Predictive model for polishing times in mould finishing

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Resumen
El pulido, proceso final utilizado en la producción de piezas, es a menudo lento y se aplica manualmente. Normalmente define la rugosidad superficial y la precisión de la pieza. Se lleva a cabo utilizando partículas duras para producir la abrasión de la superficie de la pieza a pulir. En este documento se propone un modelo para predecir el progreso del frente de pulido. El modelo tiene en cuenta la topografía resultante antes de la mecanización a través de su curva de Abbott - Firestone, la velocidad relativa del material abrasivo, la fuerza aplicada, el material de la pieza y el tamaño de los granos abrasivos. Separa las aportaciones realizadas por el material de la pieza y el tipo abrasivo de la topografía previa. También muestra los valores recomendados para los diferentes parámetros de proceso a ser utilizados en la predicción de la evolución del frente de pulido en algunos materiales. Además se estima el tiempo de pulido necesario para eliminar completamente la topografía resultante del fresado utilizando una estrategia de perforación transversal. Por último, se muestra la predicción del modelo del frente de pulido, lo que confirma que el mecanismo de abrasión caracteriza dicho proceso.

Palabras Clave: modelo de pulido, la tasa de eliminación de material, Abbott-Firestone, topografía superficial, abrasión.

Abstract
Polishing, the final process used in the production of parts, is often slow and often applied manually. It normally defines the surface roughness and part accuracy. It is performed using hard particles to produce abrasion of the workpiece surface to be polished. In this paper a model for predicting the progress of the polishing face is proposed. The model considers the topography resulting from prior mechanizing through its Abbott-Firestone curve, the relative speed of the abrasive material, the applied force, the workpiece material and the size of the abrasive grains. It separates the contributions made by the workpiece material and the abrasive type from the prior topography. It also shows recommended values for different process parameters to be used in predicting the evolution of the polishing front in some materials. Additionally, the polishing time required to completely remove the topography that results from ball milling using a cross drilling strategy is estimated. Finally, the model's correct prediction of the polished front is shown, confirming that the abrasion mechanism characterizes the sandpaper polishing process.

Keywords: polishing model, material removal rate, Abbott-Firestone, surface topography, abrasion.

1. Introduction
The last process applied to the material when manufacturing a mould is a finish usually through abrasives and often applied manually. The finishing process defines the surface roughness and accuracy of a mould and although it only modifies the surface profile, it is a very time consuming process. In order to minimize the time spent on it, systematic methods have been developed to control this process taking into account critical surface roughness variables and volume of material removed and identifying with them the process that takes less time [1].

The finishing is considered an abrasive machining process. This technique uses very hard particles to produce surface abrasion. In general, polishing employs glued abrasive particles, while buffing uses free abrasive particles suspended in a liquid or wax medium. In this case the working pressure is applied using a pad or soft cloth, only changing the surface texture and obtaining a reflecting mirror surface. For this reason, the material removed rate Q is low [2] and finishing time high.
The four most commonly accepted hypothesis in relation to the physical mechanism of material removal in the finishing process are: the abrasion hypothesis, the flow hypothesis, the chemical hypothesis, and the friction wear hypothesis [3].

A much studied finishing process is the finishing of glass, which is treated as a chemical mechanical process (CMP). The study of factors that characterize the CMP has generated various hypotheses and models. Still the basic mechanisms of material removal in CMP are not well understood [4]. The first known study on the polished glass was made by Preston [5], developing the classical theory of wear. Other studies in which models are developed to estimate the material removal rate are: Wang et al. [6] who proposes a model that describes the relation between the parameters and the polishing material removed rate based on the assumption that the probability of contact between the pad and the polishing surface to be polished depends on the thickness of the film fluid which is located between both surfaces. Brinksmeier et al. [7] proposes a grinding process for finishing optical elements using a tool made of polyamide. The material removal rates were determined using the equation of Preston. They demonstrate that this new tool increases the material removed rate when the pressure and the polishing speed increase. Furthermore, Savio et al. [4] developed a model for the glass polishing based on the Reye's wear hypothesis, in order to predict the surface roughness depending on the operating parameters. The model shows the evolution of surface roughness during the polishing process, thereby validating the experimental results obtained at the time by Preston. Jim et al. [8] developed a statistical model to predict the material removed by mechanical polishing in which there are two types of abrasive particles: type I, particles that rotate and slide between the pad and the workpiece and type II, abrasive particles that are between the pad and the workpiece. If the abrasive grain size is small, the material removed is mainly due to type II. In contrast, when the grain size is large the removed material is mainly due to type I. Another model studies how the strategy of material removal affects finishing and polished surface shape [9]. For this, different polishing paths made by non-spherical abrasive tools were studied, showing that the surface finish is highly dependent on the path defined.

