

## A CONCEPTUAL FRAMEWORK FOR ERROR REMEDIATION WITH MULTIPLE EXTERNAL REPRESENTATIONS APPLIED TO LEARNING OBJECTS

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In this paper, some of the concepts behind Intelligent Tutoring Systems (ITSs) are used to elaborate a conceptual framework for error remediation with multiple external representations (MERs) applied to learning objects (LOs). For this purpose, we have developed an LO for teaching the Pythagorean Theorem. This study analyzes the process of error remediation by classifying mathematical errors and provides support for the use of MERs in this process. The main objective of the conceptual framework is to assist the individual learner when rectifying a mistake made during interaction with the LO, due to either carelessness or lack of knowledge. First, we will classify mathematical errors and explain their relationship with MERs. Then, the concepts behind the conceptual framework will be presented. Finally, we will discuss an experiment with an LO developed with an authoring tool called FARMA, in which the conceptual framework is used to teach the Pythagorean Theorem.

**Keywords** – Intelligent learning objects, e-learning, multiple external representations, error remediation.

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**1 INTRODUCTION**

Mathematical errors can play an important part in helping learners gain knowledge, but in order for this to happen, it is necessary to analyze them correctly. Classifying mathematical errors seems to be a good method for understanding the learners' shortcomings. But the variety and complexity of mathematical errors demand specific expertise, which complicates the classification task (Peng & Luo, 2009).

Mathematical errors are considered a natural step during knowledge acquisition (Fiori & Zuccheri, 2005; Peng & Luo, 2009); they are a common phenomenon, regardless of the student's age and/or level of intelligence.

The use of learning objects as a teaching resource in the classroom improves the ability to analyze the relevant features of educational materials in a way that differs from traditional education. It can promote the use of technological resources to acquire concepts during the learning process.

The classification of errors as part of the student's learning process and how this classification can improve the acquisition of a concept included in a learning object has been discussed in several studies (Marczal & Direne, 2011; Leite, Pimentel & Petruchini, 2012; Leite, Marczal & Pimentel, 2013).

According to Silva and Fernandez (2007), a learning object must fulfill three characteristics:

- it must stimulate reasoning and critical thinking (minds-on),
- it must make students ask relevant questions (reality-on), and
- it must offer an opportunity to explore (hands-on).

The discussion about the classification of mathematical errors related to learning objects leads to a necessary discussion about error remediation in the same context. The present proposal adheres to the ITS concept according to which the learner must be supported when he/she commits an error during problem resolution. We applied this concept to learning objects using multiple external representations, enabling the learner to review the facts, rules and concepts in order to choose the correct strategy and solve the problem.

Allowing the learner to inspect the resolution path at each step is one of the key features of remediation; it is a characteristic of ITSs and can be applied to learning objects.

Error diagnosis and remediation is thoroughly discussed by Vanlehn (1987), who highlights that some ITSs already include support for error diagnosis and remediation: a well-known example is Sierra, which interacts with the learner through explanations of errors, based on concepts presented by means of textual representation.

Cognitive diagnosis and error remediation are key steps in the acquisition of a concept. The mission of cognitive diagnosis is to guide the instructional plan, in which all the teaching actions depend on a result.

A new element that has been introduced in the error remediation process is external representations, which can be understood as a constant process of "taking possession" of stable states to obtain information that can then be used in a more flexible way for other purposes.

Evidence of the benefits of error remediation to support learning has been presented in several studies (Ainsworth, 2006; Ainsworth, 2008; Cleeremans & Jimenez, 2002) that discuss its contribution to improving performance and understanding during the learning process.

A comprehensible, appropriate message or representation can be crucial to knowledge acquisition, provided that it is clear and helps communicate the remediation to the learner. Although Ainsworth (2006) presents a taxonomy of external representations, it is unclear when or how each representation is shown to the learner.

The use of MERs in learning environments has shown that students can benefit from the properties of each representation and ultimately achieve a broader understanding of the subject being taught (Ainsworth, 2008). The use of error classification in order to choose a type of remediation allows for a more adequate selection of external representations, so that the learner can review facts, rules, concepts and/or fragments that have been forgotten.

