LOWER BOUND STRATEGIES IN COMBINATORIAL NONLINEAR PROGRAMMING. A CASE STUDY: ENERGY GENERATORS MAINTENANCE AND OPERATION SCHEDULING

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The Generator Maintenance and Operation Scheduling problem is presented as large-scale mixed integer non-linear programming case. Several relaxations of the integrality condition on the variables are discussed. The optimal solution of the model based on these relaxations is viewed as the lower bound of the optimal solution in the original problem. A continuous constrained non-linear programming algorithm is used in the optimization of the relaxed formulation. Computational experience on a variety of real-life problems is provided.

NOTATION USED IN THE PROBLEM'S FORMULATION

A. Conflict maintenance scheduling constraints matrix.

b. Right hand side vector of system $AX \leqslant b$.

B. Power transmission losses matrix in function T_ℓ; it is assumed to be square, symmetric and positive definite.

B_0. Constant term in function T_ℓ.

B_i. Linear coefficient for generator i in function T_ℓ.

C. Production cost function to be minimized.

$C_{i\ell}$. Production cost function for generator i and period ℓ.

D_i. Maintenance outage duration in integral and consecutive periods for generator i.

E_ℓ. Power demand by the system at period ℓ.

i=1,...,I. A given power generator

$\phi(P_i/M_i)$. Continuous function (see eqs. (8) and figure 1) that in formulation F2 approximates constraints (5) and (6) of formulation F1.

F1. Original formulation of the problem

F2. Alternate formulation to F1, such that the Y-variables have desappeared; both formulations are equivalent for feasible solutions.

F3. Formulation obtained by relaxing the integrality condition of the X-variables in formulation F2.

F4. Formulation obtained by relaxing the integrality condition of the Y and X-variables in formulation F1.

ℓ=1,...,L and t. A given period (week) in the planning horizon.

m_i and M_i. Lower and upper bounds on the output of generator i if it is not in maintenance at a given period.

$P_{i\ell}$. Output power of generator i at period ℓ.

$Q_{i\ell} \equiv P_{i\ell}/M_i$. A continuous (0;1) variable.

$t_i^{(0)}$ and t_i. Earliest and latest available periods for beginning maintenance on generator i.

$t_i^{(2)} = \max \{t_i^{(0)}, \ell - D_i + 1\}$.

$t_i^{(3)} = \min \{\ell, t_i^{(1)}\}$.

T_ℓ. Power transmission losses function for period ℓ.

X_{it}. Binary variable such that $X_{it} = 1$ if generator i begins maintenance at period t; otherwise, $X_{it} = 0$.

X. Column vector of binary variables $\{X_{it}\}$.

$Y_{i\ell}$. Binary variable such that $Y_{i\ell} = 1$ if generator i is in maintenance in period ℓ; otherwise, $Y_{i\ell} = 0$.

Y. Column vector of binary variables $\{Y_{i\ell}\}$.

1. INTRODUCTION

The increased cost of fossil fuels used in the production of electricity has prompted the utility industry to seek more efficient operating procedures. One of the most promising of these require new methods for the automated scheduling of generators maintenance. These refined techniques will help minimize

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the cost of production.

It is expected that better generators maintenance schedule planning will result in two areas of savings. First, such planning will allow more efficient generators to be available more often during the yearly production cycle. Lessened fuel usage can amount to several million dollars a year in reduced production costs. Second, better maintenance planning may postpone generation expansion. This results in postponed capital construction costs. In addition to reduced cost saving, the maintenance crews and operating plants can be utilized more efficiently.

The purpose of this work is to find a strong lower bound to the solution of the generators maintenance scheduling problems, so that (a) an ample set of maintenance constraints is satisfied, (b) the electricity demand at the peak load hour of each period (usually, a week) is satisfied, and (c) the non-linear production cost of electricity over the planning horizon (usually, one to two years) is minimized or, at least, the difference with the optimal solution is not greater than a given value.

This paper is organized as follows. Section 2 briefly describes the problem and presents its mixed integer non-linear formulation. Section 3 describes a relaxation of this formulation. Section 4 discusses some computational experience obtained by applying a constrained non-linear programming algorithm to solve the new relaxed problem.

