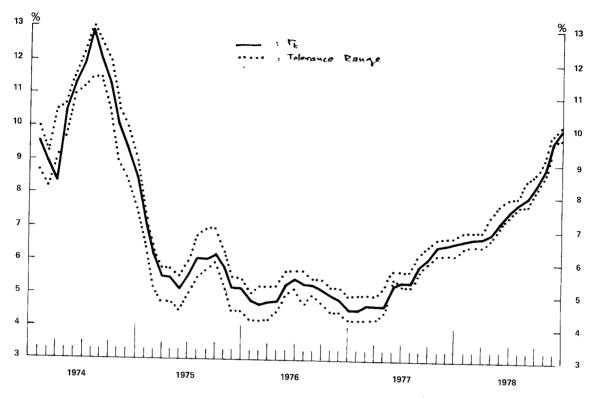


Figure 3: The series of federal funds rate

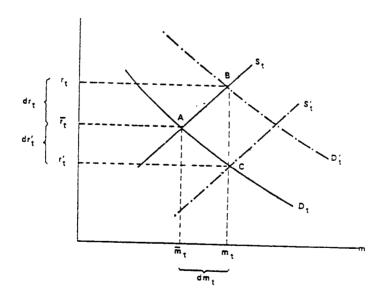


Figure 4: Demand and supply shocks

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(had they happened or been anticipated) would be incorporated.

Obviously, we cannot expect the equations to account for all variation in targets. But our objective is to estimate how preliminary-data error (noise in data) translates into errors in the setting of the targets  $\bar{m}_t$  and  $\bar{r}_t$  ("policy noise"). This we hope to capture through the distributed lags on dm..

# 4. THE MODEL IN TERMS OF PRELIMINARY DATA.

Final monetary aggretate estimates would be used in the setting of targets if this were possible, and hence equations (2a, 2b) were formulated in terms of final data. However, as described in Section 2, at time t the final values  $\mathbf{m}_t^f$  and  $\mathbf{d}_{mt}^f$  (= $\mathbf{m}_t^f$ - $\bar{\mathbf{m}}_t$ ) are unknown, as are the lagged values  $\mathbf{m}_{t-j}^f$  and  $\mathbf{d}_{t-j}^f$  for all j up through the length 12 of the DLs. (We assume that the preliminary observation on  $\mathbf{r}_t$  contains no error). Instead, at time t only a preliminary growth-rate estimate, say  $\mathbf{m}_t^o$ , is available, and the targets must be set using preliminary data.

#### 4.1 REVISION ERRORS

The relation between preliminary and final data may be expressed as

$$\mathbf{m}_{\mathsf{t}}^{\mathsf{f}} = \mathbf{m}_{\mathsf{t}}^{\mathsf{o}} + \delta_{\mathsf{t}} \quad , \tag{3}$$

where  $\delta_{t}$  is the <u>revision error</u>, or the error in preliminary data, due to seasonal and non-seasonal sources, which is corrected in subsequent revisions in the series. It is assumed that  $\delta_{t}$  and the preliminary value  $m_{t}^{O}$  are independent, and that  $\delta_{t}$  can be expressed as a moving average of future innovations of  $m_{t}$  (see Pierce /13/).

In addition to equation (3), which relates original and final data via the total revision error  $\delta_{t}$ , there are relations involving intermediate revisions of the data. In general, denote by  $\mathbf{m}_{t-k}^{k}$  the best estimate of  $\mathbf{m}_{t-k}^{f}$  available at time t (which implies, among other things, that concurrent seasonal adjustment is employed). Then the "lag-k revision error",  $\delta_{t-k}^{(k)}$  is defined by the relation:

$$m_{t-k}^f = m_{t-k}^{(k)} + \delta_{t-k}^{(k)}$$

Substracting  $\bar{m}_{t-k}$  from both sides in (2), it is then seen that all  $dm_{t-j}^f$  appearing as regressors can be written as:

$$dn_{t-k}^{f} = dn_{t-k}^{(k)} + \delta_{t-k}^{(k)}, \qquad (4)$$

where  $\text{dm}_{t-k}^{(k)}$  is a function of innovations up to time t, while  $\delta_{t-k}^{(k)}$ , the revision error still unremoved from the estimate of  $\textbf{m}_{t-k}$  at time t, only depends on future innovations. Dropping the superscript k when the context is clear, equation (2a) can be rewritten as:

$$\bar{m}_{t} = \omega(L) (dm_{t-1}^{\circ} + \delta_{t-1}) + \lambda(L) dr_{t-1} + \gamma m_{t}^{LR} + u_{t}$$

$$= \omega(L) dm_{t-1}^{\circ} + \lambda(L) dr_{t-1} + \gamma m_{t}^{LR} + u_{t}^{*} , \quad (5)$$

where

$$u_t^* = u_t + \omega(L) \delta_{t-1}$$

is in general autocorrelated but is independent of all regressors;  $^{3)}$  similarly for the equation (2b). Thus, a least squares estimation of equation (2), with or without an ARMA model for the disturbance  $u_{t}^{\star}$ , would result in consistent estimates of the parameters

#### 4.2 BENCHMARK AND SEASONAL REVISIONS

At time t, when the FOMC sets  $\bar{\mathbf{m}}_t$ , growth of  $\mathbf{M}_1$  for month (t-k) is known for  $k \ge 1$ ; hence  $\mathbf{m}_{t-k}^{\mathsf{O}}$ ,  $k \ge 2$ , is known. Hence  $\mathbf{m}_{t-1}^{\mathsf{O}}$  is not known, although it can be easily forecasted since the first month of the two-month period is already known. The series of the one-period-ahead ARIMA forecasts will be denoted  $\hat{\mathbf{m}}_{t-1}^{\mathsf{O}}$ . (Notice that at time t,  $\hat{\mathbf{dm}}_{t-1}^{\mathsf{f}} = \hat{\mathbf{dm}}_{t-1}^{\mathsf{O}}$ ). Again, since

$$dm_{t-1}^{\circ} = d\hat{m}_{t-1}^{\circ} + \varepsilon_{t} \tag{6}$$

where  $\varepsilon_{t}$  is orthogonal to  $\mathrm{dm}_{t-1}^{o}$ , the use of  $\mathrm{dm}_{t-1}^{o}$ , instead of  $\mathrm{dm}_{t-1}^{o}$  will not pose any serious estimation problem. Also, at time t the FOMC knows  $\mathrm{dr}_{t-k}$ ,  $k \ge 1$ .

Concerning the revisions in  $dm_{t}^{o}$ , it is assumed that

(1) The seasonal revision is "up to date", that is, concurrent adjustment is employed. Because of the once-a-year adjustment used in practice, the value of  $\text{dm}_{t-k}$  as used in the regression contains a component that will have been revised once. However, this would affect most the more distant regressors, which are likely to be the least important ones.

(2) The non-seasonal revision is removed from the data with an average four-month delay. In this regard, we note that benchmark revisions are made every three months with some additional months of processing involved. Also, reasonable alternative hypotheses concerning the delay were examined, with practically no effect on the results.

