AN INDEX OF DIVERSITY IN STRATIFIED RANDOM SAMPLING BASED ON THE HYPOENTROPY MEASURE

L. PARDO. D. MORALES
Universidad Complutense de Madrid

We consider a measure of the diversity of a population based on the \( \lambda \)-measure of hypoentropy introduced by Ferreri (1980). Our purpose is to study its asymptotic distribution in a stratified sampling and its application to testing hypothesis. A numerical example based on real data is given.

Keywords: Asymptotic normality, Diversity, Hypoentropy measure, Estimation, Testing hypothesis, Stratified sampling.

A.M.S. subject classification: 62B10, 94A15.

1. INTRODUCTION

When the observations from a population are classified according to several categories, the uncertainty of the population may be quantified by means of several measures in Information Theory. The diversity of the population is intuitively intended as a measure of the average variability of classes in it, based on the number of classes and their relative frequencies.

-----------


-This work was partially supported by the Dirección General de Investigación Científica y Técnica (DGICYT) under the contract PS89-0019.

-Article rebut el novembre de 1990.
Consider a finite population of $N$ individuals which is classified according to a classification process of factor $X$ into $M$ classes or species $x_1, \ldots, x_M$. We denote by $\mathcal{X}$ the set of all categories or classes

$$\mathcal{X} = \{x_1, \ldots, x_M\}$$

Let $\Delta_M = \left\{ P = (p_1, \ldots, p_M) / p_i \geq 0, \sum_{i=1}^{M} p_i = 1 \right\}$ be the set of all complete finite discrete probability distribution on the measurable space $(\mathcal{X}, \beta \mathcal{X})$.

Rao (1982) established that a measure of diversity is a function

$$\Phi : \Delta_M \rightarrow \mathbb{R},$$

satisfying the following conditions:

i) $\Phi(P) \geq 0 \ \forall P \in \Delta_M$ and $\Phi(P) = 0$ iff $P$ is degenerate.

ii) $\Phi$ is a concave function of $P$ in $\Delta_M$.

We shall refer to $\Phi(p)$ as the diversity measure within a probability space $(\mathcal{X}, \beta \mathcal{X}, P)$. The condition i) is a natural one since a measure of diversity should be nonnegative and take the value zero when all individuals of a population are identical, i.e., when the associated probability measure is concentrated at a particular point of $\mathcal{X}$. The condition (ii) is motivated by the consideration that the diversity in a mixture of populations should not be smaller than the average of the diversities within individual populations. Consequently, the entropy measures can be used as diversity indices.

Ferreri (1980) introduced a generalization of Shannon's entropy called hypentropy; the expression of this measure of uncertainty for a discrete probability distribution $P \in \Delta_M$, is given by

$$\Phi_\lambda(P) = \left(1 + \frac{1}{\lambda}\right) \log(1 + \lambda) - \frac{1}{\lambda} \sum_{i=1}^{M} \left(1 + \lambda p_i\right) \log(1 + \lambda p_i), \lambda > 0$$

where

$$\lim_{\lambda \to \infty} \Phi_\lambda(P) = H(P) = - \sum_{i=1}^{M} p_i \log p_i$$

and
Assume that a sample of $n$ members is drawn at random sampling. If we consider the estimate $\hat{\phi}_\lambda$ obtained by replacing $p_i$'s by the observed proportions $\hat{p}_i$, $i = 1, \ldots, M$, then

$$n^{1/2} \left( \hat{\phi}_\lambda - \phi_\lambda(p_1, p_2, \ldots, p_M) \right) \xrightarrow{n \to \infty} \mathcal{N}(0, \sigma^2_\lambda)$$

where

$$\sigma^2_\lambda = \sum_{i=1}^{M} p_i \log^2(1 + \lambda p_i) - \left( \sum_{i=1}^{M} p_i \log(1 + \lambda p_i) \right)^2$$

which has been used for testing several hypothesis (ref. Morales et al., 1990).