This aim of this article is to create (through the classification of mathematical errors) a conceptual framework for error remediation based on MERs and to use it with a learning object to teach the Pythagorean Theorem in primary education.

## 2 THEORETICAL REFERENCE TO CREATE A CLASSIFICATION OF ERRORS

In order to create a classification of mathematical errors, we have reviewed studies dealing with the analysis of mathematical errors. Although there is a vast body of literature on the subject, we considered it appropriate to select studies focused only on the mathematical error itself, rather than other aspects, such as its causes or consequences.

Radatz (1979) considers it difficult to clearly categorize the possible causes of an error if there is no close interaction between them. This author carried out a study of the elements of information processing theory and suggested five types of mathematical errors.

The Conceptual Fields theory is a cognitivist theory by Gerard Vergnaud (1986) that states that the process of finding a strategy for problem resolution consists of a relational calculation or a numerical calculation that appeals to the student's ability to execute mathematical operations and algorithms. Therefore, an inconsistency in any of these calculations generates one incorrect operation classified by the author as a relational or numerical error.

Movshovitz-Hadar & Zaslavsky (1987) present a qualitative analysis carried out with secondary school students from Israel during a standard exam to evaluate their mathematical knowledge. This analysis produced the

following categories: inadequate use of information, incorrect interpretation of the language, logically invalid inference, distortion of theorems or definitions, non-verified solution and technical error.

Peng and Luo (2009) present a classification based on research into the knowledge of mathematics teachers in relation to the analysis of errors committed in mathematical problems. They identified four analytical categories of errors committed by students.

Lucas (1974) uses this classification to create a rule-based model for studying how children learn to decode words. This model is based on data collected from errors in word pronunciation. This author appeals to the concept of sub-generalization to classify students' inability to consider an element as a member of a group (e.g. fish are not animals because they do not have legs), and the concept of generalization to classify occurrences of students considering an element as a member of a group when in fact it is not (e.g. a chair is an animal because it has four legs).

This classification was adapted by Ramos (2010) for application in inductive mathematical learning, and three types of conceptual errors were considered: sub-generalization, super-generalization and miscellaneous.

Research aimed at classifying mathematical errors has produced differing classifications in terms of both the nomenclature and the number of categories. Thus, it is necessary to create a classification that synthesizes this variety. This classification will be presented in Section 3.

### 3 ERROR AND MER FUNCTION CLASSIFICATION

After analyzing the research on the classification of mathematical errors described in Section 2, it is clear that a unified classification is needed. We suggest the following more logical error categories:

- Misinterpretation of the language: this type of error refers to the learner's difficulty to understand the structure of the problem, i.e. to interpret what is being asked in the problem, as a step prior to formulating a strategy.
- Directly identifiable error: errors in this group can be sub-classified as errors owing to a domain deficiency or a misuse of data; errors due to a deficiency in the use of rules, theorems or definitions; and errors regarding the logical operator.
- Indirectly identifiable error: this category includes errors resulting from faulty logic, i.e. incorrect classifications, inappropriate problem solving strategies or transformations with no progress. This type of error demands step-by-step monitoring of the learner.
- Non-categorizable solution: this category includes errors that cannot be classified into any other category. For example, the learner is too immature to assimilate a particular concept and thus proposes random strategies for problem resolution.

To take this research a step further, a study is needed that links external representations to the types of errors committed by learners. Such a study is only possible after elaborating a broad error classification that allows for a more precise remediation, since this would enable error mapping.

The Multiple External Representations (MERs) theory (Ainsworth, 2006) is a cognitivist theory that supports the use of techniques to present, organize and transmit knowledge. Ainsworth (2006) classifies MERs into categories according to their function. Those with a complementary function support or complement a cognitive process. Those with an interpretation-constraining function constrain possible interpretations that are not relevant to certain concepts. Finally, those with a deeper-understanding constructing function enable a deeper understanding through the generalization of regularities in the presented content.

Table 1 summarizes the use of MERs with different functions for error remediation, in order to contextualize this study. This table includes error types and sub-types, associated MER functions and proposed remediation mechanisms.