2. PROBLEM FORMULATION

See in Escudero et al. /5/ a full discussion of the application area, maintenance scheduling constraints and types of objective functions to be optimized. Escudero /6/ presents a methodology for dealing with the generators maintenance scheduling problem when the production cost of electricity (a non-linear function) is to be minimized subject to a large system of several thousands of linear constraints with several hundreds of binary variables and several thousands of semi-continuous variables.

In this paper we present an alternative formulation to the model described in /6/ so that (a) the semi-continuity condition on the production variables is relaxed by a non-linear formulation, and (b) a new type of constraint (the electricity transmission losses) is included. This new constraint is also non-linear. The optimal solution of the new formulation is a strong lower bound to the optimal solution of the original problem, so that the goodness of any feasible solution may be measured in terms of its maximum difference from the optimal feasible solution.

Briefly, the problem is as follows. In an electrical power system, the goal consists in obtaining the power generators maintenance and operation scheduling to minimize the cost of satisfying a prescribed demand for electric power over a given planning horizon (usually, 52 weeks). Suppose that at weeks $\ell = 1,2,\dots,L$ in the period under consideration, it is known that the power demands on the system are $E_1, E_2,\dots, E_L$. The problem is to determine appropiate outputs from the power generators $i = 1, 2,\dots, I$ at each of these weeks so as to minimize the cost of satisfying the demands. Let I be 25. Here we only consider the output, cost and the demand of the peak load hour for each week of the planning horizon.

At each week ℓ a generator may be available for the system, in which case the output, say $P_{i\ell}$, must be $m_i \leq P_{i\ell} \leq M_i$ (where m_i and M_i are given lower and upper bounds), or the generator may be unavailable for the system (it is the case when it is in maintenance, and then $P_{i\ell} = 0$). Variable $P_{i\ell}$ is termed *semi-continuous*. Let X_{it} be a binary variable such as $X_{it} = 0$ if the maintenance is not beginning in this wekk. Generator i will be unavailable for the production system in week ℓ, if $X_{it} = 1$ and $t \leq \ell \leq t + D_i - 1$, where D_i is the maintenance outage duration in integral and consecutive weeks. Let $t_i^{(0)}$ and $t_i^{(1)}$ denote the earliest and latest available weeks for beginning maintenance on generator i. Usually, generators are maintained once and only once (if any) over the planning horizon (see other variant in /6/). Then for the generators to be maintained, $\sum_t X_{it} = 1$ for $t = t_i^{(0)}, t_i^{(0)} + 1, \dots, t_i^{(1)}$ is the classical *special ordered set of type 1 or S1*. See e.g. /3/. If all generators are to be maintained, there are

I = 25 constraints of this type.

Usually, there are many exclusivity constraints among the periods in which the generators are to be maintained. The most typical constraints are (see in /5/ the details and mathematical formulation):

1) For a particular week, the total rating of generators in maintenance cannot be greater than a given amount (termed gross reserve)
2) Maintenance crews are assigned to power plants, or sets of generators, and are not available to simultaneously work on different generators. No more than one generator belonging to the same physical set may be in maintenance in the same week.
3) It is forbidden that more than a given number of generators belonging to the same special class may be out of the production system in the same week.
4) It is frequent that there are constraints, such that the elapsed time between the beginning of the maintenance in generators, say i and j, must be greater than a given number of weeks; other type of constraints requires that generator j cannot begin maintenance before a given number of weeks following the ending of maintenance in generator i; etc.

These types of restrictions may amount to several thousands of mathematical constraints. The corresponding constraints matrix is very sparse; consider that in each constraint there are involved only a few generators per each week and that different weeks produce different mathematical variables and constraints for the same type of restriction. Let $AX \leq b$ denote these constraints system, where A is the constraints matrix (it is very sparse with many 1's in its non-zero elements), X is the column vector of binary variables $\{X_{it}\}$, and b is the restriction vector (with many 1's in its non-zero elements). A typical problem involves I = 25 generators with a total of 700 possible weeks for beginning maintenance (that is, the dimension of vector X is 700), and the number of rows in matrix A varies from 52 (number of weeks in the horizon and, then, number of gross reserve constraints) to several thousands. In the case for which computational experience is reported, the number of rows is 920 with a density in matrix A of 1.02% of non-zero elements. The system $AX \leq b$ is linear with $X \in \{0;1\}$.