Thus, letting  $m_t^O$  be the rate or growth of the preliminary data corrected for non-seasonal revisions, it follows that the actual regressors in equations (2a, 2b) are contained in the vector  $[dm_{t-1}^O]$  defined by

$$\left[dm_{t-1}^{\circ}\right] =$$

= 
$$(d\hat{m}_{t-1}^{\circ}, dm_{t-2}^{\circ}, ..., dm_{t-5}^{\circ}, dm_{t-6}^{\circ}, ..., dm_{t-12}^{\circ})$$
,

where, for any t,

$$dm_{+}^{\circ} = m_{+}^{\circ} - \overline{m}_{+} .$$

# 4.3 FINAL MODEL

Two further (minor) modifications were made to the model (2). First, while  $\bar{m}_t$  behaves as white-noise,  $dm_t^O$  and  $dr_t$  have low-order autocorrelation and  $m_t^{LR}$  is trend dominated. Thus we allow for an ARMA error term; and when identified and fitted, an AR(1) error term, or equivalently a single lagged endogenous variable, was found sufficient. Second since 12 consecutive monthly releases of  $M_1$  span 13 lagged values of  $dm_t$ , an additional term is added to the two DLs. Thus, the final model can be written as the system of two stochastic difference equations.

$$(1-\phi_1 L)\widetilde{m}_t = \omega(L)d\widetilde{m}_{t-1}^\circ + \lambda(L)dr_{t-1} + \gamma m_t^{LR} + a_t / (8a)$$

$$(1-\phi_2 L) \nabla \overline{r}_t = \alpha(L) \widetilde{dm}_{t-1}^\circ + \beta(L) dr_{t-1} + \pi m_t^{LR} + b_t, (8b)$$

where the first difference  $\nabla \bar{r}_t$  is introduced to remove nonstationarity (as was in effect already done with  $M_t$  in computing rates of growth  $m_t$  and  $\bar{m}_t$ ),  $\omega(L)$ ,  $\lambda(L)$ ,  $\alpha(L)$ , and  $\beta(L)$  are the corresponding DLs, and  $(a_t,b_t)$  ~ NID $(0,\Omega)$  with

$$\Omega = \begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{pmatrix}$$

denoting the contemporaneous covariance matrix. The model (8) is a reaction function associated with a loss which depends on the deviations from both the money target and the funds rate target. This pair of equations can also be seen as a reasonable starting model within a "Comfac" approach (see Harvey /8/, Chapter 8). Notice that, since  $\overline{\mathbf{m}}_{\mathbf{t}}$  is a rate of growth, both targets appear in differenced form, one in logs and one in levels, which is sensible à priori.

The system (8) can be rewritten in the form

$$\bar{m}_{t} = \omega^{*}(L) \ \tilde{dm}_{t-1}^{\circ} + \lambda^{*}(L) \ dr_{t-1} + \gamma^{*} \ m_{t}^{LR} + u_{t}$$
 (9a)

$$\nabla \bar{r}_{t} = \alpha^{*}(L) \ \alpha \bar{m}_{t-1}^{\circ} + \beta^{*}(L) \ dr_{t-1} + \pi^{*} \ m_{t}^{LR} + v_{t}$$
 (9b)

where the asterisk denotes the modified DL. Since  $m_{\text{t}}^{LR}$  is fairly constant,  $\gamma^*$  and  $\pi^*$  can be assumed to be constant. The residuals in (9) then follow the AR(1) process

$$\begin{pmatrix} 1-\phi_1 L & 0 \\ 0 & 1-\phi_2 L \end{pmatrix} \begin{pmatrix} u_t \\ v_t \end{pmatrix} - \begin{pmatrix} a_t \\ b_t \end{pmatrix}$$

Equations (8) and (9) are two alternative representations of the model we employ in this paper. The latter has the advantage of directly yielding the distributed lag effects of the exogenous variable; hence the gains (Section 6) are given by  $\omega^*(1)$ ,  $\lambda^*(1)$ ,  $\alpha^*(1)$  and  $\beta^*(1)$ . It also removes from the explained variance that part which is attributable to residual autocorrelation.

There are some constraints that should be satisfied by this model on a priori grounds. Considering (9a), let  $m_t^{LR} \equiv m^*$  be an equilibrium constant rate of growth. When  $d\widetilde{m}_t^0 = dr_t = u_t = 0$  for all t, consistency of the short and long run targets implies  $\overline{m}_t = m^*$ ; hence  $\gamma^* = 1$ . Next, let  $v_t = 0$ . If, in the absence of external shocks, a constant level of  $m_t^*$  implies a constant level for  $r_t^*$ , then  $\nabla \overline{r}_t = 0$ ; hence  $\pi^* = 0$ .

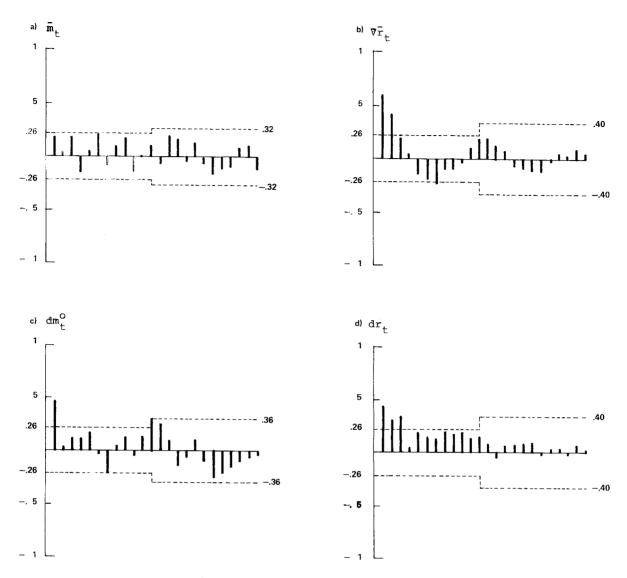


Figure 5: Autocorrelation functions

Table 1

Mean and Variance of Target and Target-Deviation Series

Series	m̄ <sub>t</sub>	∇r̄ <sub>t</sub>	dm°t	dr <sub>t</sub>
Mean	5.65 (21.7)	.01 (.08)	•24 (•29)	•05 (•98)
Variance	2.02	•28	20.40	•05

		dm <sub>t+k</sub>	dr <sub>t+k</sub>
Ξ̄t	k<0 k>0	35.4 14.5	35.7 4.4
<sup>⊽</sup> rt	k<0 k>0	40.8	45.2 24.4

Table 3
Summary of Model Estimation Results

		Money Target Equation	Funds Rate Target Equation
R <sup>2</sup>		.84	.86
F-statistic		4.51	5.30
Variance of Residuals		.35 <b>=</b> (.59) <sup>2</sup>	.020 = (.14) <sup>2</sup>
ACF of Residuals	Q(12)	8.3	11.6
	Q(24)	14.2	17.7
Lagrange Multiplier Test	$x_{m}^{2}$	28.5	35.9
	$x_r^2$	18.3	25.5
Gain of DL	For dm <sub>t-1</sub>	$\hat{\omega}^*(1) =30$	$\hat{\alpha}^{*}(1) = .22$
	For dr <sub>t-1</sub>	$\hat{\lambda}^*(1) = 1.65$	$\hat{\beta}^{*}(1) =83$
Coefficient of mt		$\hat{Y} = .73$ (4.24)	$\hat{\pi} = 0$ (.63)
Coefficient of m <sub>t-1</sub>		$\hat{\phi}_1 = .28$ (1.72)	$\hat{\phi}_2 = .27$ (1.41)

(t-values are given in parenthesis.)