<b>Error type</b>	<b>Error sub-type</b>	<b>MER function</b>	<b>Remediation</b>
<i>Misinterpretation</i>	-	<i>Complementary function</i>	<i>Proposing other ways of presenting the problem, so that the learner gets a mathematical formula instead</i>
<i>Directly identifiable error</i>	<i>Domain deficiency or misuse of data</i>	<i>Interpretation constraining function</i>	<i>Showing that although the strategy may be right, the deficiency lies in the use of information</i>
	<i>Deficiency in the use of rules, theorems or definitions</i>	<i>Deeper -understanding constructing function</i>	<i>Introducing the rule or theorem, in order to allow the learner to reorganize or generalize the concept</i>
	<i>Not choosing the correct operator</i>	<i>Deeper- understanding constructing function</i>	<i>Explaining to the learner that his/her mistake was choosing the wrong operator</i>
<i>Indirectly identifiable error</i>	-	<i>Interpretation constraining function</i>	<i>Demonstrating to the learner that the adopted logic does not lead to the solution of the problem</i>
<i>Non-categorizable solution</i>	-	<i>Deeper- understanding constructing function</i>	<i>Proposing to the learner a revision of elementary concepts or those key to the domain</i>

*Table 1. Summarizes the use of MERs with different functions for error remediation*

In order to advance the research, it is convenient to create a conceptual framework that will make up the structural part of the study, which will be carried out in the following section.

#### **4 CONCEPTUAL FRAMEWORK FOR ERROR REMEDIATION WITH MERS**

The conceptual framework of an intelligent tutoring system must dynamically combine information about the student model, the domain model and the tutor model, in order to determine what to present to the student. However, emphasis will first be placed on the modules that make up the specific conceptual framework for problem resolution.

The use of error remediation based on MERs requires a conceptual framework that can help identify the error committed by the student, classify it into the appropriate category associated with a MER function, and finally offer the learner adequate remediation.

Figure 1 represents the proposed conceptual framework for remediation based on MERs, as well as its modules: error classifier, MER function classifier and MER manager. Other components of the conceptual framework are the rule base for error classification, rule base for MER function classification and basis for external representations.

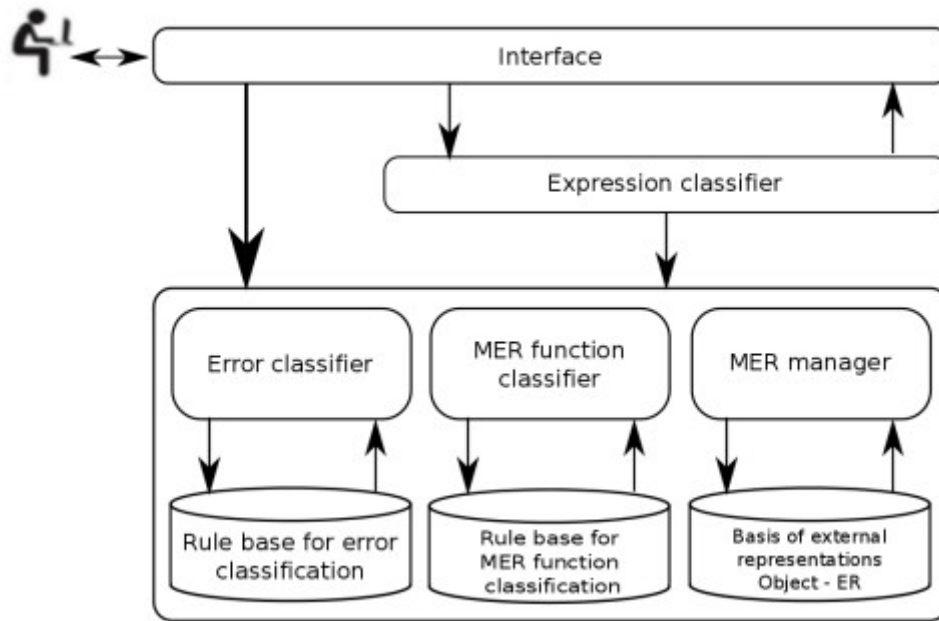


Figure 1. Conceptual framework for using MERs in error remediation applied to learning objects

Errors detected through interaction with the learner may occur at the beginning, the middle or the end of the resolution path. The action refers to the learner's current step along the resolution path and enables the error to be remediated before the final answer is given; moreover, the type of MER depends on this information. The number of attempts serves to validate that the initial MER used in the remediation was appropriate for the learner's advancement.