To account for transmission losses in the transmission network, it is necessary to derive a function of the power losses in each week in terms of the generated output powers $P_{i\ell}$; then the total power required consists of two components: the system demand E_ℓ for each week (that it is assumed to be known) and the transmission losses T_ℓ what are unknown. Most utilities use the so-called approximate B-constant formulation (see /9/-/12/) to represent transmission losses by the quadratic loss formula

$$T_\ell = B_0 + \sum_{i=1}^{I} B_i P_{i\ell} + \sum_{i=1}^{I} \sum_{j=1}^{I} P_{i\ell} B_{ij} P_{j\ell} \quad (1)$$

where the B-matrix is square, symmetric, and positive definite at least for $m_i \leq P_{i\ell} \leq M_i$. Then the formulation of the constraints that represent the relation between the output of the system and the demand to be satisfied is as follows: $\sum_i P_{i\ell} - T_\ell \geq E_\ell$ for $\ell = 1,2,\ldots,L$ There are L = 52 constraints of this type.

In the unusual case in which the output power $P_{i\ell}$ of all generators $\{i\}$ that are not in maintenance at week ℓ is their allowed minimum $\{m_i\}$, the total load (that is, output power minus transmission losses) at this week may be greater than the system demand E_ℓ; but, usually, the total load exactly covers the system demand.

Since if generator i is in maintenance in week ℓ, it is not available for the production system (then, $P_{i\ell} = 0$) and, otherwise $m_i \leq P_{i\ell} \leq M_i$, we may represent this restriction as follows: $m_i Y_{i\ell} \leq P_{i\ell} \leq M_i Y_{i\ell}$ and $Y_{i\ell} + \sum_t X_{it} = 1$ for $\ell = 1,2,\ldots,$ L and t from $t_i^{(2)} = \max\{t_i^{(0)}, \ell - D_i + 1\}$ to $t_i^{(3)} = \min\{\ell, t_i^{(1)}\}$, where $Y_{i\ell}$ is a binary variable, such as $Y_{i\ell} = 0$ if generator i is in maintenance in week ℓ (being t the week in which it begins) and $Y_{i\ell} = 1$ for $\ell < t$ or $\ell > t + D_i - 1$. At most there are $I \times L = 25 \times 52 = 1300$ variables of this type. Since $t_i^{(0)}$ and $t_i^{(1)}$ are not necessarily $t_i^{(0)} = 1$ and $t_i^{(1)} = L - D_i + 1$ for all generators, in our case the number of constraints of this type is

2028 and the number of Y-variables is 676. For $\ell < t_i^{(0)}$ or $\ell > t_i^{(1)} + D_i - 1$, Y-variables are not needed and the above constraints are substituted by (in our case) $I \times L - 676 = 624$ -- bounds of the type $m_i \leq P_{i\ell} \leq M_i$ since generator i will be always in the production -- system.

The operation cost function for the planning horizon is

$$C = \sum_{i,\ell} C_{i\ell}(P_{i\ell})$$

It has separable components, in the sense -- that at week ℓ the cost of producing the -- output $P_{i\ell}$ by generator i is independent of the other generators output. In our case $C_{i\ell}(P_{i\ell})$ is a convex function.

Thus the problem of minimizing the operating cost over the planning horizon can be expressed by

$$\text{(F1)} \qquad \min . C = \sum_{i=1}^{I} \sum_{\ell=1}^{I} C_{i\ell}(P_{i\ell}) \tag{2}$$

subject to

$$\sum_{t=t_i^{(0)}}^{t_i^{(1)}} X_{it} = 1 \qquad \forall i \tag{3}$$

$$AX \leq b \tag{4}$$

$$Y_{i\ell} + \sum_{t=t_i^{(2)}}^{t_i^{(3)}} X_{it} = 1 \qquad \forall i, \ell \tag{5}$$

$$m_i Y_{i\ell} \leq P_{i\ell} \leq M_i Y_{i\ell} \qquad \forall i, \ell \tag{6}$$

$$\sum_{i=1}^{I} P_{i\ell} - T_\ell \geq E_\ell \qquad \forall \ell \tag{7}$$

where $X_{it} \in \{0;1\}$; $Y_{i\ell} \in \{0;1\}$; and $m_i \leq P_{i\ell} \leq M_i$ if generator i must not be maintenance in week ℓ. Function T_ℓ is given by (1).