Now we suppose that the population can be divided into $r$ non-overlapping subpopulations, called strata, as homogeneous as possible with respect to the diversity associated with $X$. Let $N_j$ be the number of individuals in the $j$th stratum (so that, $\sum_{j=1}^{r} N_j = N$), and let $p_{ik}$ denote the probability that a randomly selected individual belongs to the $k$th stratum and to the class $x_i$ ($i = 1, \ldots, M$, $k = 1, \ldots, r$). Thus, $\sum_{k=1}^{r} p_{ik} = N_k / N$, $\sum_{i=1}^{M} \sum_{j=1}^{r} p_{ij} = 1$. Let $p_i$ be the probability that a randomly selected individual in the whole population will belong to the class $x_i$ ($p_i = \sum_{k=1}^{r} p_{ik}$, $i = 1, \ldots, M$). Then the hypoentropy population diversity associated with $X$ is given by

$$\Phi^*_\lambda(X) = (1 + \frac{1}{\lambda}) \log(1 + \lambda) - \frac{1}{\lambda} \sum_{i=1}^{M} (1 + \lambda p_i) \log(1 + \lambda p_i)$$

$$= (1 + \frac{1}{\lambda}) \log(1 + \lambda) - \frac{1}{\lambda} \sum_{i=1}^{M} (1 + \lambda \sum_{k=1}^{r} p_{ik}) \log(1 + \lambda \sum_{k=1}^{r} p_{ik}), \lambda > 0$$

Assume that a stratified sample of size $n$ is drawn at random from the population independently in different strata. We hereafter suppose that the sample is chosen by proportional allocation in each stratum. Assume that a sample of size $n_k$ is drawn independently at random with replacement from the $k$th stratum, where $n_k / n = N_k / N$. If $f_{ik}$ denotes the relative frequency of individuals belonging to the class $x_i$ an to $k$th stratum (and, hence $\sum_{i=1}^{M} f_{ik} = n_k / n$), and $f_i = \sum_{k=1}^{r} f_{ik}$, then the diversity sample with respect to the classification process or factor $X$ could be quantified by means of the analogue estimates, the hypoentropy sample diversity, $\Phi^*_\lambda = \Phi^*_\lambda(X)$. Following the ideas in M.A.
Gil (1989), we will study in this paper asymptotic behavior of the hypoentropy sample diversity $\hat{\phi}_\lambda^s$.

2. ASYMPOTIC DISTRIBUTION OF $\hat{\phi}_\lambda^s$

In this section, we state a general result concerning the asymptotic behavior of the hypoentropy sample diversity, $\hat{\phi}_\lambda^s$, in the stratified random sampling with proportional allocation, with replacement in each stratum and independence among different strata.

The following theorem establishes the asymptotic behavior of $\hat{\phi}_\lambda^s$.

Theorem 1

If we consider the estimate $\hat{\phi}_\lambda^s$, then

$$n^{1/2} \left( \hat{\phi}_\lambda^s - \phi_\lambda^s(X) \right) \rightarrow N(0, \sigma_{\lambda,s}^2)$$

where

$$\sigma_{\lambda,s}^2 = \frac{1}{N} \sum_{k=1}^r N_k \left( \sum_{i=1}^M t_i^2 \frac{N}{N_k} p_{ik} - \left( \sum_{i=1}^M \frac{N}{N_k} p_{ik} t_i \right)^2 \right)$$

and $t_i = -\log(1 + \lambda p_i) - 1$.