There are additional constraints related to the values of the four gain functions, which are discussed later. The model is first estimated without constraints, and the constraints are then used as checks on the reasonableness of the results.

# 5. EMPIRICAL RESULTS.

In this section some statistical characteristics of the series of data and targets are established, following which the model of section (4) is estimated.

# 5.1 DYNAMIC STRUCTURE OF THE SERIES.

The Autocorrelation Functions (ACFs) of the individual series are displayed in Figure 5. The means and variances appear in Table 1, where the numbers in parentheses represent the asymptotic t-values corrected for the presence of autocorrelation. From the figure it is seen that the money target,  $\overline{\mathbf{m}}_{t}$ , has the over-all characteristics of univariate white noise. The funds rate target,  $\overline{\mathbf{r}}_{t}$ , is highly non-stationary, with a strong trend. The variable  $\nabla \overline{\mathbf{r}}_{t}$  appears stationary, with some low-order autocorrelation.

The series  $[dm_{t}^{O}]$  measures the difference between actual and targeted growth of  $M_1$  for a two-month period. The target  $\bar{m}_+$  is set with information up to (t-1) and refers to growth over periods t and (t+1). It can be interpreted as a two-period-ahead forecast; otherwise policy would be expected to miss the target by some predetermined amount. Specifically, since at time (t+2) the best estimate available of  $\mathbf{m_t}$  is  $\mathbf{m_t^O}$  ,  $\mathbf{\bar{m}_t}$  can be seen as a two-period-ahead forecast of  $\mathbf{m_t^O}$  . However, it will not be a univariate ARIMA forecast since, when setting targets, the set of information considered by the FOMC (including its own ability to influence  $m_{\pm}^{O}$ ) is much wider than simply the past values of  $m_+^{O}$ . It follows that  $dm_{+}^{O}$ , given by

$$dm_{+}^{O} = m_{+}^{O} - \bar{m}_{+},$$

should resemble a two-period-ahead forecast error, and hence its ACF should also resemble that of an MA(1) process. This is in agreement with the ACF of the actual  $\lceil dm_+^{\circ} \rceil$ 

series, except for a small lag-12 autocorrelation (which can evidently be attributed to the fact that, 12 period later, there is a better estimate available, the first-year revision having been performed).

The series  $[dr_t]$ , a monthly series of deviations in monthly averages, can be interpreted, in a similar way, as a series of single-period forecast errors. However, although clearly stationary, the series displays low-order autocorrelation. Evidently there is reluctance in incorporating a systematic component in funds rate misses. Nevertheless, comparing Figures 2 and 3, control of the funds rate appears to have been tighter than that of  $m_t$ , although the standard deviation of  $dr_t$  (about 25 basis points) is by no means negligible.

Finally the cross correlation functions (CCFs) between the variables  $\bar{m}_{t}$  and  $\nabla \bar{r}_{t}$ , on one hand, and the variables  $dm_{t}^{0}$  and  $dr_{t}$ , on the other, were clearly one-sided, as shown in Table 2 which gives values of

$$\chi^{2}(10) = T \sum_{k=1}^{10} \hat{\rho}_{k}^{2}$$
,

 $\hat{\rho}_k$  denoting the lag-k sample cross correlation. This result is in agreement with the hypothesis of exogeneity of  $\mathrm{dm}_t^O$  and  $\mathrm{dr}_t$  with respect to the targets.

The CCFs were also in agreement with the assumptions made concerning the timing of information (in particular,  $\rho_{\text{O}}$  was close to zero in all cases).

# 5.2 ESTIMATION OF THE MODEL.

The model (8) is seen to be in the form of a SURE system, with a common set of regressors, so that OLS is a suitable procedure. Table 3 summarizes the estimation results. Comparing the variance of the two series of targets (2.02 and 0.28 - see table 1) with the variances of the two series of residuals (0.35 and 0.02), the two equations illustrate a case in which a regression model represents a substantial improvement relative to a univariate time series model for the same series. (The ACF of  $\bar{m}_t$  was seen in figure 5 to be close to that of white noise). The ACF's of the residuals  $a_t$  and  $b_t$  indicate that both series are essentially white noise, with the

corresponding Q-statistics insignificant. Lagrange multiplier tests for the four DL components were carried out, and we detail the derivation for the first of them. Letting

$$H_0: \omega(L) \equiv 0$$
  $(\omega_1 = 0, \forall i)$ ,

it is easily seen that, under Ho,

$$a_t^{\circ} = \overline{m}_t - \phi_1 \overline{m}_{t-1} - \lambda(L) dr_{t-1} - \gamma m_t^{LR}$$
,

$$\frac{\partial a_t}{\partial \phi} \Big|_{H_0} = -\bar{m}_{t-1}$$
,  $\frac{\partial a_t}{\partial \omega_i} \Big|_{H_0} = -d\bar{m}_{t-1}^{\circ}$ 

$$\frac{\partial a_t}{\partial \lambda_i} \Big|_{H_0} = -dr_{t-1}$$
, and  $\frac{\partial a_t}{\partial \gamma} \Big|_{H_0} = -m_t^{LR}$ 

Then, in the regression of  $a_t^O$  on  $\overline{m}_{t-1}$ ,  $[dm_{t-1}^O]$  ,  $[dr_{t-1}]$  and  $m_{t}^{LR}$  ,

$$TR^2 \sim \chi_{12}^2$$
 .

Thus, it is seen in table 3 that deviations in money growth with respect to its target are highly significant (at the 1 percent level) in the  $\bar{\rm m_t}\text{-}{\rm equation}$ , while deviations in the funds rate are borderline at the 10 percent level. For the  $\bar{\rm r_t}\text{-}{\rm equation}$ , both DL components are highly significant.

# 5.3. ROLE OF THE FEDERAL FUNDS RATE.

The difference in the significance of fundsrate deviations in the two equations may have a reasonable explanation. Assume, for example, that at time (t-1),  $\bar{m}_{t-1}$  and  $\bar{r}_{t-1}$ are set and that, being in equilibrium,  $\bar{m}_{t-1} = m_t^{LR}$  . Further assume that, shortly after the meeting, incoming data indicate that  $\mathbf{m}_{\mathbf{t-1}}$  will be larger than desired. If by increasing the funds rate (within the tolerance range) the growth of  $M_1$  is brought back to the desired path, then there would be no reason to modify  $\bar{m}_{t}$  at the next meeting, assuming  $m_{\rm t}^{\rm LR}$  remains unchanged. What could be expected is  $\bar{m}_t = m_t^{LR}$  and  $\bar{r}_t > \bar{r}_{t-1}$ . In this case,  $m_{t}$  would not depend on  $dr_{t-1}$ , while obviously  $r_+$  would.

On the other hand, if the increase in the funds rate needed to bring  $\mathbf{m}_{\mathsf{t}}$  to the desired path is judged too large, then some deviation

in money growth would be accepted and, likely,  $\bar{m}_t < \bar{m}_{t-1}$  (in order to meet  $m_t^{LR}$ ) and  $\bar{r}_t > \bar{r}_{t-1}$ . Thus, although  $\bar{m}_t$  may depend on  $dr_{t-1}$ , this dependence is stronger for the case of  $\bar{r}_t$ . If, eventually, the target is not met and  $dm_{t-1} \neq 0$ , then both targets would be modified. Hence deviations in money growth should be significant in both equations.