In our case the dimensions of problem F1 are number of rows: 3025; number of X-variables: 707; number of Y-variables: 676; number of P-variables: 1300 (there are 624 that are explicitly bounded).

3. RELAXED FORMULATIONS

It is clear that F1 is a very sparse non-linear constrained problem with continuous and binary variables. In order to reduce the inconvenience of dealing with binary variables in non-linear problems, let us approximate - constraint types (5) and (6) by the following formulation suggested by Biggs /2/. Let $\phi_i(P_i/M_i)$ be a continuous function for $P_i \geq m_i$ having the following properties for

$$\phi_i(P_i/M_i) = P_i/M_i \tag{8a}$$

$$\frac{\delta \phi_i(P_i/M_i)}{\delta(P_i/M_i)} = 1 \tag{8b}$$

and it is desired that $\phi_i(P_i/M_i)$ is very -- small for $P_i < m_i$. Figure 1 shows the general form that is required for ϕ_i.

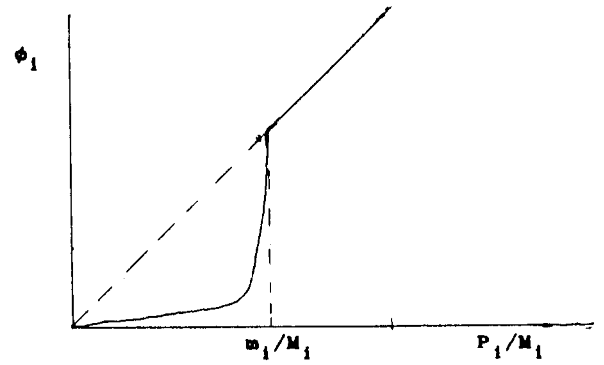

Figure 1. A continuous approximation to constraints (5) and (6)

Now let (5), (6) and (7) be replaced by the following continuous constraints

$$\frac{m_i}{M_i} + \left(1 - \frac{m_i}{M_i}\right) \sum_{t=t_i^{(2)}}^{t_i^{(3)}} X_{it} \leq Q_{i\ell} + \sum_{t=t_i^{(2)}}^{t_i^{(3)}} X_{it} \leq 1 \tag{9}$$

and

$$\sum_{i=1}^{I} M_i Q_{i\ell} - T_\ell \geq E_\ell \qquad \forall \ell \tag{10}$$

where

$$Q_{i\ell} = \begin{cases} P_i/M_i, & P_i \geq m_i \\ \phi_i(P_{i\ell}/M_i), & P_i < m_i \end{cases} \tag{11a}$$

and

$$T_\ell = B_0 + \sum_{i=1}^{I} B_i M_i Q_{i\ell} +$$

$$+ \sum_{i=1}^{I} \sum_{j=1}^{I} Q_{i\ell}(M_i B_{ij} M_j) Q_{j\ell} \qquad (11b)$$

Note that $0 \leq Q_{i\ell} \leq 1$. Biggs suggests the --- following expression for $\phi_i(P_i/M_i)$

$$\phi_i(P_i/M_i) = (a/M_i)P_{i\ell} + (b/M_i)P_{i\ell}^{n-1} -$$

$$- (c/M_i)P_{i\ell}^{n+1} \qquad (12)$$

for n>>1. Specifically, a = 0.01, b = = 0.000297, c = 0.0000198, and n = 6. Biggs made the following remark that it is also important in our context: the linear term must be present in $\phi_i(P_{i\ell}/M_i)$, since it is needed a non-zero derivative when $P_{i\ell} = 0$; otherwise, the system will be completely insensitive to any generators that are out of the -- production system.