Proof

Let us define

$$G_\lambda^s(f^*) = \hat{\phi}_\lambda^s(f) (\hat{\phi}_\lambda^s(f) \equiv \hat{\phi}_\lambda^s(X))$$

where $f^* = (f_{11}, \ldots, f_{(M-1)r}), \ldots, f_{1r}, \ldots, f_{(M-1)r}$ and $f = (f_{11}, \ldots, f_{M1}, \ldots, f_{1r}, \ldots, f_{Mr})$. Now we consider the Taylor expansion for $G_\lambda^s(f^*)$

$$G_\lambda^s(f^*) = G_\lambda^s(p^*) + \sum_{i=1}^{M-1} \sum_{k=1}^r \frac{\partial G_\lambda^s(p^*)}{\partial p_{ik}} (f_{ik} - p_{ik}) + R_n$$

in a neighbourhood of
\[
p^* = (p_{11}, \ldots, p_{(M-1)1}, \ldots, p_{1r}, \ldots, p_{(M-1)r})
\]

where

\[
G^*_\lambda(p^*) = \phi^*_\lambda(p) (\phi^*_\lambda(p) \equiv \phi^*_\lambda(X)), \text{ and } p = (p_{11}, \ldots, p_{M1}, \ldots, p_{1r}, \ldots, p_{Mr})
\]

and \(R_n\) is the Lagrange rest.

As,

\[
G^*_\lambda(p^*) = \left(1 + \frac{1}{\lambda}\right) \log(1 + \lambda) - \frac{1}{\lambda} \sum_{i=1}^{M-1} \left(1 + \lambda \sum_{k=1}^r p_{ik}\right) \log \left(1 + \lambda \sum_{k=1}^r p_{ik}\right) - \\
- \frac{1}{\lambda} \left((1 + \lambda(1 - \sum_{k=1}^r \sum_{i=1}^{M-1} p_{ik})) \log \left(1 + \lambda(1 - \sum_{k=1}^r \sum_{i=1}^{M-1} p_{ik})\right)\right)
\]

we have

\[
\frac{\partial G^*_\lambda(p^*)}{\partial p_{ik}} = -(\log(1 + \lambda p_i) + 1) + (\log(1 + \lambda p_{M_i}) + 1).
\]

Then,

\[
G^*_\lambda(f^*) = G^*_\lambda(p^*) + \sum_{i=1}^{M-1} \left(- \log(1 + \lambda p_i) - 1\right) (f_i - p_i) + \sum_{i=1}^{M-1} \left(\log(1 + \lambda p_{M_i}) + 1\right) (f_i - p_i) + R_n
\]

\[
= G^*_\lambda(p^*) + \sum_{i=1}^{M-1} \left(- \log(1 + \lambda p_i) - 1\right) (f_i - p_i)
\]

\[
+ (\log(1 + \lambda p_{M_i}) + 1) (1' - f_{M_i} - (1 - p_{M_i})) + R_n = \\
= G^*_\lambda(p^*) + \sum_{i=1}^{M} \left(- \log(1 + \lambda p_i) - 1\right) (f_i - p_i) + R_n
\]
Therefore, as

\[ G^*_\lambda(p^*) = \phi^*_\lambda(p) \quad \text{and} \quad G^*_\lambda(f^*) = \phi^*_\lambda(f) \]

it follows that, the random variables

\[ \left( \hat{\phi}^*_\lambda - \phi^*_\lambda(X) \right) \quad \text{and} \quad \sum_{i=1}^M \left( - \log(1 + \lambda p_i) - 1 \right) (f_i - p_i) \]

converge in law to the same distribution because \( R_n \) converge in probability to zero. From now on, we denote \( t_i = (- \log(1 + \lambda p_i) - 1) \).

The random vectors,

\[ \left( \frac{n}{n_k} f_{1k}, \ldots, \frac{n}{n_k} f_{(M-1)k} \right), \quad k = 1, \ldots, r \]

are independent and distributed as a multinomial distribution of parameters \( \left( \frac{N}{n_k} p_{1k}, \ldots, \frac{N}{n_k} p_{(M-1)k} \right) \), respectively.

