

HPC and edge elements for geophysical electromagnetic problems: an overview

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Abstract- In Finite Element Methods for solving electromagnetic problems, the use of Nédélec Elements has become very popular. In fact, Nédélec Elements are often said to be a cure to many difficulties that are encountered, particularly eliminating spurious solutions, and are claimed to yield accurate results. In this paper, we present our first steps in developing a Nédélec Elements code for simulation of geophysical electromagnetic problems and first ideas on how implement the key issues of Edge Elements in an efficient way on HPC platforms.

I. INTRODUCTION

Electromagnetic Methods (EM) are an established tool in geophysics, finding application in many areas such as hydrocarbon and mineral exploration, reservoir monitoring, CO2 storage characterization, geothermal reservoir imaging and many others. The marine controlled-source electromagnetic method (CSEM) has become an important technique for reducing ambiguities in data interpretation in the offshore environment and a commonplace in the industry. In the traditional configuration, the sub-seafloor structure is explored by emitting low-frequency signals from a high-powered electric dipole source towed close to the seafloor. By studying the received signal, the subsurface structures could be detected at scales of a few tens of meters to depths of several kilometers.

On the other hand, in the Finite Element Method for solving electromagnetic field problems, the use of Edge-based elements (Nédélec elements) has become very popular. In fact, Nédélec elements are often said to be a cure to many difficulties that are encountered, particularly eliminating spurious solutions, and are claimed to yield accurate results. In Edge elements, the simulated quantity is interpolated by vector shape functions that provide several characteristics important for the representation of the electric field.

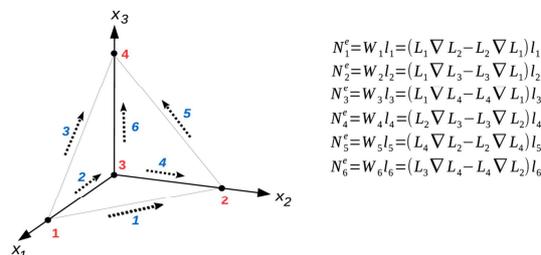
Based on previous ideas and considering the societal value of exploration geophysics, since this process is essential to among others, we present the key issues of edge element to simulate geophysical electromagnetic problems, namely, we present our first steps in developing a Nédélec Elements code for simulation of geophysical electromagnetic problems and first ideas on how implement the key issues of Edge Elements in an efficient way. Strong emphasis is placed on three aspects not easily found in the literature: vector shape functions, orientation of geometric entities and the technique to avoid the source singularities in CSEM simulations. Finally, we make some concluding remarks and show the performance of the code in terms of efficiency and accuracy.

II. NÉDÉLEC ELEMENTS FORMULATION

The Nédélec Finite Element Method uses vector basis functions defined on the edges of the corresponding elements. Similar to the conventional node-based finite element method, the modeling domain can be discretized using rectangular, tetrahedron, hexahedron or other complex elements [1].

Fig. 1 (left) is an illustration of the tetrahedron grid that we used with node (red numbers) and edge indexing (blue numbers). Following the work of [1], the tangential component of the electric field is assigned to the center of each edge. The edge basis functions defined by [1] are divergence free but not curl free. The vector basis functions are also continuous at the element boundaries. As a result, the divergence free condition of the electric field in the source free region and the continuity conditions are imposed directly in the Nédélec Finite Element formulation. Thus, the element vector basis functions for the field associated with the edges are given by Fig. 1 (right) where L_i are defined in [1] and l_i represents the length of edge i .

Fig. 1. Nédélec element description with nodes and edges (left). Vector basis functions of linear edge elements right).

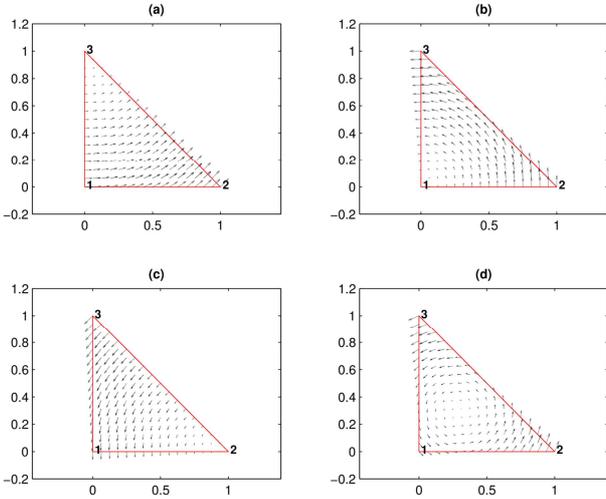


In order to guarantee global continuity of tangential components, special care has to be taken with regard to the edge direction. In short, the edge direction strategy works as follows. A global index is assigned to each node of the mesh. If an edge connects two nodes n_i and n_j , we define the direction of the edge as going from node n_i to node n_j if $i < j$. This gives a unique orientation of each edge. The same philosophy is used locally to determine the directions of the local edges on each element.