Hence an alternative formulation of the generator maintenance and operation scheduling - problem F1 is

$$\text{(F2)} \quad \min. C = \sum_{i=1}^{I} \sum_{\ell=1}^{L} C_{i\ell}(M_i Q_{i\ell}) \qquad (13)$$

subject to constraints (3), (4), (9) and (10) where $X_{it} \in \{0;1\}$ and $Q_{i\ell}$ and T_ℓ are given by (11). The dimensions of the new problem are: number of rows: 2349; number of X-variables: 707; and number of Q-variables: 1300 continuous non-linear variables (624 have the bounds m_i/M_i and 1, and the rest have the bounds 0 and 1).

Note that formulations F1 and F2 are equivalent in the sense that a feasible solution - to one of them is also feasible to the other sharing the same objective function. The advantage of F2 over F1 is that the Y-variables have desappeared and its LP relaxation is -- tighter than the LP relaxation of F1.

4. COMPUTATIONAL EXPERIENCE

In this section we report some computational experience for obtaining lower bounds to the optimal solution of a variety of problems F1 with very similar dimensions.

The following definitions will be useful later: F3 is the formulation obtained by relaxing the integrality condition of the X-variables in formulation F2 and F4 is the formulation obtained by relaxing the integrality condition of the X and Y-variables in formulation F1. Then, both F3 and F4 are LP formulations.

We may see that the dimensions of problem F2 are smaller than the dimensions of problem F1; but it has higher non-linearities. We may try to solve problem F2 by using the --- branch-and-bound approach or some mixed integer non-linear programming algorithm (e.g. see /1/ and its references). But given the dimensions of problem F2, this approach is - not practical. Instead, and by exploiting -- the special structure of problem F2, we use - the following approach. First, we may note that for a given maintenance schedule $\{X_{it}\}$, problem F2 is converted in L different problems, each of which has a convex non-linear separable objective function, the convex non linear Knapsack constraint (10), and the corresponding variables $Q_{i\ell}$ in week ℓ whose generators are not in maintenance (being, $m_i/M_i \leq Q_{i\ell} \leq 1$). Then we may solve independently the L non-linear convex continuous Knapsack problems that are associated to each feassible maintenance node in the implicit enumeration approach (see the details in /5/and /6/). Before using the implicit enumeration algorithm, we may obtain a lower bound of -- the optimal solution to problem F1: a) by relaxing the maintenance constraints (3) and (4), and (b) by solving the corresponding L Knapsack problems may be non-convex, and some Q-variables are semi-continuous. With - this approach we obtain feasible solutions such that the best value obtained by using the implicit enumeration algorithm is not -- greater than 8% of this lower bound of the - optimal solution, in the cases with which we have experimented.

We obtain a stronger lower bound to the optimal solution of problem F1, by relaxing the integrality constraint of the X-variables of formulation F2. Let F3 denote the new problem. It should be noted that a schedule -- produced by F3 will contain values of $Q_{i\ell}$ lying between 0 and m_i/M_i. Because of -- the form of the constraints, such values -- tend to lie close to zero or close to m_i/M_i

and the optimum value of F3 may be a stronger lower bound of the optimum value of F1. If this solution is feasible in F1, the problem is solved; if it is infeasible its value is also a measure of the goodness of the best - current implicit enumeration solution.

In /6/ we describe an algorithm whose problem has not the component of transmission losses (1). The above approach also may be used here, since function $C_{i\ell}$ is convex and separable; in this specific case the lower bound - is very strong (the difference between the - current best feasible solution and this bound is by average not greater than 2%).

Anther lower bound to the optimal solution of problem F1 is obtained by relaxing the integrality condition on its binary variables X and Y, and solving the continuous non-linear programming problem (1)-(7) with $0 \leq X, Y \leq 1$ and continuous. Let F4 denote this new problem. Clearly, formulation F3 has a strictly smaller solution than formulation F4. The - former problem is tighter than the latter since eq. (11) allows a stronger reduction of - the possibility $0 < P_{i\ell} < m_i$; note that this value is not allowed in the original problem F1. Also note that constraints (5)-(7) and (9)-(10) are equivalent for $P \equiv MQ$. In any case, problem F1 may be solved by using its --- LP relaxation together with a branch-and-bound approach; see /1/.