Applying the \((M-1)\)-dimensional Central Limit Theorem, we obtain

\[ n_k^{1/2} \left( \left( \frac{n}{n_k} f_{1k} - \frac{N}{n_k} p_{1k} \right), \ldots, \left( \frac{n}{n_k} f_{(M-1)k} - \frac{N}{n_k} p_{(M-1)k} \right) \right) \xrightarrow{n_k \uparrow \infty} N(0, \Sigma(K)) \]

where

\[ \Sigma(K) = \begin{pmatrix} \frac{N}{n_k} p_{1k} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \frac{N}{n_k} p_{(M-1)k} \end{pmatrix} - P(K)P(K)^t \quad k = 1, \ldots, r, \]

and

\[ P(K)^t = \left( \frac{N}{n_k} p_{1k}, \ldots, \frac{N}{n_k} p_{(M-1)k} \right), \]

i.e.,
\[ n_k^{1/2} \frac{N}{N_k} ((f_{1k} - p_{1k}), \ldots, (f_{(M-1)k} - p_{(M-1)k})) \xrightarrow[n_k \uparrow \infty]{} N(0, \Sigma(K)) \]

as

\[ n^{1/2} = n_k^{1/2} \left( \frac{N}{N_k} \right)^{1/2} \]

we have

\[ n^{1/2} \left( \frac{N}{N_k} \right)^{1/2} ((f_{1k} - p_{1k}), \ldots, (f_{(M-1)k} - p_{(M-1)k})) \xrightarrow[n_k \uparrow \infty]{} N(0, \Sigma(K)) \]

Therefore

\[ n^{1/2} \left( \frac{N}{N_k} \right)^{1/2} \sum_{i=1}^{M-1} (t_i - t_{M})(f_{ik} - p_{ik}) \xrightarrow[n_k \uparrow \infty]{} N(0, T^t \Sigma(K)T) \]

with

\[ T = (t_1 - t_M, \ldots, t_{M-1} - t_M)^t \]

As,

\[ \sum_{i=1}^{M-1} (t_i - t_M)(f_{ik} - p_{ik}) = \sum_{i=1}^{M} t_i (f_{ik} - p_{ik}) \]

we have

\[ n^{1/2} \sum_{i=1}^{M} \sum_{k=1}^{r} t_i (f_{ik} - p_{ik}) = \]

\[ = n^{1/2} \sum_{i=1}^{M} t_i (f_{ik} - p_{ik}) \xrightarrow[n \uparrow \infty]{} N \left( 0, \frac{1}{N} \sum_{k=1}^{r} N_k T^t \Sigma(K)T \right) \]
Therefore,

\[
\sigma_{\lambda,s}^2 = \frac{1}{N} \sum_{k=1}^{r} N_k T_i \Sigma(K) T = \\
= \frac{1}{N} \sum_{k=1}^{r} N_k \left( \sum_{i=1}^{M} t_i^2 \frac{N}{N_k} p_{ik} - \left( \sum_{i=1}^{M} \frac{N}{N_k} p_{ik} t_i \right)^2 \right)
\]

3. APPLICATIONS ON TESTING HYPOTHESIS

In the stratified random sampling with proportional allocation with replacement in each stratum and independence among different strata, the hypothesis can be used for testing the following hypothesis:

i) \( H_0 : \phi_{\lambda}^* = D_0 \) against \( \phi_{\lambda}^* > D_0 \). Under \( H_0 \), the statistic

\[
Z = \frac{n^{1/2} (\phi_{\lambda}^* - D_0)}{\sigma_{\lambda,s}}
\]

has approximately a standard normal distribution for sufficiently large \( n \). Clearly, we reject \( H_0 \) at level \( \alpha \) if \( z > z_\alpha \), where \( z_\alpha \) is such that \( P(Z \geq z_\alpha) = \alpha \). Similar arguments may be applied in the remaining cases, i.e., \( H_1 : \phi_{\lambda}^* < D_0 \) and \( H_1 : \phi_{\lambda}^* \neq D_0 \).

