Application of Fuzzy Techniques to the Design of Algorithms in Computer Vision

Eduard Montseny* and Pilar Sobrevilla**
*Dept. Enginyeria Sistemes, Automàtica i Informàtica Industrial
e-mail: montseny@fabrique.upc.es
**Dept. Matemàtica Aplic. II. e-mail: pilars@grec.upc.es
Univ. Politécnica de Catalunya,
Pau Gargallo 5, 08028 Barcelona, Spain

Abstract
In this paper a method for the design of algorithms is presented which use fuzzy techniques in order to achieve a better vagueness treatment.
A base of rules will be developed in order to design the algorithms. Data fuzzification problem is solved by using probability density functions and probability distribution functions, whereas data analysis is set out associating, to each one of the “analysis rules”, a fuzzy set which will be obtained by applying an aggregation function which will be defined by using an OWA operator.
The proposed design provides a solution to the data value fuzzification problem, which is a quite well solved problem for applied control algorithms, but, up to now, displayed great difficulties for vision ones.
Moreover, the proposed data analysis method provides a solution for non intrinsic problems from vision algorithms.
Keywords: Computer vision, image processing, fuzzy techniques, aggregation functions, membership functions, OWA operators.

1 Introduction
The great majority of algorithms implementing low level image processing have to analyze vague data. The vagueness is present at the images due to several factors hardly avoidable with techniques nowadays used for image captation and digitation. So, since Li, Yang and Pao [9, 14] introduced the existence of a vagueness, no randomness, component in the computer vision systems variables, a lot of researchers have treated to solve these problems by means of fuzzy techniques.
Prewit, in 1970, suggested that segmentation results were fuzzy subsets of the plane, but solutions he searched for were crisp.
Also Haralick [8], in 1985, assured that whichever contour extraction algorithm works well in the absence of noise. It is known that noise is one of the factors introducing more vagueness at the images.
In the 90's many algorithms have been designed introducing, in the low level process, a treatment of the vagueness. Although all of them present the basic modules that must construct an algorithm based on fuzzy techniques, are missed both, the model allowing to define data fuzzification and the way with which data are analyzed. So, in the algorithms, these processes appear implemented by means of hardly justifiable equations.

Many works using fuzzy techniques formalize the algorithm by means of rules, and implement these rules using a set of equations. That is why, at the present work, this way of design has been used and the exposed method propose the formalization of the algorithm by means of rules, introducing guidelines in order to implement them.

2 Structure of the Algorithm

The proposed method presents the design of computer vision algorithms allowing vagueness treatment. These algorithms are constituted by five modules, such as it can be seen at the diagram in figure 1.

![Diagram](image)

Figure 1

Next the function accomplished by each one of the modules is described.
2.1 Base of rules

In this module will be described, in rules form, the several steps that must be followed in order to attain the objective of the algorithm. With this rules the characteristics, which must be analyzed in the several stages of the algorithm, will be shaped.

Given the structure of the proposed algorithm, a good solution in order to define the rules is to use those of Mamdani type.

Furthermore, the way in which each one of the analyzed characteristics will be evaluated must be introduced in this module.

2.2 Evaluation of the characteristics

Here the mathematical functions or, as the case may require, the way in which the characteristics must be evaluated, will be defined according to what has been introduced in module 2.1.

The characteristics will have to be evaluated for each one of the pixels in the image, whether having into account its own information or the one in a local neighbourhood of it. That’s the way to do it because we are trying to implement low level algorithms.

2.3 Fuzzification

In this module, due to the type of rules used on the description of the algorithm (module 2.1), fuzzy sets “fi” associated to each characteristic “ci” must be defined.

The definition of the membership functions associated with these fuzzy sets is carried out by using the probability density function obtained from the evaluation of the characteristics over a pattern image where such characteristic is given.

2.4 Data analysis and aggregation functions

In this module the rules concerning with the analysis of the characteristics are implemented by means of aggregation functions. So, for each named rule, the following map is defined

\[ f_{ai} : [0, 1]^n \rightarrow [0, 1] \]

These functions will be defined using OWA operators, the weights of which are obtained analyzing the evaluations performed over a pattern image.

2.5 Defuzzification

Final results are obtained from the corresponding fuzzy sets by means of a defuzzification, which is performed accomplishing an \( \alpha \)-cut, where \( \alpha \) is equal to 0.5.
3 Base of rules

This is the most important module, because is where the algorithm will be described.

In order to carry out the algorithm description the structure in figure 2 has been proposed. This structure allows us fix both, the characteristics that must be analyzed and the way in which such analysis have to be performed.

![Figure 2](image)

The node-root is associated to the element trying to detect/locate inside the image. This element is described by means of lower level elements, and so on. In this way each level in figure represents elements with more simple structure from the root to the leaves, these elements having basic structure are present in the image if certain characteristics are given in a local neighbourhood.

All tree nodes are fuzzy sets, the $f_{a_i,j}$ are described by means of rules and the leaves $f_{ci}$ are the fuzzy sets associated with the characteristics.

In the analysis process the way in which the fuzzy sets $f_{a_i,j}$ are obtained will be described by means of Mamdani rules, so an element is in the image if constituent elements are present, in a local neighbourhood. Also the way in which the characteristics have to be evaluated will be specified in the analysis process.

4 Fuzzyfication

With the aim of know de degree with which each pixel in image verify the characteristics, we’ll associate a fuzzy set to each one of them.

In order to obtain the membership functions associated with these fuzzy sets,
the following “fuzzification functions” have been defined:

\[ f_{c_k}: \mathbb{R} \to [0,1] \]

these functions will be calculated starting from the probability density function of
the data obtained evaluating the corresponding characteristic over the pixels, or
regions, in a pattern image where such characteristic is given.