For solving the continuous non-linear problem F3, we use the constrained non-linear programming algorithm described in /4/. Some remarks are in order:

(1) The Biggs approach (11a) and (12) to be dealt with the semi-continuous variables $\{Q_{i\ell}\}$ is quite satisfactory; although we must be aware of the possible inestabilities of parameters a, b and c.

(2) The initial point $X^{(0)}$ to be used by the constrained non-linear programming algorithm is feasible and it is provided by the implicit enumeration algorithm (see /5/ and /6/) applied to problem F1; it needs an average of 2.30 m of CPU time - in an IBM 370/158 computer to find the - first feasible solution. It is interesting to note that the time is only 0.22 m if the transmission loss component is deleted in constraint (7). Note that a -- feasible point in problem F1 is also feasible in problem F3.

(3) It is very fast to obtain by the algorithm the set J of active constraints -- and its Jacobian matrix $\hat{A}$, (see Appendix 1). The first estimate $\mu^{(0)}$ of the Lagrange multipliers vector for the set J, is obtained by minimizing $\| g_L^{(0)} \|^2$ where $g_L^{(0)}$ is the Lagrangian gradient vector; (see /4, eq. (3.7)). For obtaining $\mu^{(0)}$ we use the procedure described in (4, Sec. 4), such that $\mu_i^{(0)}$ is set to zero if -- the i-th active inequality constraint has a negative solution in the minimization of $\| g_L^{(0)} \|^2$. It is interesting to note that there is only an average of 11% inequality constraints that are active in - each feasible point of problem F1.

(4) The Lagrangian Hessian matrix $B^{(0)}$ at -- point $X^{(0)}$ is analytically evaluated; see /4, eq. (1.11)/ and Appendix 1. Note -- that $B_i^{(0)} = 0$ for constraints (3) and -- (4) independently of the value of X. Note also that $\mu_i^{(0)} = 0$ for the non-active inequality constraints. Matrix $B^{(0)}$ is scaled with formulation (3.10) of /4/; in anycase, it is very sparse and does not need so much storage capability.

(5) We use the direct BFGS approximation --- (see /4, eq. 5.8/ and Appendix 1) to obtain the Hessian matrix $B^{(k)}$, without -- using the Powell modification /4, eq. -- (5.10); our main concern was to preserve the sparsity condition, instead of keeping the positive definite property. Then we use the Shanno procedure /4, eq. (6.3)/ to keep the sparsity condition, being -- $\hat{B}^{(k)}$ the new matrix.

(6) At iteration k, see Appendix 1, we use the sparse updated matrix $\hat{B}^{(k)}$ to obtain the new matrix $B^{(k+1)}$ at the following - iteration. Procedure (7.1) of /4/ tests if $\hat{B}^{(k)}$ is positive definite; if it is, $\hat{B}^{(k)}$ is used to obtain the search direction $\delta^{(k+1)}$. If $\hat{B}^{(k)}$ is not positive definite, we use procedure (7.3) of /4/ to modify it, so that the resulting matrix $\bar{B}^{(k)}$ is used to obtain $\delta^{(k+1)}$

(7) Only in one iteration, it was found inconsistent the quadratic programming problem that is used to obtain the step direction δ, (see Appendix 1) in step (3) of the given algorithm. In this case we used the approach described in /4, Sec.9/

(8) It was detected that setting directly -- $\alpha^{(k)} = 1$ (see Appendix 1), often avoids to use in step (4) of the given algo---- rithm the approximate line search for obtaining the steplength. The reason is - that $X^{(0)}$ is very close to the optimum $\overset{*}{X}$

(9) In the cases with which we have experimented, it was not frequent, when criteria t_1 and t_2 described in /4, Sec. 10/, (see Appendix 1) were satisfied, $\mu_i < T_3$ for any active inequality constraint i. Usually, point $X^{(k)}$ is feasible in problem F3. T3 is a given tolerance; these results were obtained for $T3 = 10^{-4}$.