On the basis of this result we can obtain a method to test the null hypothesis:

\( H_0 : p_1 = p_2 = \cdots = p_M = 1/M \)

This hypothesis is equivalent to test

\( H_0 : \phi_{\lambda}^* = \phi_{\lambda,\text{MAX}} \)

where
\[ \phi^*_\lambda, \text{MAX} = (1+1/\lambda) \log(1+\lambda) - \frac{M+\lambda}{\lambda} \log(1+\lambda/M). \]

In this case we reject \( H_0 \) at the level \( \alpha \) if

\[ \hat{\phi}_\lambda^* < \phi^*_\lambda, \text{MAX} - \frac{z_{\alpha/2} \sigma_{\lambda,s}}{n^{1/2}} \quad \text{or} \quad \hat{\phi}_\lambda^* > \phi^*_\lambda, \text{MAX} + \frac{z_{\alpha/2} \sigma_{\lambda,s}}{n^{1/2}} \]

Since in the non–null case

\[ \frac{n^{1/2}[\hat{\phi}_\lambda^* - \phi^*_\lambda(P)]}{\sigma_{\lambda,s}(P)} \xrightarrow{n \uparrow \infty} \mathcal{N}(0,1), \]

the asymptotic power function in \( P = (p_1, p_2, \ldots, p_M) \) is given by

\[
\beta_\lambda(P) = \Psi \left( \frac{n^{1/2}(\phi^*_\lambda, \text{MAX} - \phi^*_\lambda(P)) - \sigma_{\lambda,s} z_{\alpha/2}}{\sigma_{\lambda,s}(P)} \right) + 1
- \Psi \left( \frac{n^{1/2}(\phi^*_\lambda, \text{MAX} - \phi^*_\lambda(P)) + \sigma_{\lambda,s} z_{\alpha/2}}{\sigma_{\lambda,s}(P)} \right)
\]

where \( \Psi \) denotes \( P(X \leq x) \) when \( X \) is normally distributed with mean 0 and variance 1.

\text{ii)} \( H_0 : D_1 = D_2 \) (diversities of two independent populations are equal) against one–sided or two sided alternatives. Now under \( H_0 \), the statistic

\[ Z = \frac{(n_1n_2)^{1/2}(\hat{\phi}_{\lambda,1} - \hat{\phi}_{\lambda,2})}{(n_2\sigma^2_{\lambda,s,1} + n_1\sigma^2_{\lambda,s,2})^{1/2}} \]

has approximately a standard normal distribution, where subscript \( i \) has been used to denote population \( i \) and \( n_i \) denote the sample size in population \( i \), \( (i = 1, 2) \).
Remark

From theorem 1, an approximate $1 - \alpha$ level confidence interval for $\hat{\phi}_\lambda^*$ is given by

$$
\left( \hat{\phi}_\lambda^* - \frac{\lambda, \hat{z}_\alpha/2}{n^{1/2}}, \hat{\phi}_\lambda^* + \frac{\lambda, \hat{z}_\alpha/2}{n^{1/2}} \right);
$$

Furthermore, the minimum sample size guaranteeing a specified limit of error $\epsilon$ with a small risk is

$$
n = \left[ \frac{\lambda, \hat{z}_\alpha^2}{\epsilon^2} \right] + 1
$$

4. CONNECTIONS WITH THE RANDOM SAMPLING

Stratification provides a method of utilizing supplemental information to get greater precision in our sample estimates. Auxiliary information may be used to divide the population into groups, called strata, such that the elements within each group are more alike than are the elements in the population as a whole. Then a sample is selected from each stratum and the sample results from the different strata are pooled in order to arrive at an estimate for the whole. If there are large differences between the units in the different strata the accuracy of the overall estimates will be substantially increased as strata will be represented in their correct proportions, whereas in a random sample these proportions will be subject to sampling errors. We are now going to formulate the comments above for the asymptotic variances as well as for their analogue estimates.