III. SUMMARY OF RESULTS

The following images are the result of our first steps in developing a Nédélec Elements code for simulation of geophysical electromagnetic problems.

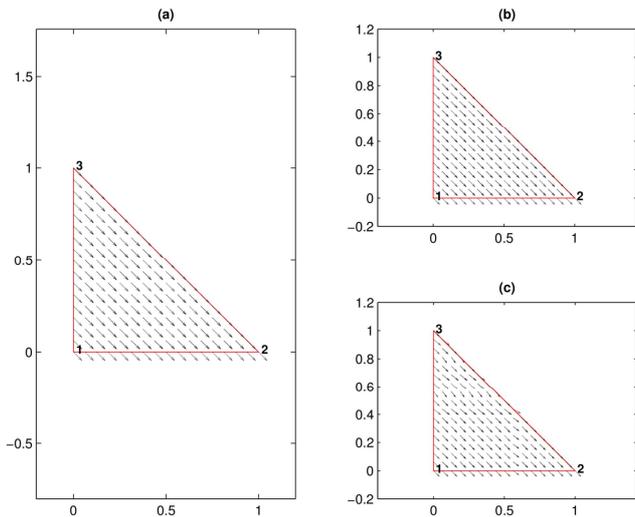
Fig. 2 Vectorial basis functions. Subplot (a) describes the shape function along edge 1 – 2, Subplot (b) describes the shape function along edge 2 – 3, Subplot (c) describes the shape function along edge 3 – 1 and subplot (c)



In geophysical simulations with CSEM, the primary field is calculated analytically for a background layered-earth model. The secondary field is discretized using edge-elements. The primary field computation stage is a classic example of embarrassingly parallel workload because no exists dependency or communication between those parallel tasks.

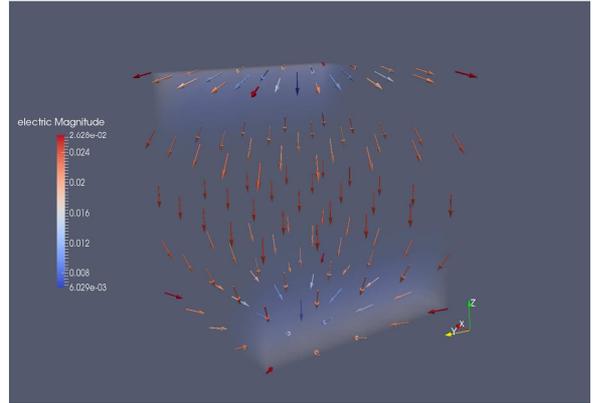
On the other hand, numerical modeling of geophysical electromagnetic problems imposes a significant

Fig. 3 Ability of vectorial basis functions to interpolate a given field. (a) constant field to test, (b) field within the element, (c) error of approximation (maximum error=0.5551 - 0.3311 e-15)



computational burden. In solving large-scale models, single processor-based methods are limited and often incapable of managing the required large memory and computational time requirements. Therefore, we are exploring parallel environments such as MPI and OpenMP in order to improve the performance of these simulations.

Fig. 4 Primary field of a z-directed dipole in a homogeneous domain with 384 elements.



# T	# E	Assembly time	Solver time	h	L ²	O _L
128	208	7.66e-03	6.76e-03	3.3e-01	3.969e-01	---
512	800	6.75e-03	7.72e-03	2.0e-01	9.831e-02	2.016
2,048	3136	2.22e-02	4.47e-02	1.1e-01	2.430e-02	2.017
8,192	12,416	1.82e-01	2.08e-01	5.8e-02	6.001e-03	1.999
32,768	49,408	4.27e-01	6.68e-01	3.0e-02	1.501e-03	2.004
131,072	197,120	1.51e00	2.27e00	1.5e-02	3.741e-04	2.002
524,288	787,456	5.83e00	6.71e00	7.8e-03	9.340e-05	2.002
2,097,152	3,147,776	21.32e00	24.7e00	3.9e-03	2.333e-05	2.001

Table 1: Summary of results for the 2D case. Number of elements (T), number of edges (E), assembly time (seconds), solver time (seconds), mesh spacing (h), L² error and convergence order.

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