In this paper a model of metal mould polishing is developed, with the purpose of evaluating the effect of process variables on the polishing time. Variables such as polishing speed, polishing force, abrasive-surface effective contact area, workpiece material and type of abrasive. The proposed model is constructed on the analysis of the Abbott-Firestone curve as an estimate of the surface texture, from which the planar polishing surface advance is predicted. The mechanism of material removal is the abrasion caused by the relative motion between sanding paper and workpiece material. Variables: workpiece material and type of abrasive are incorporated into the $K$ constant of the model.

2. Model

The classical wear theory and experiments developed by Preston [5] allow assuming that the material removed volume per unit of time $Q$ is proportional to the energy dissipated per unit of time $W$. In turn, the dissipated energy is a function of the cutting speed $V_c$ and the frictional force between the abrasive and the material. The cutting speed is the maximum relative velocity between the abrasive and the material. Assuming that the frictional force is proportional to the polishing force $F_p$, which is acting normal to the surface, the removed material rate reduces to:

$$Q = K \cdot V_c \cdot F_p$$  \hspace{1cm} (1)

where $K$ is the proportionality constant.

In Fig.1 the polishing of a surface previously machined with a spherical milling cutter with feed rate $f$ small and sidesteps $b$, can be seen. This process generates a texture characterized by semicircular canals with period $b$. 

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Moreover, the material removed rate is the product of the polishing face feed, multiplied by the effective contact area $Ac$, or what is the same:

$$ Q = -\frac{dz}{dt} \cdot Ac $$

Replacing (2) into (1) an equation that predicts the planar surface position of polished feed $z$ can be obtained.

$$ \frac{dz}{dt} = -K \cdot Vc \cdot \frac{Fp}{Ac} $$

The effective dimensionless contact area is a function that is known as a bearing ratio function of material area $A(z)$, defined by the Abbott-Firestone Curve [10]. It can be interpreted as a cumulative probability function of the ordinate $z(x,y)$, to a certain evaluation area.

$$ A(z) = \frac{Ac(z)}{At} $$

where $At$ is the total area or effective contact area in $z=0$.

Replacing (4) into (3) an equation that predicts the planar surface position as a function of the Abbott-Firestone curve among other variables.

$$ \frac{dz}{dt} = - \frac{K \cdot Vc \cdot Fp}{At} \cdot \frac{1}{A(z)} $$

Equation (5) solution, using the initial condition (6) allows determining how the planar surface position progresses in time for any bearing ratio function.

$$ z(t = 0) = Rt $$

where $Rt$ is the maximum peak valley roughness.

In this model, the polishing problem depends explicitly on the operation conditions: cutting speed $Vc$ and polishing force $Fp$. It also depends on the texture left by prior milling characterized by the Abbot-Firestone curve $A(z)$; as well as on other features of the polishing process such as the abrasive used and the mechanical properties of the material summarized in the $K$ constant, which must be determined experimentally for a specific material and abrasive.

### 2.1 Model constant determination

To determine the model constant, a least squares fit of a particular analytic solution of equation (5) is performed. This situation corresponds to a semicircular channel machining, developed with a spherical milling cutter, where feed $f$ is very small and sidestep $b$ relatively large compared to the feed, see Fig.2.
Analyzing a half period in the direction of the sidestep, an analytical expression for the bearing ratio function of material area $A(z)$ can be deduced. In this case, the curve defining the surface texture is a small circular arc, which can be approximated by a parabolic curve with error $O(x/r_H)^2$.

$$z = \frac{x^2}{2 \cdot r_H}$$  \hspace{1cm} (7)

where $r_H$ is the radius of the spherical milling cutter.

The effective contact area depending on ordinate $z$, and therefore the bearing ratio function of material area $A(z)$ or Abbott-Firestone Curve depending on ordinate $z$, can be obtained from Fig. 3.

$$A(z) = 1 - \frac{2}{b} \sqrt{2 \cdot r_H \cdot z}$$  \hspace{1cm} (8)

Moreover, the maximum peak valley roughness in the surface texture shown in Figure 3 can be estimated as follows:

$$Rt = \frac{b^2}{8 \cdot r_H}$$  \hspace{1cm} (9)

Substituting (9) into (8), an expression for the Abbott-Firestone Curve of semicircular channel milling, such as those shown in Figure 2, is obtained.

$$A(z) = 1 - \frac{z}{\sqrt{Rt}}$$  \hspace{1cm} (10)

