

Fuzzy Sets in Computer Vision: an Overview

P. Sobrevilla*, E. Montseny**

*Applied Mathematics II Dep.

**Comp. Eng. and Industrial Systems Dep.

Technical University of Catalonia.

{pilar.sobrevilla, eduard.montseny}@upc.es.

Abstract

Every computer vision level crawl with uncertainty, what makes its management a significant problem to be considered and solved when trying for automated systems for scene analysis and interpretation. This is why fuzzy set theory and fuzzy logic is making many inroads into the handling of uncertainty in various aspects of image processing and computer vision.

The growth within the use of fuzzy set theory in computer vision is keeping pace with the use of more complex algorithms addressed to solve problems arisen from image vagueness management.

Due to the natural linguistic capabilities of high-level computer vision, it is a very appropriate place for applying fuzzy sets. Moreover, scene description, i.e., the language-based representation of regions and their relationships, for either humans or higher automated reasoning provides an excellent opportunity.

With this overview we want to address the various aspects of image processing and analysis problems where the theory of fuzzy sets has so far been applied. On the other hand, we will discuss the possibility of making fusion of the merits of fuzzy set theory, neural networks theory and genetic algorithms for improved performance. Finally a list of representative references is also provided .

1 Introduction

Computer vision is the study of theories and algorithms for automating the process of visual perception. It involves tasks such as noise removal, smoothing, and sharpening of contrast (low-level vision); segmentation of images to isolate objects and regions and description and recognition of the segmented regions (intermediate-level vision); and finally interpretation of the scene (high-level vision).

Uncertainty abounds in every phase of computer vision. Therefore, a computer vision system, to be robust, must have at its disposal the machinery

allowing vagueness representation. Moreover, from a theoretic viewpoint, the techniques used during the system development must help to anticipate vagueness effects, so that a correct interpretation of the obtained results can be carried out.

To develop flexible computer vision systems capable of representing the uncertainty at the different decision levels, two approaches have been traditionally followed: the probabilistic and the possibilistic.

Usually uncertainty has been considered as a result of some random component of the variables involved within the system. This is why uncertainty has been analyzed using probabilistic methods. However, there exist a lot of cases wherein source of uncertainty is not only random by nature, but it is also dependant of other kind of factors. Some of these sources of uncertainty [43], [51] are:

Projection of a 3D scene over a 2D image.

The set up of the illumination, the conversion of the light energy into an electronic signal, and the digitization

The discretization process of the spatial coordinates.

Corruption and distortion of image features during the acquisition process.

Lack of knowledge about image quality. The image quality definition itself is based on human perception, what turns it into a subjective parameter.

Imprecision in computations and vagueness in class definitions and concepts as usual in computer vision as contour, vertex, homogeneity region, and so on.

Object definitions are not always crisp, knowledge about the objects in the scene can be described only in vague terms, and the output of low level processes provide vague, conflicting, or erroneous inputs to higher level algorithms.

Ambiguities in interpretations of the obtained results, and ill-posed questions.

Fuzzy set theory and fuzzy logic are ideally suited for dealing with these types of uncertainty. That's why from the middle of the 80's some researchers started to use qualitative models in computer vision problems.

Traditionally, image processing techniques have been used for automating tasks related to low and intermediate computer vision levels, while pattern recognition techniques were applied for designing algorithms capable of automate the tasks connected with high-level computer vision.

Problems come up from the not handling of uncertainty at low and intermediate levels made still more difficult high level tasks automation as, for example, scene interpretation. The not handling of uncertainty at low and intermediate levels had an influence on high level tasks, making even more difficult their automation, as was the case of scene interpretation. It compelled researchers to look for solutions with which designing computer vision algorithms of which the models were close to the two Marr's principles [45]:

Principle of least commitment: "Don't do something that may later have to be undone".

Principle of graceful degradation: "Degrading the data will not prevent the delivery of at least some of the answer".

Seeking for tools and methods allowing improving the automation of high-level computer vision tasks, some researchers included, within their algorithms, techniques based on artificial intelligence, expert systems, neural networks and fuzzy logic. Later on, for intermediate-level in the first place and shortly after for low-level, it was observed that, for improving the results of algorithms implementing related tasks, it was necessary a restatement of the design of the image processing techniques used up to then. As a consequence, new models based on artificial intelligence, expert systems, neural networks and fuzzy logic were also developed for implementing image processing techniques allowing the automation of low-level and intermediate-level tasks.