The use of probability density functions is due to the fact that the gray level of
each pixel in the image is affected by:

1. the presence of white noise in the images.
2. the digitation process.
both having random space distribution.

So, in order to obtain the fuzzification functions, the process works in accord-
ance with the following rules:

- **R1**: \( f_{c_k}(p(i,j)) \) increases when the probability density function, for the eval-
uated value of characteristic \( k \) at the pixel \((i,j)\), is close to the maximum.

- **R2**: \( f_{c_k}(p(i,j)) \) increases when the evaluated value of characteristic \( k \) at the
pixel \((i,j)\), is close to the median of the distances.

These rules are implemented in the following way:

**Rule R1**: \[ f_{c_k}(p(i,j)) = \frac{p_{c_k}(p(i,j))}{\max (p_{c_k})} \]

where:

- \( p_{c_k}(p(i,j)) \) is the value of the probability density function associated with
the characteristic \( c_k \) for the pixel \( p(i,j) \).

- \( \max (p_{c_k}) \) is the greatest value of the probability density function associated
with the characteristic \( c_k \).

**Rule R2**: \[ f_{2c_k}(p(i,j)) = \min \{ 1, \frac{P_{c_k}(p(i,j))}{0.25} \} \]

where:

- \( P_{c_k}(p(i,j)) \) is the value of the probability distribution function associated
with the characteristic \( c_k \) for the pixel \( p(i,j) \).

Finally \( f_{c_k}(p(i,j)) \) is evaluated in the following way:

\[ f_{c_k}(p(i,j)) = 0.5 \times f_1 c_k(p(i,j)) + 0.5 \times f_2 c_k(p(i,j)) \]

5 Data Analysis and Aggregation Functions

In this module the data analysis is carried out according to the information con-
tained in the base of rules. Each rule contained in the mentioned base, give rise to
a fuzzy set $f_{a_{i,j}}$, which will be obtained by means of the aggregation of the fuzzy sets named at the corresponding rule, following the guidelines pointed out in it.

Having into account the nature of the analyzed problem, will be accepted that the membership degree of each element $(i, j)$ to the new fuzzy set satisfies:

$$\min \{ f_{n,p}(i, j) \} < f_{a_{i,j}} < \max \{ f_{n,p}(i, j) \}$$

where:

$f_{n,p}(i, j)$ are the membership degrees of the element $(i, j)$ to the fuzzy sets which will be aggregated.

$f_{a_{i,j}}$ is the membership degree of the element $(i, j)$ to the new fuzzy set.

In order to define the aggregation functions we have considered, in addition to the previous section, that all the membership degrees of the element $(i, j)$ to the antecedents $f_{n,p}$ contribute in the same way in the obtention of the membership degree of this element to the consequent $f_{a_{i,j}}$. So a good solution is to use the OWA operators introduced by Yager.

The problem when OWA operators are used is the weights obtention. With the aim to solve this problem, for a given rule like:

if $(i, j)$ is $f_{n,p}(i, j)$ is $f_{n,p+1}, \ldots, (i, j)$ is $f_{n,p+q}$ then $(i, j)$ is $f_{a_{i,j}}$.

of which the antecedents $(f_{n,p}, f_{n,p+1}, \ldots, f_{n,p+q})$ and the way in which are obtained are known, the procedure is the next:

- Inside of a pattern image have been located the elements which constitute the consequent in order to obtain their membership degree to the antecedents $(f_{n,p}, f_{n,p+1}, \ldots, f_{n,p+q})$.

- Previous information is analysed by means of several steps, in such way that operators weights are obtained in consecutive approximations.

Finally the weights obtained must satisfy that, at least the 85% of the elements classified a priori as belonging to the consequent set, will have got a membership degree equal or bigger than 0.5.

6 Results

The proposed method has been applied in the design of three classes of algorithms which analyze the characteristics of images in a low level, these algorithms are:

- Contour detection.
- Texture detection.
- Natural texture analysis.

At the three cases the results have been very satisfactory, both by the easiness of the algorithm design and the results which have been obtained.

Although is difficult evaluate the results and to contrast them with those furnished by the existent algorithms, nevertheless can be pointed out that:

a) In the case of texture detection published results haven’t been found neither about the kind of detected textures nor the kind of analyzed characteristics. It
must be pointed out that the 90% of the texture has been well detected (must be
had into account that analyzed textures have a great variability of characteristics)
and, when the noise has been added into the image, has been observed a very
robust behavior of the algorithm, in such a way that its efficiency only has been
reduced to the 83%.

b) When natural textures are analyzed the algorithm works very well with the
characteristics’ variability. In this case the results are considered like very good
because of its efficiency is 90%.
c) In the case of contours extraction it has been observed that, with the designed
algorithm, contours with smaller contrast have been detected, and the location has
been accomplished with a bigger precision.

7 Conclusions

As conclusions must be said that the presented method make easy the design of
algorithms which use fuzzy techniques for the treatment of vagueness.

Two contributions have to be pointed out:

- Guide-lines are introduced for the fuzzification of characteristics’ values, which
  adapt to the kind of vagueness present into the images
- The method accomplish the data analysis by using OWA operators, which
  implement the functions described at the rules which have to be applied in
  the treatment of the data.

Acknowledges

This work has been, in part, financed by the CICYT, project (plan): BIO95-0916-
C02-01.

References

Sets Theory, Journal of Mathematical Analysis and Applications 93, 1 (1983),