#### 5.4 ESTIMATED LAG DISTRIBUTION.

The shapes of the four DLs are given in Figure 6. The  $\omega^*$ -weight tends to decrease as the corresponding lag increases, except for a small peak at lag 12.4). The  $\lambda^*$ -weights behave following a more erratic pattern, in accordance with the fact that  $\lambda(L)$  was estimated with less precision. The  $\alpha^*$  and  $\beta^*$  -weights both gradually decrease, exhibiting negative correlation between adjacent values. This correlation is also present in  $\omega(L)$  and  $\lambda(L)$  and can be attributed to the lag-1 autocorrelation in the  $[dm_{+}^{O}]$  and  $[dr_{+}]$  series. The correlation between adjacent coefficient estimates within a particular DL is not a matter of concern to us since we shall not be interested in individual coefficient estimates. Of more interest to us are the values of the gain functions, or total multipliers.

### 6. THE GAIN FUNCTIONS

Deviations of monetary aggregates from their targets were seen in Section 3 to be caused by different types of unanticipated shocks, for which different policy responses were appropriate. During the period we consider, FOMC intended behavior was, for a supply shock, adherence to the monetary aggregate target, offsetting therefore the money growth deviation (see Lindsey /10/ and references in section 3.1). For a shock originating on the demand side, the Fed considered it a more adequate response to accommodate, at least partly, the change.

Assume an equilibrium situation, satisfied at all times before t, when there have been no shocks, and  $dm_{t}^{O}$ , =  $dr_{t}$ ,=0, t'<t. Such a system is growing at the rate

$$\bar{m}_{+}$$
, =  $m_{+}$ ,  $\bar{l}R = m^{*}$ ,

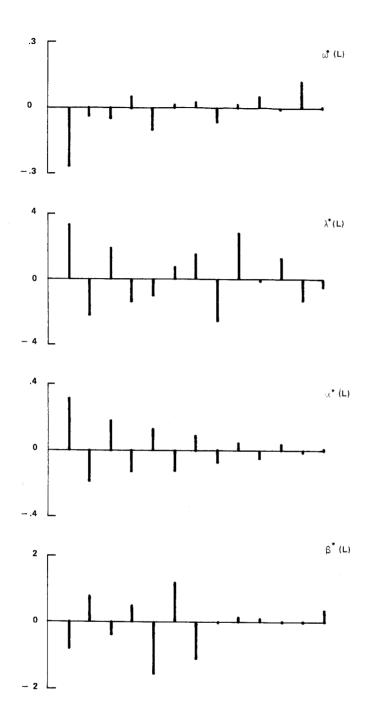


Figure 6: Distributed lag profiles

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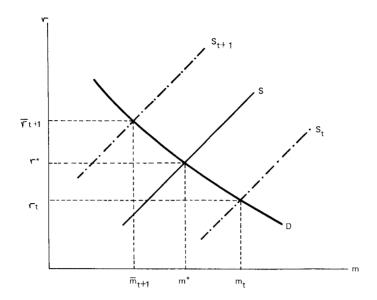


Figure 7: Response to money supply shocks

with the funds rate set at the level

$$\overline{r}_{t}, = r_{t}, = r^{*},$$

so that  $\nabla \bar{r}_{t}$ ,=0. The values (m\*,r\*) are constant and are the equilibrium values associated with an underlying supply-demand system, S and D, such as in Figure 7.

At time t,  $\bar{m}_t = m^*$  and  $\bar{r}_t = r^*$ , but assume there is an unexpected (one-period) shift in supply, so that S moves to  $S_t$ . The new equilibrium values will be  $m_t$  and  $r_t$ , hence

$$dm_{t}^{O} = m_{t} - m^{*} > 0$$
,  $dr_{t} = r_{t} - r^{*} < 0$ . (11)

Assume that m is computed over periods of one month, and that deviations with respect to targets are also offset in one month. The exogenously given long-run target (measured quarterly) remains unaffected, so that in period (t+1) the monetary authority will have to decrease the money supply in such a way as to compensate for the undesired supply shocks. Thus the authority will attempt to move the supply towards  $S_{t+1}$  and the new targets will be  $\bar{r}_{t+1}$  and  $\bar{m}_{t+1}$ . Therefore,  $\bar{r}_{t+1} > r^*(=\bar{r}_t)$  and  $m_{t+1} < m^*(=m_t^{LR})$ 

$$\nabla \bar{r}_{t+1} > 0$$
,  $\bar{m}'_{t+1} < 0$  , (12)

where  $\bar{\mathbf{m}}_{t+1}' = \bar{\mathbf{m}}_{t+1} - \mathbf{m}^{LR}_{t+1}$ . Moving from the comparative statics framework to our model written as (9), expressions (11) and (12) imply that the total multipliers should satisfy the constraints

$$\omega^*(1) < 0$$
 ,  $\lambda^*(1) > 0$ 

and

$$\alpha^*(1) > 0$$
 ,  $\beta^*(1) < 0$  .

If the unexpected shift is in demand, insofar as it is partly offset, similar reasoning yields

$$\omega^*(1) < 0$$
 ,  $\lambda^*(1) < 0$ 

and

$$\alpha^*(1) > 0$$
 ,  $\beta^*(1) > 0$  .

Thus in both cases the gain of the DL which applies to deviations in money growth has the same sign whether the unexpected shift

is in demand or in supply, while the one corresponding to deviations in the funds rate has different signs in each case. Therefore:

1. The value of  $\omega^*(1)$  should be less than zero. However, it is difficult to specify a priori a numerical value, since such a value depends on the relative importance of the deviations that are accomodated. We shall simply require

$$-1 \leq \omega^*(1) \leq 0$$
 . (13a)

2. Since the numerical value of  $\alpha^*(1)$  depends on the units of measurement of money growth and interest rates, we simply require

$$\alpha^{*}(1) \geq 0 \quad . \tag{13b}$$

3. The expressions  $\lambda^*(1)$  and  $\beta^*(1)$  should have opposite signs, although which is positive depends on whether supply shocks or demand shocks dominate in the short run. Thus

$$sgn |\lambda^{*}(1)| = -sgn |\beta^{*}(1)|$$
 (13c)

Estimated values of the four gains are displayed in Table 3, where it is seen that the constraints (13) are satisfied.

Also, from the signs of  $\lambda^*(1)$  and  $\beta^*(1)$ , it is evident that the short run is mostly characterized by supply shocks.<sup>5)</sup> In fact, it is for this type of disturbances that intermediate money stock targeting is more adequate (se Davis /4/.

Given that in the short run money supply is more volatile than money demand, we would expect negative correlations both between the residuals  $\mathbf{a}_t$  and  $\mathbf{b}_t$  of (8) and between the targets  $\bar{\mathbf{m}}_t$  and  $\nabla \bar{\mathbf{r}}_t$ . In fact the estimated cross-correlation between  $\mathbf{a}_t$  and  $\mathbf{b}_t$  is  $\hat{\rho}_0 = -.22$  and that between the targets is  $\hat{\rho}_0 = -.30$ .