According to Keller [34], "Rule-based systems have gained popularity in computer vision applications, particularly in high level vision activities". This assertion was said in 1995, and nowadays these systems have spread to every computer vision level. In the same paper Keller already stated that rule-based systems were suitable for modeling algorithms related with low and intermediate levels of computer vision systems, asserting that "fuzzy logic offers numerous approaches to translate such rules and to make inferences from the rules and facts modeled similarly".

Fuzzy set theory and fuzzy logic provide the tools needed for modeling the algorithms according with aforementioned Marr's principles. Although it doesn't mean that just the use of membership functions guarantee that algorithms designed using them will preserve those principles, we can state that "fuzzy set theory contains natural modeling mechanisms, calculus for computation involving uncertain information, and intuitively pleasing interpretation. Moreover, fuzzy set theory offers one of the best overall frameworks within which to formulate, model and solve problems in computer vision".

The aim of this paper is to briefly state the activities involved in computer vision. We will attempt to point out how fuzzy set theory can be, and has been, applied for solving problems within this domain along with some of the relevant references.

2 High level: Fuzzy sets in pattern recognition and scene description

Identification of objects and scene interpretation/description are the main tasks of computer vision at its high-level. The relevance of the fuzzy set theory in pattern recognition and scene description problems has adequately been addressed in the literature [4],[32],[53],[5],[9].

According to W. Pedrycz [58] "It is evident that fuzzy sets have placed pattern recognition into a completely new perspective by developing an innovative methodological and algorithmic framework to cope with complex and ill-defined systems". Probably, one of the greatest fuzzy set theory's contributions to pattern recognition algorithms development consists in making easier the design of algorithms based on interpretable models.

Algorithms based on probabilistic models are opaque to interpretation, making difficult to include expert knowledge. However, humans have used vision for a long time and, in this sense, we can consider that all of us are experts recognizing and identifying objects within a scene. Obviously, when performing these tasks we are classifying objects in classes and we are even capable of explaining where a particular classification is based on. Is in this context that fuzzy set theory has at its disposal the mechanisms for allowing and facilitate a satisfactory use of all the knowledge that experts can represent by means of a base of rules.

Conventional approaches to image analysis and recognition ([63], [24] and [45]) consist of segmenting the image into meaningful regions, extracting their edges and skeletons, computing various features/properties (e.g. area, perimeter, centroid, etc.) and primitives (e.g. line, corner, curve, etc.) of and relationships among the regions, and finally, developing decision rules/grammars for describing, interpreting and/or classifying the image and its sub-regions. In a conventional system each of these operations involves crisp decisions to make regions, features, primitives, properties, relations and interpretations crisp.

As the regions in an image are not always crisply defined, uncertainty can come up within every phase of the aforementioned tasks. Moreover, it has to be taken into account that decisions made at a particular level will have repercussions on all higher activities. That is why a recognition system should have at its disposal the necessary mechanisms for representing and manipulating the uncertainty involved at every processing level; so that the system be able of keeping as much of the information content of the data as possible. The ultimate output of a system endowed with the adequate mechanisms will own minimal uncertainty.

Natural scene understanding/description is an important aspect of computer vision. However, although it has received considerable attention, up to now the results haven't been as promising as would be desirable. It has been partly due to two reasons: the need for sophisticated world models and the large quantity of uncertainty that has to be handled when reasoning at high levels. Early approaches exposed the difficult nature of scene interpretation. These systems were mainly constructed for locating objects within a given scene, without explicitly modeling uncertainty.

The basic focus of attention was on creating structures to effectively carry out scene analysis tasks. Following this idea, Antony in [2] addressed the possibility of incorporating fuzzy set concepts into constraints such as "near", and used quadtree

representations to determine crisp areas of an image that would correspond to spatial concepts like “northeast”.

In high-level computer vision, spatial relations among image objects are very significant for getting an accurate scene description. Although it has been made quite clear that human intuition varies considerably, it is wholly accepted that humans can judge the spatial relationship between two objects, e.g., “B is to the right of A”. So, the vague concepts of what spatial relationships should mean, as well as the uncertainty of how they can model differing human perceptions, make very problematic both automated calculation and the use of this important information.