The aim of the expression classifier module is to establish a connection between the LO and the system. This module is responsible for the initial communication and receives data to determine the degree of correctness of the learner's answer. If the answer is correct, the other modules will not be launched; otherwise, the error classifier module will be launched.

The aim of the error classifier module is to classify the error made by the learner, for which purpose the incorrect data or expression is identified by the expression identifier module. This module receives an error from the expression classifier module and classifies it using the rules contained in the rule base for error classification, which includes the error classification presented in this study (misinterpretation, directly identifiable error, indirectly identifiable error and non-categorizable solution).

The aim of the rule base for error classification is to lay the basis for classifying the error detected. The action and the number of attempts are stored as individualized remediation elements for the learner, in order to track him/her during the resolution process. The action will depend on the learner's current resolution step. Furthermore, the number of attempts will help determine the adequacy of the MER used for remediation.

At this stage, input is received (error, action, number of attempts and error type) and the error type is confirmed against the rule base for MER function classification. This rule base is aimed at determining the MER function.

The MER manager seems to be one of the most relevant modules, since it determines the type of remediation needed for the learner to improve his/her problem-solving strategy. This module receives the following inputs: error, action, number of attempts and MER function. Note that in this module it is not necessary to continue storing the error type, as it has already been used for MER classification.

The role of the MER manager is to decide which MER to use to remediate the error once it has been classified. It should store the last representation used in order to improve the learner's effectiveness.

The MER manager has a sub-module, Object-ER, which is responsible for searching for external representations according to basic criteria, such as error persistence, success of certain external representations in previous situations, and degree of complexity of the situation faced by the learner.

## 5 LEARNING OBJECT FOR TEACHING THE PYTHAGOREAN THEOREM

An ITS is based on the assumption that a student's thinking process can be modeled, tracked and corrected (Self, 1999). This enables not only teaching, but also using different teaching methods, as well as discovering the paths that the learner follows to acquire the desired knowledge.

A learning object for mastering the Pythagorean Theorem was developed for application in a public school in Curitiba, State of Paraná, Brazil, during the 9th grade of primary education.

In order to develop this LO, we used a Web-based authoring tool called FARMA (an authoring tool for error remediation with learning disabled students), which allowed for the application of the conceptual framework presented in Section 4.

FARMA's characteristics made it suitable for implementing the conceptual framework; these characteristics are:

- less effort required to create educational materials, whose main feature is their intuitiveness,
- fewer skills required to manage content outside the author's field of expertise, and
- easy and quick prototyping, i.e. it is possible to validate the final draft of the LO in real time.

These aspects contributed to the completion and validation of the present study, as they facilitated the implementation of the architecture for mathematical error remediation based on MERs (Marczal & Direne, 2011).

The Adaptive Control of Thought (ACT) theory by John Anderson (1983) is a unified theory on information processing that states that learning mechanisms are closely related to the way the content is transmitted to the learner, particularly the way in which it is presented.

In general, proposals based on the ACT theory consist of four premises: a model, in other words, a model of production rules for basic skills, in this case, the ideal-student model; actions on the right path, i.e. correct actions by the student, which are part of a set of solutions included in the model; actions on the wrong path, which enable verifying whether the student continues on the right path towards the solution; and answers about errors and assistance systems, which refer to the system's interaction instruction (Anderson, 2005).

Another relevant aspect is that the ACT theory explores the decomposition of a goal into sub-goals, allowing the student to proceed in stages, which is one of the characteristics of the use of production rules (Anderson, 2005).

This theory has been used as a mechanism for submitting representations for error remediation, in order to optimize the student's knowledge acquisition during the resolution process.