The following theorem establish that the asymptotic variance of the statistic $n^{1/2}(\hat{\phi}_\lambda^* - \phi_{\lambda}(X))$ is smaller than the asymptotic variance of $n^{1/2}(\hat{\phi}_\lambda - \phi_{\lambda}(P))$.

Theorem

It is verified that

$$
\sigma_{s, \lambda}^2 \leq \sigma_{\lambda}^2
$$

with equality if and only if $r = 1$ or
\[ \sum_{i=1}^{M} \log (1 + \lambda p_{i_0}) \frac{N}{N_k} p_{ik} \]

does not depend on \( k (k = 1, \ldots, r) \).

**Proof**

Since

\[ \sigma^2_{\lambda,s} = \frac{1}{N} \sum_{k=1}^{r} N_k \left( \sum_{i=1}^{M} t_i \frac{N}{N_k} p_{ik} - \left( \sum_{i=1}^{M} \frac{N}{N_k} p_{ik} t_i \right)^2 \right) \]

with \( t_i = \log(1 + \lambda p_{i_0}) - 1 \), we have

\[ \sigma^2_{\lambda,s} = \frac{1}{N} \sum_{k=1}^{r} N_k \left( \sum_{i=1}^{M} \left( \log(1 + \lambda p_{i_0}) + 1 \right)^2 \frac{N}{N_k} p_{ik} - \left( \sum_{i=1}^{M} \frac{N}{N_k} p_{ik} \log(1 + \lambda p_{i_0}) \right)^2 \right) = \]

\[ = \sum_{k=1}^{r} \sum_{i=1}^{M} \log^2(1 + \lambda p_{i_0}) p_{ik} - \frac{1}{N} \sum_{k=1}^{r} N_k \left( \sum_{i=1}^{M} \frac{N}{N_k} p_{ik} \log(1 + \lambda p_{i_0}) \right)^2 = \]

\[ = \sum_{i=1}^{M} \log^2(1 + \lambda p_{i_0}) p_{i_0} - \frac{1}{N} \sum_{k=1}^{r} N_k \left( \sum_{i=1}^{M} \frac{N}{N_k} p_{ik} \log(1 + \lambda p_{i_0}) \right)^2 \]

Now we consider the random variable \( Z \), taking the values

\[ \sum_{i=1}^{M} \frac{N}{N_k} p_{ik} \log(1 + \lambda p_{i_0}) \quad k = 1, \ldots, r. \]

with probabilities \( \frac{N}{N_k} \), respectively. Let us consider the convex function,

\[ \phi(x) = x^2 \]

21
then by using Jensen’s inequality, we have

\[
\left( \sum_{i=1}^{M} \log(1 + \lambda p_i.) p_i. \right)^2 \leq \frac{1}{N} \sum_{k=1}^{r} N_k \left( \sum_{i=1}^{M} \frac{N_k}{N} p_{ik} \log(1 + \lambda p_i.) \right)^2
\]

therefore,

\[
\sigma_{x,\lambda}^2 \leq \sigma_{\lambda}^2
\]

5. A NUMERICAL EXAMPLE

In the Natural Farm Survey of England and Wales (Ministry of Agriculture, 1944, G) for the country of Hereford the farms sample were classified into the following domains:

<table>
<thead>
<tr>
<th>Percentage of arable land</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A Mainly grass</td>
<td>0 – 29.9</td>
</tr>
<tr>
<td>B Intermediate</td>
<td>30 – 49.9</td>
</tr>
<tr>
<td>C Mainly arable</td>
<td>50 – 100</td>
</tr>
</tbody>
</table>

The population of farms over 25 acres was divided into three size–groups (25–100, 100–300, over 300). The basic data is given in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>Size group (acres)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 – 100</td>
<td>72</td>
<td>62</td>
<td>39</td>
<td>173</td>
</tr>
<tr>
<td>100 – 300</td>
<td>79</td>
<td>55</td>
<td>108</td>
<td>242</td>
</tr>
<tr>
<td>over 300</td>
<td>13</td>
<td>61</td>
<td>25</td>
<td>99</td>
</tr>
</tbody>
</table>