Due to the importance of spatial relations and its connection with human scene understanding, this concept has been considered from linguistic and psychological points of view to automated definition and reasoning systems [19], [62], [72], and [17]. Spatial relations such as ABOVE, RIGHT, and others defy precise definitions, and seem to be best modeled by fuzzy sets [71], [48], [39], [46], [11] and [20]. However, the subjectivity and complexity of these concepts turn the objective definition of spatial relations into a very difficult task, as can be deduced by the large quantity of fuzzy definitions available.

Basic principles and operations of image processing and recognition in the light of fuzzy set theory are available in [53], [9], [6], [57], [73], [61], [47], [18], [37], [59], [42].

3 Low and intermediate level: Fuzzy sets in image processing

To begin with let us to explain the difference between digital image processing and digital image analysis. Image processing can be thought of as a transformation that takes an image into an image, i.e. starting from an image a modified (enhanced [65], [66]) image is obtained. On the other hand, digital image analysis is a transformation of an image into something different from an image, i.e. it produces some information representing a description or a decision.

Interest in digital image processing methods stems from two principal application areas: improvement of pictorial information for human interpretation, and processing of scene data for autonomous machine perception. For computer vision systems, the only purpose of image processing consists in producing images that not only simplify the subsequent analysis but also make it more reliable. In particular, the low-level image analysis phase should facilitate the extraction of information. Traditionally the computer vision researchers are not at all interested in how “well” the image looks. Introduction of fuzzy set theory has made extremely easier the knowledge representation and, consequently, the exploitation of experts’ skills for image processing and analysis. It allows to model the way in which experts

perform image analysis process, taking into account that a good image quality to the eye is one of their requirements.

Traditionally, digital image processing has had two main thrusts to its development. One is the natural extension of one-dimensional (temporal) digital signal processing to two (spatial) dimensions. Consequently, two-dimensional signal processing was approached from a mathematical basis, allowing a great deal of rigorous manipulation to be performed, using classical linear system theory. The second, more heuristic, thrust considers digital images as a set of discrete sample points, performing arithmetic operations on the individual points. This contrasts with the signal processing approach, which treats images as a discrete representation of a continuous two-dimensional function.

Probably is at low and intermediate levels where the effects of data uncertainty and vagueness within the considered concepts are more noticeable. In the attempt of designing algorithms best fitted to the Marr's principles, researchers were compelled to make use within their algorithms of pattern recognition and image analysis techniques. As a consequence, the concept of contour went from "a simple gray-level discontinuity" to "a shape that must be described by a set of features" and, from "a characteristic of a given image pixel" to "a shape present at a local image region".

As everybody knows, features always are fulfilled at a certain degree, and natural objects' shapes display certain similarity degree. In the same way that an improvement of the results obtained from a features' analysis process is easier within an interpretable model than in a non-interpretable, it is also easier to design an algorithm following and imitating a known operative model than if such a model isn't at our disposal.

Fuzzy sets theory and fuzzy logic provide with the necessary set of tools for representing, easy and clearly, the degree to which an element satisfies a feature. Likewise, they furnish with methods for analyzing, according with a set of rules, a set of features so as to get the degree to which two shapes are similar.

Previous reasons joined to the fact that with breathtaking pace, computers are becoming more powerful and at the same time less expensive, have impelled many researchers to use fuzzy set theory and fuzzy logic for solving problems connected with low and intermediate computer vision levels.

When the expert's knowledge is considered within a base of rules for carrying out a specific kind of analysis, an usually intuitive way of behavior (heuristic rules), highly non-linear by nature and hardly describable using traditional mathematical models, is being introduced. Moreover, it is possible to design more flexible and adaptable systems combining heuristic rules with traditional methods.

4 Boundary detection and representation.

Boundary detection is a very important process in image processing. Its importance arises from the fact that boundaries carry one of the most important image informations. The boundaries indicate the location of the objects and describe their shape.

In an ideal image edges correspond to object boundaries, and so edge detection provides an effective way of segmenting the image into meaningful regions. However, the definition of what constitutes an edge is rather vague, heuristic, and even subjective. So, in [31], Jain et al state that: *“An edge point locates a pixel where there is significant local intensity change; an edge fragment is a collection of edge points; and an edge detector produces either a set of edge points or edge fragments”*.