(10) In the quadratic programming problem QP (note that the Hessian matrix B of the - objective function is positive definite) to be solved in step (3) of the given algorithm for obtaining the step direction $\delta^{(k)}$ (see Appendix 1), we force $\delta = 0$ to be the first solution of $\delta^{(k)}$ (usually, it is feasible in QP). Since $X^{(k-1)}$ is close to $\overset{*}{X}$ and based on the strategy devised for obtaining the variables that - in QP will be basic, superbasic and nonbasic /7/, the execution of QP is very - fast. In fact, most of the basic variables are slack variables (they correspond to inequality constraints), most of the structural variables are nonbasic , and most of these variables do not change their status during the QP execution.

(11) A typical QP to be solved at iteration - k = 1 requires about 32 m of CPU time or so, and involves about 2800 inner iterations. During the first 3 or 4 major iterations, the performance is very similar. The subsequent QP's (of which 20 - or so are required) involve very few inner iterations (about 300). The total CPU time required for solving problem F3 lies between 3 and 3.50 hours in an IBM -- 370/158 computer using the operating system VM/CMS, with the routines being written in PL/1 Optimizer, and using the MPSX -- system /8/ in the inner iterations of -- the very sparse system of linear equa -- tions of the Shanno procedure /4, eqs. (6.3)/.

(12) Although we cannot know before solving - F3 how close is the initial point $X^{(0)}$ - (that is feasible in F1) to the point $\overset{*}{X}$, in the cases with which we have experi - mented the deviation of the production - cost for $X^{(0)}$ is not greater than 3% of the production cost for the point X that satisfies the stopping criteria described in /4, Sec. 10/.

(13) Finally, it must be remarked that the main drawback of the *optimum* solution of F3 - is that condition (11a) permits the existence of multiple local minima and, if - point $X^{(0)}$ is not close to the global optima of F3, it cannot be guaranteed that the optimum of problem F3 is a true lower bound of the optimum of problem F1.

5. CONCLUSION

A general formulation of the Generator Maintenance and Operations Scheduling problem is described. The problem is viewed as a large--scale mixed integer non-linear programming case, since the energy production cost objec - tive function and the energy transmission -- losses constraints are non-linear functions.

A relaxation of the integrality condition on certain type of semi-continuous variables is obtained by introducing a new non-linear --- function in the constraints system. A continuous constrained non-linear programming algorithm used in the optimization of the relaxed model has been proved to be quite satisfactory. The optimal value of its objective function may be a strong lower bound of the optimal value of the objective function in - the original problem, if the initial solu -- tion is close to the optimal point.

Since the new problem has multiple local minimum points, if the initial solution is not close to the optimum then the obtained local minimum is not necessarily a lower bound to the optimal solution value of the original - problem. By exploiting the special structure of the original problem (it is a multiperiod problem with non-strong linkages among the periods), an ad-hoc implicit enumeration algorithm provides an initial feasible solution to the continuous non-linear problem -- that it is also feasible in the original problem.

Since it is required much more time for obtaining the optimum solution to the original problem than for obtaining the initial feasi ble solution and the optimal solution of the relaxed problem, it is worthy to optimize -- this problem. If the difference on the production cost value of both solutions is -- small, it does not mean that the feasible solution is close to the optimum point in the original problem; but it was very close in - the real-life cases with which this approach was tested.

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7. APPENDIX 1.

Notation used int he nonlinearly constrained nonlinear programming algorithm used for solving problem F3,

$\alpha^{(k)}$. The steplength of the descent step direction at iteration k, such that

$$X^{(k)} = X^{(k-1)} + \alpha^{(k)} \delta^{(k)}$$

where $\delta^{(k)}$ is the descent step direction.

$A \equiv A(X^{(k)})$. Jacobian matrix of the constraints functions in formulation F3. $\hat{A}$ is - the corresponding submatrix of the active - constraints.

$B^{(k)}$. Hessian matrix of the Lagrange function $L(X,\mu)$, such that

$$L(X,\mu) = F(X) - c(X)^t \mu$$

where $F(X)$ is the objective function, $c(X)$ is the column vector of the constraints functions, and μ is the column vector of the Lagrange multipliers. $B^{(k)}$ is evaluated (or approximated) at point $X^{(k)}$ for the Lagrange multipliers estimates vectors $\mu^{(k)}$. Note -- that

$$B^{(k)} = G(X^{(k)}) - \sum_{i \in j} \mu_i^{(k)} B(X^{(k)})_i$$

where $G(X^{(k)}) \equiv G^{(k)}$ is the Hessian matrix of the objective function $F(X)$ evaluated (or -- approximated) at point $X^{(k)}$.