Finally, the coefficients of  $\mathbf{m}_{t}^{LR}$  are also in agreement with the "a priori" values. For the  $\mathbf{m}_{t}$  equation,  $\hat{\gamma}$  = .72,  $\hat{\phi}_{1}$  = .28 and  $\hat{\gamma}(1)$  = 1.02, so that  $\gamma^{\star}$  = 1. For the  $\nabla \bar{\mathbf{r}}_{t}$  equation,  $\pi^{\star}$ = 0, hence  $\pi^{\star}$  = 0.

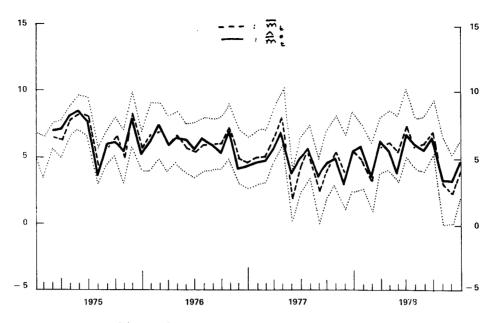


Figure 8: Actual and fitted values of  $\mathbf{m}_{\mathsf{t}}$ 

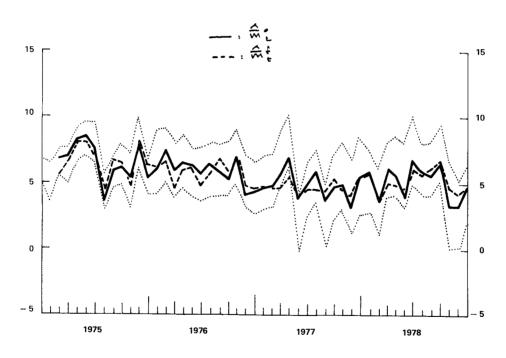


Figure 9: Fitted values with preliminary and final data (series  $\mathbf{m}_{\mathbf{t}})$ 

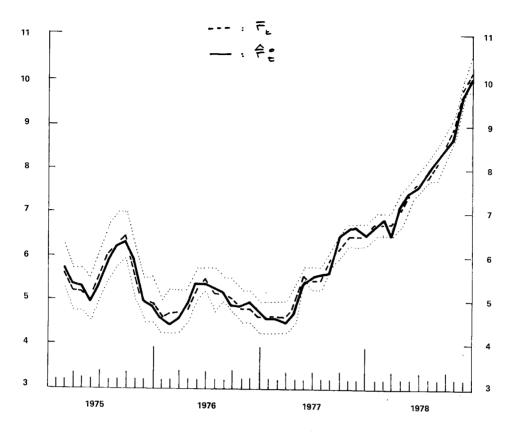


Figure 10: Actual and fitted values of  $\boldsymbol{r}_{\text{t}}$ 

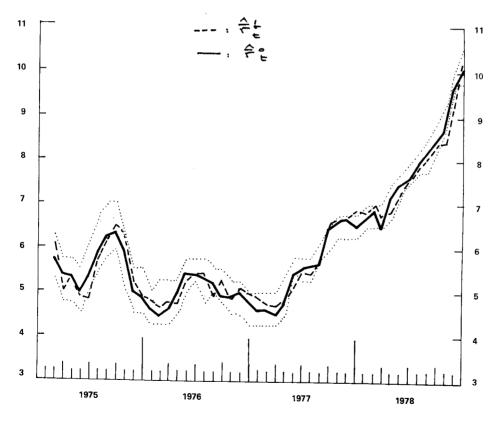


Figure 11: Fitted values with preliminary and final data (series  $\boldsymbol{r}_{\boldsymbol{t}})$ 

# Z. THE TARGETS WITH PRELIMINARY AND FINAL DATA.

Orthogonality of the revision error and the preliminary money stock measure imply that the model estimated with preliminary data can be applied to the final data. Thus we can infer what the targets would have been if the final data had been known. First, for the preliminary data, using the sequence  $[d\tilde{m}_{t}^{O}]$  and setting  $\phi_{1}=.30$ ,  $\phi_{2}=0$ , fitted values of  $\bar{m}_{t}$  and  $\bar{r}_{t}$ , denoted  $\bar{m}_{t}^{O}$  and  $\bar{r}_{t}^{O}$ , were computed through (9) with  $u_{t}=v_{t}=0$ .

Second, replacing the sequence  $\{dm_{t}^{O}\}$  in (7) with the corresponding one for final data (the first element being  $d\hat{m}_{t-1}^{f}$ , the one-period ahead ARIMA forecast, since for final data M1 is still assumed unknown next month), we obtain estimates of the targets that would have been set if final data had been available,  $\bar{m}_{t}^{f}$  and  $\bar{r}_{t}^{f}$ . Figures 8 and 9 show the series of actual and fitted targets, together with the (actual) tolerance ranges.

In order to assess the effect of the revisions, we note first that the difference between the two sets of fitted values is

$$\mathbf{x}_{\mathsf{t}} = \widetilde{\mathbf{m}}_{\mathsf{t}}^{\mathsf{f}} - \widetilde{\mathbf{m}}_{\mathsf{t}}^{\circ} = -\omega \star (\mathbf{L}) \delta_{\mathsf{t}}$$
 (15)

Similarly, for the funds rate the estimated revision effect is

$$y_{t} = \overline{r}_{t}^{f} - \overline{r}_{t}^{\circ} = -\alpha^{*}(L)\delta_{+} . \qquad (16)$$

Note that in (15) and (16),  $\delta_{\rm t}$  is  $\delta_{\rm t}^{\rm O}$  and  ${\rm L}^{\rm j}\delta_{\rm t}$  is  $\delta_{\rm t-j}^{\rm (j)}$ .

Figures 10 and 11 compare  $[m_t^O]$  with  $[\bar{m}_t^f]$  and  $[\bar{r}_t^O]$  with  $[\bar{r}_t^f]$ , respectively. Practically all the targets that vould have been set if final data had been available lie within the tolerance range, set when only preliminary information on recent money growth is available.

Comparing Figures 2 and 10, it is seen that although preliminary and final data often give conflicting signals as to whether growth of M1 is as desired or not, the effect of these conflicting signals on the setting of short-run targets is rather small. The targets would not have been much different if

the (revision) error in preliminary data had not been present, in spite of its size.

This smoothing effect is also evidenced by the fact that, while the standard deviations of  $\delta_{\rm t}$  and  $\delta_{\rm t}^{\rm S}$ , the total and seasonal revision errors, are 3.57 and 2.72, the standard deviation of the difference in M-targets is .69. The smoothing, according to (15), is due to  $\omega^{\star}({\rm L})$ . To get a better understanding of this mechanism, let us assume first that all deviations  ${\rm dm_{t}}$  are fully offset (a "pure" monetary aggregate targeting policy). If  $z_{\rm t}$  denotes the annualized monthly rate of growth of M1 (in percent points), so that

$$z_t = 1200 \, V \log M_{t+1}$$
 ,

then since

$$m_t = 600(1+L)(1-L)\log M_{t+1}$$
,

it follows that

$$m_t = \frac{1+L}{2} z_t$$
.