In order to exemplify the application of the ACT theory concepts, the error remediation process and the use of MERs, we first presented the students with a traditional exercise on the Pythagorean Theorem and then we redesigned it according to the aforementioned approaches. The description of the exercise is:

*An acrobatic bike rider wishes to cross from one building to another along a rigid steel cable with a special bike. The height of the building from which he will depart is 75 meters and the height of the building to which he will arrive is 25 meters. The distance between the two buildings is 120 meters.*

**Question:** *What is the minimum length required for the cable? (The correct answer is 130 meters.)*

Such an exercise requires a great capacity for abstraction on the part of the learner, especially to understand the problem, retrieve the data and then perform the necessary calculations. It must be noted that the learner's ability to do mathematical calculations is not enough to solve the problem: he/she must reflect on the description of the problem, retrieve the correct data, identify the concepts involved and finally calculate the answer. Another downside of this approach is the inability to offer relevant error-based feedback to the student, as it is only possible to validate the final answer and the tutor cannot determine at which point the learner made a mistake (he/she may have misunderstood the problem, misused the data or even chosen an answer at random).

The ACT theory supports the monitoring and analysis of the entire interaction between the student and the LO (the ACT theory-based LO developed in this research is called Pythagoras Max) as a mechanism for defining when to use an external representation for remediating an error committed by the student. The objective is to



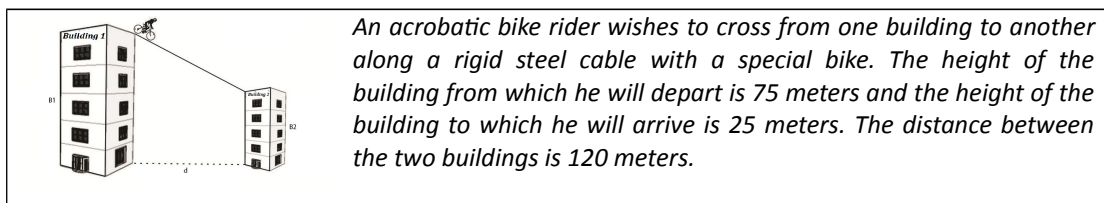
optimize the path the student takes when acquiring a concept, for which purpose a tracing model is used to enable the exploration of each step in the student’s resolution process in an individualized manner. In this case, it was necessary to divide the problem into steps in order to find out the point at which the student deviated from the ideal path, and then establish an adequate error remediation plan based on an external representation.

In the case of the exercise above, which was designed to compare two teaching approaches, a traditional approach (Pythagoras Mix learning object) and an ACT theory-based approach that includes error remediation using MERs (Pythagoras Max learning object), the student would have to abstract all information in order to answer the question. However, in the Pythagoras Max LO, the exercise was divided into sub-steps in order to track the student’s resolution path. Moreover, this second approach allows for a gradual development of the learner’s capacity for abstraction.

In the exercise included in the Pythagoras Mix LO, typical feedback would be "try again", "wrong answer" or signaling in different colors, which would simply draw attention to the student's misconception. The Pythagoras Max LO allows for a very different situation, as the division into stages makes it possible to monitor the students' progress during problem resolution, identify the mistake, classify it in association with a MER function, and finally use a MER for error remediation.

Therefore, the advantage of the Pythagoras Max LO is that it enables gradual learning by breaking down the problem-resolution process into steps, which allows for the application of ACT concepts such as tracing models.

Below, the exercise is divided into steps and the MERs to be used (which depend on the learner’s number of attempts) are presented.



**Question 1:** What are the measurements shown in the figure? (The correct answers are 75, 25 and 120 meters.)

The MERs for error remediation at this stage of the exercise are presented in Figure 2.

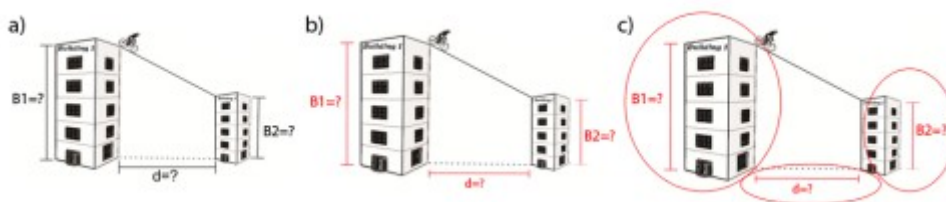


Figure 2. Remediation for question 1: a) used after 1st unsuccessful attempt, b) used after 2nd unsuccessful attempt and c) used after 3rd unsuccessful attempt

**Question 2:** What is the difference in height between the two buildings? (The correct answer is 50 meters.)

The MERs for error remediation at this stage of the exercise are presented in Figure 3.