Replacing (10) in (5), the differential equation that predicts the advancing of the polishing front can be obtained. This equation in a dimensionless form gives a compact expression that determines the advancing of the planar surface position, as shown in the following equation

$$\frac{dz^*}{dt} = \frac{P}{1 - \sqrt{z^*}}$$  \hspace{1cm} (11)

where the dimensionless ordinate value $z$ is

$$z^* = \frac{z}{Rt}$$  \hspace{1cm} (12)
and the constant $P$ is

$$P = \frac{-K \cdot Vc \cdot Fp}{Rt \cdot At}$$

(13)

The unit of constant $P$ is $s^{-1}$. That is why the dimensionless time is defined by

$$t^* = P \cdot t$$

(14)

Finally, the polishing model (11) is completely defined by specifying the initial condition of the problem:

$$z^*(t = 0) = 1$$

(15)

As per equation (13), number $P$ allows to find the dimensionless time and thereby adimensionalize the polishing model. This number is a fundamental part of this model and we will refer to it further on. $P$ explicitly encloses working conditions (cutting speed and polishing force); surface properties such as the maximum peak and valley roughness $Rt$; and polished surface area $At$. In turn, the type of material and the type of abrasive are embedded in the $K$ constant.

Analytically solving equation (11) with the initial condition (15), an equation that predicts the polishing front advance depending on time is obtained, as shown below.

$$\left(1 - z^*\right) - \frac{2}{3} \left(1 - z^{3/2}\right) + P \cdot t = 0$$

(16)

Once planar surface advance is known, an experimental data set ($z^*_i, t_i$) can be adjusted by least squares and thereby obtain the value of the constant $P$ that best fits the experimental data to equation (16).

$$P = \frac{\sum \left(\frac{2}{3} \left(1 - z_i^{3/2}\right) + z_i^* - 1\right) t_i}{\sum t_i^2}$$

(17)

With $i = 1, 2, ..., n$, where $n$ is the number of experimental points used in the fit.

Substituting (14) in (16) and clearing the variable $t^*$, to $z^*$ equal to zero, the dimensionless polishing time required to remove the surface topography left by a prior milling of semicircular channels can be estimated.

$$t^*(z^* = 0) = \frac{1}{3}$$

(18)

3. Experiments and results

The $K$ constant of the wear model has been determined on the basis of experimental data. To achieve this, workpieces with semicircular channels are machined, as shown in Figure 2. The tool used, the machining conditions and the workpiece dimensions are: an 8mm diameter spherical milling cutter; spindle speed of the milling machine 2500rpm, feed rate 500mm/min; 3 passes each with 0.5mm of cutting depth are made and a side step of 2.2mm. Specimen dimensions were 30mm diameter and 15mm height. These specimens are later polished with a STRUERS automatic polishing LaboPol-5.

The experimental determination of the planar surface advance requires successive measurements of the surface topography at different times. Figure 4a shows the position of the polishing profile of X5CrNi18-10 polished with abrasive P180, at three different time instants $t=0s$, 15s, and 30s. Surface profile shown in Figure 4a is obtained with a Mitutoyo SJ-210 surface roughness instrument. With the experimental points and equation (16), the value of the constant $P$ of the analytical model was obtained. Figure 4b shows a graph with the experimental points and analytical model for the mentioned conditions.
Equation (13) is used to obtain the $K$ constant. To achieve this, it is necessary to:

- Estimate the cutting speed $V_c$ from the average relative speed between the workpiece and the abrasive; measure
- The force applied $F_p$ and the planar surface position in the milled workpieces $z(t=0)$. The magnitude order of these quantities is: $F_p=15\text{N}$, $V_c=0.33\text{m/s}$ and $R_t=0.12\text{mm}$. The materials studied are:
  - AlCu4PbMg aluminum, 30CrNiMo8 low alloy steel, X5CrNi18-10 stainless steel and C45E carbon steel. The abrasives used are discs BUEHLER CarbiMet2 with granulometry P180, P240, P320, P400, P600 and P1200. They are associated with average grain sizes 82, 58.5, 46.2, 35, 25.8 and 15.3μm, respectively.

A set of experiments to study the effect of grain size and the material of the workpiece on the planar surface advance was performed. Fig. 5 shows the relationship between the $K$ constant and the diameter of the particle or grain size ($D_p$) in the 4 materials tested. In general $K$ shows a linear behavior with respect to the variation of the grain size. If the grain size increases, the $K$ constant increases and thereby increases the material removed rate. Moreover, changing materials defines a new linear relation between $K$ and $D_p$. This situation is repeated for the four materials tested. Figure 5 also shows the regression lines made for each material tested.
Table 1 shows the linear regression constants for the four materials tested. The coefficient $R^2$ adjusted is equal or greater than 0.993 for all four materials tested. Therefore, assuming that $K$ has a linear behavior with respect to the variation of the abrasive grain size used is very reasonable.