If implicit in the two-month targets there are monthly targets, then

$$dm_t = \frac{1+L}{2} dz_t , \qquad (17)$$

where  $dz_{t}$  represents the monthly deviation. Hence the term " $\omega^{\star}(L)\,dm_{t}$ " of (9a) becomes

$$W(L)dz_t = \frac{1}{2}(1+L)\omega^*(L)dz_t$$
 (18)

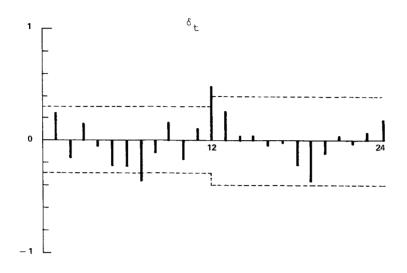
It is easily seen that, if an undesired change in the level of M1 is offset by exactly the same amount, then

$$W(1) = -1$$
.

Hence setting L = 1 in (18) yields

$$\omega^*(1) = -1$$
.

However, our estimate of (1) was -.30, which implies that, for a deviation of 1, only .30 of it would eventually be offset  $^6$ . It seems quite unlikely that money demand shocks that should be accomodated can account for 70 percent of target misses. An explanation of the numerically low value of  $\omega^*(1)$  is given in the next section.



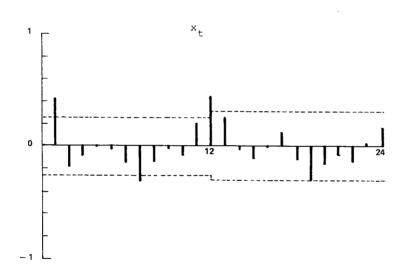


Figure 12: Autocorrelation functions of  $\boldsymbol{\delta}_{t}$  and  $\boldsymbol{x}_{t}$ 

# 8. NOISE EXTRACTION.

Growth of the money supply is subject to erratic, transitory movements that tend to cancel out over relatively short periods (see Pierce et.al. /14/. Such movements are present in both demand and supply shocks. In terms of policy, it could be reasonable to ignore them, focusing instead on a smoother component, presumably some type of trend. Similar approaches have been taken for other variables followed closely by policy makers--see, for example, Blinder /2/ and Davidson /3/.

Thus, assume the FOMC intends to react to a signal  $\mu_{+}^{\,f}$  in the final data, where

$$m_t^f = \mu_t^f + n_t^f ,$$

and the noise  $n_{\mathsf{t}}^{\mathsf{f}}$  is orthogonal to the signal. If consequently the targets are set for the signal, then

$$dm_{+}^{f} = d\mu_{+}^{f} + n_{+}^{f} ,$$

where  $d\mu_t^f = \mu_t^f - \bar{m}_t$ . In terms of the preliminary data,

$$dm_{t}^{\circ} = d\mu_{t}^{\circ} + n_{t}^{\circ} , \qquad (19)$$

where  $d\mu_{\text{t}}^{\text{O}}$  and  $n_{\text{t}}^{\text{O}}$  are the undesired deviation of, and the noise in, the preliminary signal. From (5), if  $\nu_{\text{t}}$  is the signal component in  $\delta_{\text{+}}$ , then

$$d\mu_t^f = d\mu_t^o + \nu_t .$$

The noise-reduction effect then follows easily: the revision in the data is large, but the revision in the signal contained in it is relatively small. Since targets are set for the signal, the difference in targets induced by revisions in the data is also small  $^{7}$ ). This explanation is also in agreement with the dynamic features of the series of differences in targets,  $\mathbf{x}_{t}$ , and the series of revisions,  $\delta_{t}$ . Figure 12 compares the ACF of  $\mathbf{x}_{t}$  and  $\delta_{t}$ . The shape of both functions is similar; hence the large difference in variance can be attributed to a large noise component in the revision series.

If the preliminary growth measure is not taken at face value and the targets express

the desired growth of a signal contained in the data, then in equation (9a) the variable  $dm_{t}^{O}$  should be replaced by  $d\mu_{t}^{O}$ ; and similarly for (9b). Having used  $dm_{t}^{O}$ , we have incurred in a traditional Errors-in-Variables (EIV) situation, and the error  $u_{t}$  is correlated with  $dm_{t}^{O}(through\ n_{t}^{O})$ . Therefore, our parameter estimates are inconsistent. However since the regressors  $[dm_{t}^{O}]$ ,  $[dr_{t}]$ , and  $[m_{t}^{LR}]$  are approximately orthogonal, the estimates of  $\lambda^{*}(L)$  and  $\gamma^{*}$  will be essentially unaffected by the presence of error in  $dm_{t}^{O}$  (see the Appendix).

Of greater importance, since our purpose was to compare fitted values obtained with (9a), our interest is in estimating  $\mathbf{E}_{+}^{\phantom{\dagger}}(\mathbf{\bar{m}}_{+}^{\phantom{\dagger}})\,,$  which is the conditional expectation of  $[dm_{t-1}^{O}]$  ,  $[dr_{t-1}]$  , and  $[m_{t}^{LR}]$  . It is shown in the Appendix that this conditional mean is correctly estimated by using OLS on (9a), and more generally, that EIV assumptions do not cause any harm to either the fits or the forecasts. In other words, the OLS inconsistent estimates applied to the noisy data do provide consistent estimates of  $E_+(\vec{m}_+)$ . Therefore, the comparison we performed between  $\bar{m}_{t}^{O}$  and  $\bar{m}_{t}^{f}$  (i.e., the change in targets if the final data had been available) is still valid, despite the EIV structure of the model.

As for the estimates of the parameters in  $\omega^*(L)$  and of the gain  $\omega^*(1)$ , it is also shown in the Appendix that the effect of the EIV is to reduce their values by a factor inversely proportional to the signal-to-noise ratio.

An analysis of the noise component of the Ml series, based on univariate statistical techniques, is contained in Pierce et.al./14/, where it is assumed that whatever part of the Ml series is serially uncorrelated should be considered transitory noise. While such a method of noise extraction is unlikely to be used in the conduct of monetary policy, where considerations other than past values of Ml are also relevant, it is interesting to compare our results with their findings. For the month-to-month rates of growth of Ml (seasonally adjusted), the estimated standard deviation of the noise is 4.5; when the twomonth rates of growth are considered, this estimate becomes 2.5. The resulting variance ratio (see equation A.8) is .31, so that the

plim of the estimate of the gain would be attenuated by a factor of (1-.31), or .69.

Consequently, for a unit unexpected deviation in the rate of growth of M1, .30 would be offset and .31 could represent irrelevant transitory movements. The rest represents deviations due to money demand shocks that should be accommodated, plus biases due to other sources of error such as the use of once-a-year rather than concurrent seasonal factor estimation. (Based on results reported in Bayer and Wilcox (1981), a reasonable value for the asymptotic downward bias on tne estimate of the gain would be in the order of .1.) Therefore, even if most transitory deviations represent supply shocks, the proportion of demand shocks that are accommodated seems large. Since accommodation of (non-transitory) demand shift would eventually show up in changes in  $m_{t}^{LR}$ , the steady decrease of  $m_t^{LR}$  over our period may be partly related to a downward move of money demand associated with institutional and technological changes in financial markets (see Simpson and Porter, 1980).