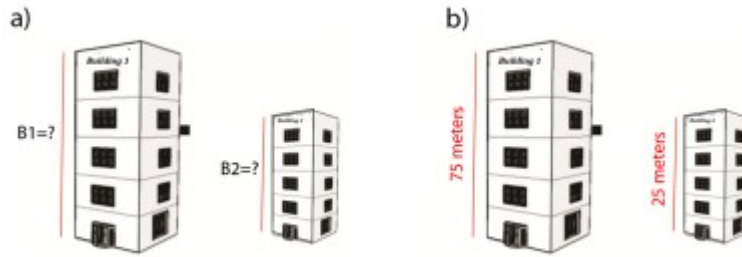


Figure 3. Remediation for question 2: a) used after 1st unsuccessful attempt and b) used after 2nd unsuccessful attempt

**Question 3:** In order to determine the minimum length of the steel cable linking both buildings, it is necessary to draw a triangle between them and then apply the Pythagorean Theorem. What are the measurements of this triangle? (The correct answers are 50 and 120 meters.)

The MERs for error remediation at this stage of the exercise are presented in Figure 4.

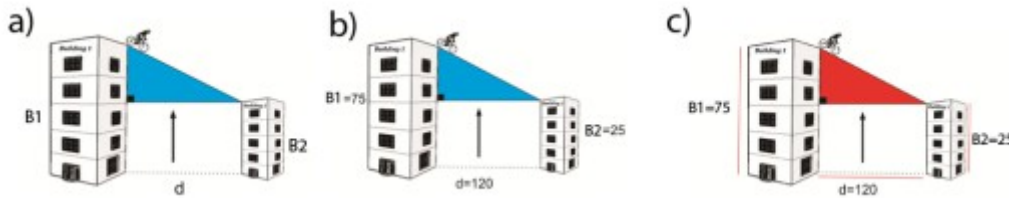


Figure 4. Remediation for question 3: a) used after 1st unsuccessful attempt, b) used after 2nd unsuccessful attempt and c) used after 3rd unsuccessful attempt

**Question 4:** Assuming that the length of the steel cable is “a”, write the Pythagorean Theorem’s equation for the bike rider figure in order to calculate the minimum length required for the steel cable stretching from one building to the other. (The correct answer is  $a^2 = 50^2 + 120^2$ .)

The MERs for error remediation at this stage of the exercise are presented in Figure 5.

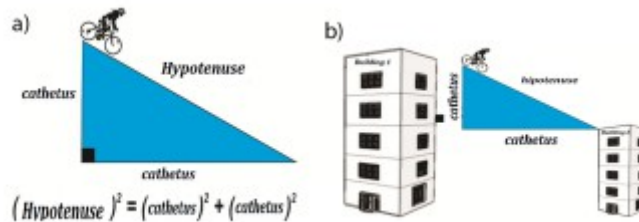


Figure 5. Remediation for question 4: a) used after 1st unsuccessful attempt and b) used after 2nd unsuccessful attempt

**Question 5:** What is the minimum length required for the steel cable? (The correct answer is 130 meters.)

The MERs for error remediation at this stage of the exercise are presented in Figure 6.



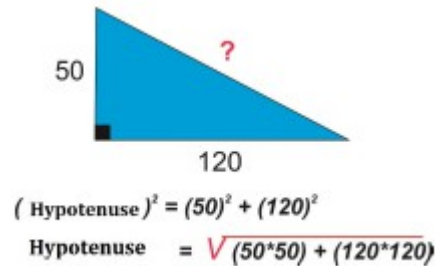


Figure 6. Remediation for question 5: a) used after 1st unsuccessful attempt

Thus, remediation is achieved through new representations of the problem, in this case by images intended to provide the student with a better understanding of how to solve the problem.