$B(X^{(k)})_i \equiv B_i^{(k)}$. Hessian matrix of the constraint function $c_i(X^{(k)})$ evaluated (or approximated) at point $X^{(k)}$. Note that $G^{(0)}$ and $B_i^{(0)}$ are analitically evaluated at the initial point $X^{(0)}$. For $k > 0$, $G^{(k)}$ and $B_i^{(k)}$ are not evaluated, but $B^{(k)}$ is approximated by using the direct BFGS Quasi-Newton formulation; see /4, eq.(5.8)/.

$\hat{B}^{(k)}$. The Lagrange Hessian approximation - obtained by modifying matrix $B^{(k)}$ at iteration k. This modification satisfies the --- sparsity condition on the exact Lagrange Hessian matrix. See in /4, eq. (6.3)/ the formulation for $\hat{B}^{(k)}$.

$\bar{B}^{(k)}$. The Lagrange Hessian positive definite approximation obtained by modifying ma -- trix $\hat{B}^{(k)}$ at iteration k, so that the step direction at the next interation is obtained. See in /4, procedures (7.1) and (7.3)/ the - procedure for obtaining $\bar{B}^{(k)}$.

$\delta^{(k)}$. The descent step direction of the solution at iteration k.

$f^{(0)}$. It is any vector or matrix evaluated before iteration 1 of the given algorithm; $f^{(k)}$ is the same vector or matrix evaluated at iteration k.

$g_L^{(k)}$. Gradient column vector of the Lagrange function $L(X,\mu)$, such that

$$g_L^{(k)} \equiv g_L(X^{(k)}) = g(X^{(k)}) - \hat{A}\mu^{(k)}$$

where $g(X^{(k)})$ is the gradient vector of function $F(X)$. Vector $g(X^{(k)})$ is usually approximated by finite differences for $k > 0$; it - is analitically evaluated for $k=0$.

J. Set of active (i.e. strictly satisfied - and violated) constraints in formulation F3. Constraint $\hat{a}_i^t X \geqslant 0$ is active if $\hat{a}_i^t X \not> 0$. Note that a_i is the ith column vector of matrix $\hat{A}$ and X is the unknown column vector.

k. A given iteration of the algorithm.

$\mu^{(k)}$. Column vector or the Lagrange multipliers estimates at iteration k. $\bar{\mu}^{(k)}$ is -- the corresponding subvector of the inequality constraints.

QP. Quadratic programming problem to be solved at iteration k for obtaining the des - cent step direction $\delta^{(k)}$, such that QP can be written

$$\min\{g^{(k)t}\,\delta^{(k)} + 1/2\delta^{(k)t}\,B^{(k-1)}\delta^{(k)}$$

$$\text{subject to } \hat{A}^{(k)t}\delta^{(k)} \geqq -\hat{c}^{(k-1)}$$

where $\hat{c}^{(k-1)}$ is a column vector whose indexes belong to the set J updated at iteration k-1. Note that it is assumed that the general formulation of problem F3 can be written

$$\min\{F(X) \quad \text{subject to } c(X) \gtreqqless 0\}$$

t_1 and t_2. Stopping criteria to be satisfied by the current point $X^{(k)}$; if these criteria, among others, are satisfied it is assumed that $X^{(k)}$ is the optimum point $\overset{*}{X}$. Criteria t_1 and t_2 are related to the feasibility of point $X^{(k)}$; see /4, sec. 10/.

T3. A given tolerance of the Lagrange multipliers, such that if $\bar{\mu}_i \geqslant T3$ for $\forall_i \in J$ it is assumed that the stopping criterion t3 is satisfied. Usually, $T3 = 10^{-4}$. Note that it is required that the Lagrange multipliers estimates vector $\hat{\mu}^{(k)}$ of the active inequality constraints must be positive at optimum - point $\overset{*}{X}$, except for the degenerate case; see /4, sec. 1/.

$X^{(k)}$. Point (solution vector) at iteration k.

$\overset{*}{X}$. Optimum (locally strong) point of formulation F3.