<table>
<thead>
<tr>
<th>Material</th>
<th>$m*1000$ (mm$^3$/Ws $\mu$m$^{-1}$)</th>
<th>$b*1000$ (mm$^3$/Ws)</th>
<th>Adjusted $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlCuMgPb</td>
<td>1.58±0.05</td>
<td>-4.7±2.4</td>
<td>0.993</td>
</tr>
<tr>
<td>30CrNiMo8</td>
<td>0.55±0.01</td>
<td>-1.9±0.4</td>
<td>0.999</td>
</tr>
<tr>
<td>X5CrNi18-10</td>
<td>0.44±0.01</td>
<td>7.4±0.7</td>
<td>0.994</td>
</tr>
<tr>
<td>C45E</td>
<td>0.25±0.00</td>
<td>10.6±0.2</td>
<td>0.999</td>
</tr>
</tbody>
</table>

4. Topography defined by the previous milling

The previous milling, which defines the surface topography, plays an essential role on the planar surface advance. The surface profile effect on the model has been introduced with the bearing material rate function $A(z)$. At the equation (5) shows that the planar surface advance and $A(z)$ are inversely proportional relationship. If it is calculated the dimensionless values of (12), (13) and (14), using (5), an equation that estimate the dimensionless planar surface position is obtained. This equation depends on the bearing material rate and the dimensionless time.

$$\frac{dz^*}{dt^*} = -\frac{1}{A(z^*)}$$ (19)

The influence of the polishing time and the bearing rate is studied with a surface topography simulation. The surface topography simulation is defined by a theoretical milling process, using a spherical milling tool for a cross mill strategy. This topography model assumes that the feed rate is small and the removal material is defined by the geometrical intersection of the spherical mill tool on the material. The theoretical computed topography is used to estimate the Abbott Firestone curve. That curve will be introduced at (19) to estimate the polishing front rate. This simulation let us determine the relative effect of the lateral step movement on the polishing front rate. The simulation showed at the Fig. 6 represents a limit situation, where the feed rate is infinitely small and the previous milling marks were eliminated. The figure represents the surface topography made with 1:1 relative sidestep milling process.

![Figure 6](image_url)

Figure 6. Surface topography of a cross milling done with a spherical milling cutter and relative lateral sidestep 1:1

Figure 7 shows the behavior of the Abbott-Firestone curve for different sidesteps of theoretical milling, defined in terms of the relation between $bx$ and $by$, sidesteps on X and on Y respectively. These figures show the results of surface topographies made with a sidestep on $bx:by$ of: 1:1, 5:1
and $\infty$: 1 (or parallel canals). Parallel canals, Fig.2, can be understood as a limit relation between $bx$ and $by$. If the relative lateral step increases, the bearing ratio increases. For example, for a dimensionless position of the polishing profile $z^*$ equals to 0.4, the bearing ratio to a relative steps 1:1, 5:1, and parallel; are 0.20, 0.29 and 0.39 respectively.

![Figure 7. Abbott-Firestone curves for different cross-milling strategies](image1)

**Figure 7. Abbott-Firestone curves for different cross-milling strategies**

Figure 8 shows the planar surface advance for the different size steps relations (for the three aforementioned machining strategies). In this figure it can be seen how the prior machining strategy influences on the polishing dimensionless time $t^*$. Polishing time required to remove the topography produced by prior milling $t^*_{max}$ depends on the machining strategy selected. The polishing time $t^*_{max}$ for sidesteps 1:1, 5:1 and parallel is 0.21, 0.27 and 0.33 respectively. Therefore, the 1:1 cross milling strategy is the most favorable to minimize polishing time.

![Figure 8. Planar surface advance for different prior cross milling](image2)

**Figure 8. Planar surface advance for different prior cross milling**

In general, the developed model allows estimating the polishing time required to remove topography left by any previous machining. To achieve this it is necessary to have the bearing ratio of the topography defined by the Abbott-Firestone curve.

The model shows the correct prediction of the surface profile advance. This situation is shown in Figure 4b, which corresponds to a topography defined by a parallel milling of semicircular channels.

### 5. Conclusions

A model that predicts the surface profile advance, based on the mechanism of abrasion has been developed successfully. The model separates the contributions made by the abrasive and the workpiece material from the contribution made by the prior milling. Model parameter varies linearly with the size of the abrasive grain. With this model, reference values to estimate polishing time on a surface made with a cross spherical milling are obtained.

### 6. References


7. Acknowledgements

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