To summarize, roughly 1/3 of M1 target misses can be attributed to transitory noise, 1/3 to deviations that are offset, and 1/3 to deviations that are accommodated. The extraction of noise and the accommodation of the demand-induced deviations explain why relatively large revisions in the data have relatively small impact on the setting of targets.

# 9. A FINAL COMMENT ON THE MEASUREMENT OF THE GROWTH OF M1.

It has been seen that the effective series on which monetary policy is based can be viewed as the result of a smoothing of a two-month rate of growth of seasonally adjusted Ml. Since self-cancelling, transitory noise should be removed irrespective of the source of the shock, it makes sense, first, to remove the noise and seasonality, extracting from the series a signal (presumably, some type of trend), and second, to identify which part of the deviation in the signal should be accommodated.

The point may be quite relevant. For example,

if the two-month targets are assumed to hold for the first of the two months (all targets expressed as annualized percent points), then the ACF of the series of monthly deviations in preliminary data resembles that of white noise, with variance of 38.85. Using as an estimate of the variance of the noise the one in Pierce et.al./14/, equal to 20.25, the ratio of the signal variance to the series variance is .52. If the series of deviations is white noise and the signal and noise independent, the latter two also have to be white noise. Hence, by a well-known result,

#### $E(d\mu_t^e|dm_t^e) = .5 dm_t^e$

where  $\mathrm{d}\mu_{\,\mathrm{t}}^{\,\mathrm{O}}$  and  $\mathrm{d}m_{\,\mathrm{t}}^{\,\mathrm{O}}$  are as in (16), but for month-to-month deviations. Thus, prior to any policy response, a new preliminary measured monthly deviation should be cut in half.

Finally, while seasonal adjustment relies heavily on statistical estimation, noise extraction is mostly judgemental. However, in general, signal (or trend) extraction within a model based approach offers several advantages. First, it facilitates systematic analysis, hence methodological improvements. Second, it could simplify seasonal adjustment. avoiding possible inconsistencies in the present procedure. Finally, it makes "political bias" more difficult to use. This bias is reflected in a tendency to consider a large undesired increase in M1 a statistical aberration when interest rates are high, and a large decrease an indication of the FOMC commitment to anti-inflation policy in periods of high inflation.

# 10. APPENDIX: SOME RESULTS ON EIV MODELS

Let the model be

 $y = X\beta_1 + Z\beta_2 + u ,$ 

where Z represents a set of variables observed without error and X is not directly observable. Instead, observations are available on a variable W related to X by

$$W = X + V . (A.1)$$

The shock u is assumed NID(0, $\sigma_{\rm u}^2$ ), uncorrelated with X, Z, and V. The errors in V are uncorrelated with X and Z. All random variables are

assumed Normal with zero mean. The variables X, V, and Z have finite limiting variance-covariance matrices, denoted  $\Sigma_{\rm X}$ ,  $\Sigma_{\rm V}$ , and  $\Sigma_{\rm Z}$ . Let  $\Omega$ ,  $\Sigma$ , and  $\Sigma_{\rm W}$  denote the covariance matrices of (W,Z), (X,Z) and W, respectively.

Let  $\underline{b} = (\underline{b}_1', \underline{b}_2')'$  be the OLS estimators of  $\underline{\beta} = (\underline{\beta}_1', \underline{\beta}_2')'$  in the regression of y on (W,Z). Then it is well-known (see, for example, Levi /9/ that

plim  $\underline{b} = \Omega^{-1} \Sigma \underline{\beta}$ .

If W and Z are uncorrelated, it is easily seen that

plim 
$$\underline{b} = \text{plim} \begin{pmatrix} \underline{b_1} \\ \underline{b_2} \end{pmatrix} = \begin{pmatrix} \Sigma_w^{-1} & \Sigma_x & \underline{\beta_1} \\ \underline{\beta_2} \end{pmatrix}$$
 (A.2)

and hence the OLS estimator of  $\underline{\beta}_2$  is consistent. In what follows, we shall not consider variables measured without error.

#### A.1 Consistency of OLS Fits

Let the true model be:

$$y = X\beta + u$$
,

where (A.1) holds, together with the relevant assumptions of the previous section. For a particular set of observations W, consider the estimation of E(y|W), and let  $\underline{b}$  denote the OLS estimator of  $\underline{\beta}$  in the regression of y on W.

Lemma:

E(y|W) = W plim b.

Proof:

W plim  $\underline{b} = W \text{ plim}(W'W)^{-1}W'(X\underline{B} + u) =$   $= W \text{ plim} \{(W'W)^{-1}W'X\}B .$ 

Since (W,X) are jointly Normal,

 $E(X|W) = W \Pi$ ,

where  $\Pi$  is estimated consistently by  $(W'W)^{-1}W'X$ , (i.e., by an OLS regression of X on W). Hence

W plim  $b = W\pi\beta = E(X|W)\beta = E(y|W)$ , q.e.d.

The results tells us that, although an EIV assumption produces inconsistent OLS parame-

ter, it does not cause much harm to the OLS fits as estimators of the conditional mean of y. The inconsistent parameter estimators applied to the noisy data are consistent estimators of the expected value of the endogenous variable, for a given set of observations. (For the case in which X has one variable,

 $E(y|W) = \beta E(X|W) = \beta (\sigma_y^2/\sigma_w^2)$ .

Since plim b =  $\beta(\sigma_X^2/\sigma_W^2)$ , E(y|W) = plim b W.)

The proof is easily extended to show that

$$E(y|W_f) = W_f \text{ plim } \underline{b}$$
,

where  $W_{\mathbf{f}}$  represents out of sample values of the exogenous variables. Thus the Lemma applies equally to forecast computation.

Effect on Individual Parameters and on the Gain.

From the first subset of equations in (A.2),

$$plim \ \underline{b} = \Sigma_{w}^{-1} \ \Sigma_{x} \ \underline{\beta} \quad , \tag{A.3a}$$

and, since  $\Sigma_{\rm W}$  =  $\Sigma_{\rm X}$  +  $\Sigma_{\rm V}$ , this can also be expressed as

$$plim \ \underline{b} = (I - \Sigma_{w}^{-1} \ \Sigma_{v})\underline{\beta} . \tag{A.3b}$$

For the model we consider in the paper, W denotes lagged values of the observed variable dm $_{\mathsf{t}}^{\mathsf{O}}$ . The true unobservable variable X is the signal d $\mu_{\mathsf{t}}^{\mathsf{O}}$ , and V is the noise  $n_{\mathsf{t}}^{\mathsf{O}}$ , where

$$dm_{t}^{\circ} = d\mu_{t}^{\circ} + n_{t}^{\circ} , \qquad (A.4)$$

with uncorrelated signal and noise. In section 5 we saw that  $\mathrm{dm}_{\mathsf{t}}^{\mathsf{O}}$  could be assumed to be an MA(1) process. In fact, since the lag-1 autocorrelation of  $\mathrm{dm}_{\mathsf{t}}^{\mathsf{O}}$  is approximately .5, the MA parameter should be equal to -1. This is also in agreement with the following argument. From (17),

$$dm_t^{\circ} = \frac{1}{2} (dz_t + dz_{t-1})$$
,

and hence

$$dm_t^{\circ} = \frac{1}{2} [(z_t - \overline{z}_t) + (z_{t-1} - \overline{z}_{t-1})]$$
.