Therefore, the main advantages of this approach are:

- Providing immediate feedback on errors, giving the learner guidance throughout the entire process, not just in the final step
- Minimizing the memory effort, since the problem can be analyzed in stages
- Enabling the student to advance towards the solution one step at a time
- Monitoring the student's action against the expected correct path and determining through MERs whether he/she needs to review his/her strategy
- Possibility of reducing the cognitive load by using MERs together with the error classification as relevant error-remediating tools

In order to validate this approach, we carried out an experiment with 20 ninth-grade students from a public school in Brazil. They were divided into two groups: the experimental group and the control group. The experimental group used the Pythagoras Max LO, which includes error remediation based on MERs. The control group used the Pythagoras Mix LO, with identical exercises, but without employing the ACT concepts or error remediation based on MERs.

The students were distributed into the groups based on the results of a pre-test which consisted of 6 questions about the Pythagorean Theorem. A t-test was used to analyze the data in order to identify any significant gain in learning.

The experiment was aimed at confirming the adequacy of MERs for error remediation. Thus, we expected to obtain significant results from the use of the Pythagoras Max LO (which included error remediation based on MERs).

The results confirmed that the use of error remediation based on MERs related to error categories contributes to increase student's knowledge. The hypothesis of the experiment was that the Pythagoras Max LO helps the learner acquire concepts (Leite et al., 2013).

The performance of participants using the Pythagoras Max LO led us to reject the null hypothesis (level of significance: 0.05%) and confirmed with a confidence of 95% that this LO facilitated the acquisition of mathematical concepts (Leite et al., 2013).

The null hypothesis for the Pythagoras Max LO stated that the post-test average results would be inferior or equal to the pre-test average results. However, they were significantly higher, which demonstrates a gain in learning. To confirm this, we used a t-test, since the sample included less than 30 subjects. With a confidence level of 95% ( $\alpha = 0.05$ ), we obtained  $p = 0.000412178$  ( $t = 4.9202$ ,  $df = 9$ ). Thus, as  $p < \alpha$ , we can reject the null hypothesis in terms of concept acquisition.

The statistical results and the details of the experiment, as well as other evidence on the effectiveness of the Pythagoras Max LO as compared to the Pythagoras Mix LO can be found in the studies already validated by the scientific community (Leite et al., 2013).

## 6 CONCLUSIONS

This paper presents two important aspects of ITSs that can be applied to LOs: error remediation and MERs. Later on, it proposes a conceptual framework for applying these two elements to LOs.

With the aim of creating this conceptual framework, we reviewed the literature on mathematical errors committed by learners in order to gain a deeper understanding of how knowledge acquisition works. We concluded that error remediation based on MERs can greatly improve knowledge acquisition, especially when the learner stagnates at a particular error, either due to lack of understanding or lack of problem-solving skills.

Then, using the conceptual framework created, we developed an exercise for teaching the Pythagorean Theorem (this, in turn, forms part of the LO used to confirm our hypothesis). This exercise highlighted the advantages of the use of MERs for error remediation, as compared to traditional exercises.

Finally, we explained an experiment on the adequacy of MERs for error remediation, for which we created an LO based on the ACT theory with the authoring tool called FARMA. The results confirm that this approach brings significant gains in comparison to the traditional method for elaborating and using exercises.

We expect that the proposed framework, as well as the authoring tool FARMA, will be used extensively by authors of educational content in the future.

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Exploring careers in Computer Science and Teaching (College Professor), Maici earned three earn college degrees following BSc. Informatics, Pedagogy and Mathematics (Catholic University of Pelotas, UCPel, Brazil). Also she concluded her graduate degree for Mathematics Education Systems (Federal University Foundation of Rio Grande, FURG, Brazil). After that, she obtained a master degree in Computer Science with specialisation in Design of Educational Interfaces in Mathematics (Federal University of Pernambuco, UFPE, Brazil). Currently, she completed her doctoral degree in Informatics, conducting independent research in Artificial Intelligence (Federal University of Paraná). As professor since 1998, research about Development of Learning Objects using concepts of Intelligent Tutoring Systems, which collaborates for several research groups in Brazil. Since 2003, she is Associate Professor in Informatics at Federal Technological University of Paraná, Brazil, and respected member of OPTICS research centre.

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