Two variables can be distinguished within previous definition: intensity and location. These variables are related to concepts defined in an imprecise way: 1. When can we say that a local intensity change is sufficiently meaningful as to take the decision that it is an edge? 2. If an edge fragment is made up of a set of pixels, Under which conditions can we assert that an edge is located in a specific pixel and not in one of its neighborhoods?

Obviously, in the ideal case of correct image data, the answers to these questions could be given performing an analysis wherein the only considered information was provided by the pixel itself and the one of a local neighborhood (let's say a 3x3 window). However, as it is known image data contain vagueness. This is why it must be taken into consideration that from the analysis of data provided by the pixel and its neighborhood we can only get information with regard to the degree to which a significant level intensity change can be noticed at the given pixel. Later on a more regional analysis ought to be performed for checking if really exists a set of pixels allowing confirming the existence of an edge fragment.

Many researchers have described algorithms for edge detection wherein image vagueness is taken into account, as for example: Russo and Ramponi [64], Bezdek et al. [8], Law et al. [41], Garcia-Barroso et al. [21], Tizhoosh [70], Dave [15].

5 Segmentation and region representation.

The objective of segmentation is to divide an image into (meaningful) regions. These regions have to display a uniform behavior with regard to one or several features. Once again, two imprecisely defined concepts appear within this definition. The first one makes reference to the feature or features showing uniform behavior but, when can we say that a behavior gives up being uniform? On the other hand we are faced again with the location problem because, as objects have uniform

boundaries, for marking the boundaries of a region, and locate the uniform ones, the uniformity degree of neighboring pixels should be considered

Image segmentation is one of the most critical components of the computer vision process. Errors made in this stage will impact all higher level activities. Therefore, methods that incorporate the uncertainty of object and region definition and the faithfulness of the features to represent various objects (regions) are desirable. The first connection of fuzzy set theory to computer vision was made by Prewit [60] who suggested that the results of image segmentation should be fuzzy subsets rather than crisp subsets of the image plane.

As pointed out previously, in a segmented image each region should be homogeneous with respect to some characteristics or features, as gray level or texture. Moreover, these characteristics or features should be significantly different for adjacent regions (Haralick and Shapiro [28]). In many cases features considered for determining homogeneity may not have sharp transitions at region boundaries, what makes task of determining if a pixel should belong to a region or not into a hard problem. This situation is mainly given when features are computed over a local region (a local 3x3 or 5x5 window).

For alleviating problems as previously described fuzzy set theory and fuzzy logic are being introduced within the traditional image segmentation techniques - thresholding, clustering, supervised segmentation and rule-based segmentation- for improving segmentation results [9] and [55]. The usual process consists in associating a fuzzy set to every region and obtaining the degree to which each pixel belongs to each region-fuzzy set. Afterwards, for getting the final segmentation, over previously obtained membership degrees, fuzzy techniques are applied to methods of: thresholding [67], [52], [50], [54], [49], [56] and [10], clustering [25], [12], [16] and [30], supervised segmentation [3], [38], [33], and rule-based segmentation [7], [44], [14], [68], [36] and [40]

6 Improving fuzzy set theory performance.

A large number of researchers are merging the advantages of fuzzy set theory with the merits of neural networks theory for improving the results of computer vision algorithms. Systems obtained combining these two theories have been applied for solving different computer vision problems providing accurate results. However, it must to be taken into account that these systems waste one of the most relevant advantages of fuzzy set theory, that is: Its interpretability. A good review explaining the merits of fusing these two technologies can be found in [5], other interesting papers are available in: [26], [38], [22].

Genetic algorithms have also been used for solving pattern recognition problems involving adaptive and optimization processes. Unlike many conventional search algorithms, that consider a single point in the search space, genetic

algorithms consider many points simultaneously. It allows reducing the possibility of converging to local optima. Moreover, instead of using deterministic rules, genetic algorithms use probabilistic rules to guide their searching process.

With regard to handling uncertainty, these algorithms may be helpful in determining the appropriate membership functions, rules, and parameters space, and in providing a reasonably suitable solution. A good review explaining the merits of combining these two technologies can be found in [29] and [27], other interesting papers are available in: [23], [1], [69].

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