Since both are successive 1-period ahead

forecast errors, it follows that  $dm_{t}^{O}$  has approximately the MA(1) representation

$$dm_t^o = \frac{1}{2} (a_t + a_{t-1})$$
, (A.5)

where  $a_t$  is white-noise. Thus  $dm_t^O$  is a non-invertible MA(1) process.

The three variables in (A.4) are monthly series of two-month periods. Hence each series "overlaps" one month. The noise contained in one month appears in two successive values of  $n_{\rm t}^{\rm O}$ . Thus  $n_{\rm t}^{\rm O}$  should also be an MA(1) process. Since  $dm_{\rm t}^{\rm O}$  is also an MA(1), the same should be true of the signal  $d\mu_{\rm t}^{\rm O}$ . Therefore, the three variables in (A.4) are MA(1)'s. The following Lemma allow us to identify uniquely the parameters in the signal and noise process.

<u>Lemma:</u> Let  $y_t$  be the sum of several independent components, each an MA(1) process. If  $y_t$  is non-invertible, then all components are also non-invertible.

Proof: Write

$$y_t = \sum_i x_t^{(i)} ,$$

where

$$x_t^{(i)} = a_t^{(i)} + \theta_i a_{t-1}^{(i)}$$

with  $a_t^{(i)}$  ~ NID(0,  $\sigma_i^2)\,.$  Obviously  $y_t$  is an MA(1) process, of the type

$$y_t = b_t + \theta b_{t-1} .$$

Since  $y_t$  is non-invertible,  $\theta$  = 1 or  $\theta$  =-1. Consider first the case  $\theta$  = 1. Then:

$$\rho_{y}(1) = \frac{\sum_{i=1}^{\infty} \theta_{i} \sigma_{i}^{2}}{\sum_{i=1}^{\infty} (1+\theta_{i}^{2}) \sigma_{i}^{2}} = \frac{1}{2} . \tag{A.6}$$

Letting  $k_i = \sigma_i^2/\sigma_1^2$ ,  $i \ge 2$ , expression (A.6) yields

$$(1-\theta_1)^2 + \sum_{i>2} (1-\theta_i)^2 k_i = 0 . \tag{A.7}$$

Since  $k_i > 0$ ,  $\forall i$ , this implies  $\theta_i = 1$ ,  $\forall i$ .

When  $\theta$  = -1, the 1/2 of expression (A.6) becomes -1/2 and (A.7) is replaced by

$$(1+\theta_1^2) + \sum_{i\geq 2} (1+\theta_i)^2 k_i = 0 ,$$

which implies  $\theta_i = -1$ ,  $\forall i$ , finishing the proof.<sup>8)</sup>

Applying the Lemma to our model, we have that the three series in (A.4) are MA(1) processes with unit root, and parameter equal to 1. Therefore, expressions (A.3a and b) can be greatly simplified. The three matrices  $\boldsymbol{\Sigma}_{\mathbf{W}}$ ,  $\boldsymbol{\Sigma}_{\mathbf{X}}$ , and  $\boldsymbol{\Sigma}_{\mathbf{V}}$  can be expressed as hH, where, in all cases, H is the matrix:

$$H = \begin{pmatrix} 1 & .5 & 0 & ... & 0 \\ .5 & 1 & .5 & 0 \\ .5 & ... & ... & ... \\ 0 & .5 & ... & ... \\ \vdots & ... & ... & ... \\ 0 & .0 & .5 & 1 \end{pmatrix}$$

and h is  $\sigma_w^2$ ,  $\sigma_x^2$ , and  $\sigma_v^2$ , respectively. Thus, after simplification (A.3) becomes

plim 
$$\underline{b} = \frac{\sigma^2}{\sigma^2}$$
  $\underline{\beta} = \underline{\beta} \left( 1 - \frac{\sigma^2}{\sigma^2} \right)$ 

Therefore the EIV assumption has an identical effect on each of the parameters, namely, to shrink the numerical value toward zero.

In terms of the gain, letting

$$g_b = 1'b$$

$$g_{\beta} = 1'\beta$$
 ,

where  $\underline{1}' = (1...1)$ , it is easily seen that

plim 
$$g_b = \frac{\sigma^2}{\frac{x}{\sigma^2}} g_\beta = g_\beta \left( 1 - \frac{\sigma^2}{\frac{v}{\sigma^2}} \right)$$
, (A.8)

so that the same shrinking effect takes place. The net effect is seen to depend on the relative contributions of the signal and noise to the variance of the observed series. Since

$$\frac{\sigma^2}{\sigma^2} = \frac{1}{1+\nu} \quad ,$$

where  $v = \sigma_X^2/\sigma_V^2$ , the asymptotic bias can be expressed in terms of the signal-to-noise ratio. The smaller the signal, the larger the bias will be.

In the terminology used in the paper,  $g_b = -.30$ ,  $g_\beta = \omega^*(1)$ ,  $\sigma_w^2 = \mathrm{Var}\ (\mathrm{dm}_t^0)$  and  $\sigma_v^2 = \mathrm{Var}\ (n_+^0)$ .

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#### 12. NOTES.

- 1) A typical statement reads "If it appears that the growth rate over the two-month period will deviate significantly from the midpoint of the indicated range, the operational objective for the Federal funds rate shall be modified in an orderly fashion..."
- 2) The original series and the computation of  $m_t^0$ ,  $m_t^f$  and the intermediate series are described in Maravall and Pierce /12/.
- 3) The orthogonality of u<sup>\*</sup><sub>t</sub> and the regressors in (5) is based on the assumption of concurrent adjustment whereas in practice seasonal revisions are computed once a year. This will introduce some inconsistency in the parameter estimates, but the effect is

- 3) likely to be small (see section 9).
- 4) This peak could be attributed to the fact that some of the more distant lagged dm<sup>O</sup><sub>t-j</sub> values would not be used as such when setting targets, since the first year seasonal revision would already be available. Roughly, what is likely to happen is that some corrections are made after the preliminary measured deviations in money growth are modified after 12 months of additional data have become available.
- 5) Consistent with this finding, Poole /15/ states "my guess is that the vast bulk of weekly and monthly money-growth surprises reflect money-supply disturbances rather than either IS or money-demand disturbances".
- 6) The standard deviations of the estimates of  $\omega^*(1)$  and  $\alpha^*(1)$  were .15 and .04, respectively.
- 7) This interpretation is, on occasion, contained in press coverage of monetary policy. For example, the lack of reaction to self-cancelling noise is implied by the following quotation: "The Fed is viewing the April M-1 growth as an aberration, and is willing to give it some time to be reversed in coming weeks" (International Herald Tribune, May 3, 1982). The use of a signal which is less affected by revisions is implicit in the following excerpt from an editorial in The Washington Post: "The rule of wisdom,... for people who make policy, is to pay more attention to general trends over the months than to the latest flash number. An unexpected number may mean that a trend is changing. Then again, as time passes, it may also be the number that gets changed", (March 17, 1982).
- 8) An obvious corollary is the following: under the same assumptions, if there is at least one invertible component, the aggregate will also be invertible.