We use data from Table I to test for deviation from uniformity for \( \alpha = 0.05 \) and \( \lambda = 10 \). As
\[ \phi_{\lambda \text{ MAX}} = 0.73145, \quad \phi_{\lambda} - \frac{\hat{\sigma}_{\lambda} \times 1.96}{n^{1/2}} = 0.51660, \quad \phi + \frac{\hat{\sigma}_{\lambda} \times 1.96}{n^{1/2}} = 0.89854 \]

we have,

\[ 0.51660 < 0.73145 < 0.89850. \]

Therefore, we do not reject \( H_0 : p_A = p_B = p_C \).

In the following tables we have computed powers of test deviation from uniformity for \( M = 3, r = 3 \) and \( \alpha = 0.05 \), when there are 130 elements in the first stratum, 110 elements in the second stratum and 140 elements in the third stratum. We calculate powers of test \( H_0 : p_1 = p_2 = p_3 = 1/3 \), i.e., the population is homogeneous, for different sets of probabilities \( (p_1, p_2, p_3) \), where \( p_i \) is the probability of class \( x_i (i = 1, 2, \ldots, M) \). We consider the following values of \( \lambda : 0.01, 0.1, 1, 10, 100, 1000, 10000 \) and 100000. Observe that results for \( \phi_{1000} \) can be interpreted as results for Shannon entropy.

\[ \begin{array}{c|c|c|c} \lambda & 0.29474 & 0.36842 & 0.33684 \\ \hline 0.010 & 0.15971 & & \\ 0.100 & 0.15085 & & \\ 1.000 & 0.15003 & & \\ 10.000 & 0.14941 & & \\ 100.000 & 0.14913 & & \\ 1000.000 & 0.14909 & & \\ 10000.000 & 0.14908 & & \\ \end{array} \]

\[ \begin{array}{c|c|c|c} \lambda & 0.77368 & 0.16579 & 0.06053 \\ \hline 0.010 & 1.00000 & & \\ 0.100 & 1.00000 & & \\ 1.000 & 1.00000 & & \\ 10.000 & 1.00000 & & \\ 100.000 & 1.00000 & & \\ 1000.000 & 1.00000 & & \\ 10000.000 & 1.00000 & & \\ \end{array} \]
<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$P_i$</th>
<th>$P_i$</th>
<th>$P_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.46053</td>
<td>0.42105</td>
<td>0.11842</td>
</tr>
<tr>
<td>0.010</td>
<td>0.99999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>1.00000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.000</td>
<td>0.99999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.000</td>
<td>0.99997</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.000</td>
<td>0.99993</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000.000</td>
<td>0.99991</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10000.000</td>
<td>0.99991</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$P_i$</th>
<th>$P_i$</th>
<th>$P_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.06579</td>
<td>0.07368</td>
<td>0.86053</td>
</tr>
<tr>
<td>0.010</td>
<td>1.00000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>1.00000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.000</td>
<td>1.00000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.000</td>
<td>1.00000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.000</td>
<td>1.00000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000.000</td>
<td>1.00000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10000.000</td>
<td>1.00000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$P_i$</th>
<th>$P_i$</th>
<th>$P_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.32368</td>
<td>0.32895</td>
<td>0.34737</td>
</tr>
<tr>
<td>0.010</td>
<td>0.43509</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>0.06137</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.000</td>
<td>0.06009</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.000</td>
<td>0.06007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.000</td>
<td>0.06008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000.000</td>
<td>0.06009</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10000.000</td>
<td>0.06010</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From these tables we see that the asymptotic powers of these seven tests are not very different. Also we observe that no test is uniformly better than the others.
6. ACKNOWLEDGMENT

The authors thanks the referees for their valuable suggetions.

7. REFERENCES


