Observation of Chaotic Behavior in Automatic Tuning Loops for Continuous–Time Filters

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Resumen
El apropiado modelizado dinámico lineal de filtros de tiempo continuo (CTFs) con lazos de sintonía automática debe llevarse a cabo para poder asegurar la estabilidad cuando queremos diseñar controladores de lazo mejorados. Con este objetivo, a partir de un análisis general y sistemático que permite obtener un modelo incremental linealizado equivalente para pequeña señal, se derivan las funciones de transferencia entre las variables de salida y las tensiones de control. Ello permite, a continuación, un posterior diseño de los compensadores de lazo con mejora en la estabilidad y en las prestaciones dinámicas. Este procedimiento sistemático permite conseguir mejores controladores para los dos lazos de control involucrados. Sin embargo, los CTFs con lazos de sintonía o ajuste automático son sistemas realimentados no lineales que tienden a ser inestables. Es más, fenómenos no lineales, que no pueden predecirse con un enfoque mediante modelos de pequeña señal orientados al diseño, pueden ser observados en este tipo de sistemas de sintonía. El propósito de este trabajo es poner de manifiesto que cuando se varían los parámetros de control, el sistema puede presentar diferentes tipos de fenómenos no lineales dinámicos tales como bifurcaciones y comportamiento caótico, que no pueden ser predichos por el modelo en pequeña señal orientado al diseño.

Palabras Clave: Filtros de tiempo continuo, lazos de sintonía automática, modelos incrementales equivalentes linealizados para pequeña señal, sistemas realimentados no-lineales, fenómenos no-lineales, bifurcaciones y comportamiento caótico.

Abstract
The appropriate linear dynamic modeling of continuous–time filters (CTFs) with automatic tuning loops should be obtained to assure stability in case an improved design of the loop controllers is to be carried out. With this aim, starting from a general and systematic analysis in order to obtain an equivalent small–signal linearized incremental model, from which transfer functions between output variables and control voltages are derived, the subsequent design of compensated loops with enhanced stability and dynamic performance is required. This systematic procedure allows obtaining improved controllers for the two involved control loops. However, CTFs with automatic tuning loops are nonlinear feedback systems with potential instability. What is more, nonlinear phenomena, which cannot be predicted by a design-oriented small signal modeling approach, are observed in this kind of tuning systems. The purpose of this work is to highlight that when control parameters are varied, the system could present different kinds of dynamical nonlinear phenomena such as bifurcations and chaotic behavior, which cannot be predicted by the small signal design-oriented model.

Keywords: Continuous–time filters (CTFs), automatic tuning loops, equivalent small–signal linearized incremental model, nonlinear feedback systems, nonlinear phenomena, bifurcations and chaotic behavior.

1. Introduction

Filters with tuning capabilities are adaptive filtering stages that incorporate tuning input signals aimed to directly modify the parameters of the original circuit structure. They typically exhibit a non–linearity of the bilinear type, which is well–known in several system modeling areas as in average models for switching power converters [1]. This bilinear behavior is caused by terms containing the product of state variables and control inputs in equations that describe their behavior. An additional source of modeling complexity of filters with tuning capability is caused by the sinusoidal nature of the reference input signal to the control loops, which requires the dynamic model to be obtained for low frequency baseband equivalent signals (envelope and phase shift).

In particular, starting from the modeling technique proposed in [2] for filters with automatic tuning, the corresponding transfer functions from the quality factor control signal to the amplitude of the output signal, as well as from the central frequency control signal to the phase–shift of the output signal are required. As a result, these transfer functions allow designing both enhanced non–adaptive and adaptive controllers that improve the performance of previous controllers based in dominant pole compensation for tuning both the quality factor (Q) and the central frequency (ω_O). Note that the design of loop controllers is, without the knowledge of the involved transfer functions, blind in relation to the considered system. In addition to this, the aim of the self–tuning subsystem might consist not only in correcting component tolerances and drift DC errors, but also in dynamically varying Q and ω_O parameters of the CTF. In such applications the loop bandwidth is critical.

However, nonlinear phenomena that can not be predicted by a design-oriented small signal modeling approach are observed in this kind of tuning systems. As a matter of fact, this bilinear behavior and time-varying nature of the reference input signal to the control loops make the system prone to exhibit nonlinear phenomena that cannot be predicted by a design-oriented small signal modeling approach. The prediction of such behaviors can only be predicted by a suitable model that retains the nonlinearity and the time-variance of the system.

That is, while an appropriate small signal linear dynamic modeling of the tunable filter should be obtained for design purpose, its ability to predict the real large-signal nonlinear dynamic behavior of the system is very limited. To overcome this problem, a general and systematic procedure has to be used in order to obtain a large signal nonlinear model.

The purpose of this work is to highlight that, when control parameters are varied, the system could present different kinds of dynamical nonlinear phenomena such as bifurcations and chaotic behavior, which cannot be predicted by the small signal design-oriented model. In addition, these nonlinear phenomena are also shown in high efficiency filters with tuning capabilities. Thus, the study of this behavior could be used to improve tuning control loops in electronically tunable switch-mode high-efficiency adaptive band-pass filters intended for energy harvesting applications [3].

The rest of the paper is organized as follows. Section II presents the system description and the mathematical dynamic model of CTFs with tuning capability. Section III shows the description of and application example, in which a discrete implementation of CTF with tuning capability is considered. Finally, in Section IV, non-chaotic and chaotic behaviors are observed. Different tools are combined to identify the dynamical behavior of the system. Finally, some concluding remarks are drawn in the last section.

2. Generic Dynamic Modeling of Tunable Filters

The conventional tuning strategy consists of an indirect adjustment based on the so–called master–slave scheme [2], [4]. The main filter (slave circuit) performs the filtering process for the incoming signal (Figure 1). The master filter, which is embedded within the ω_O and Q tuning control loops, receives a reference sine wave $v_{REF}(t)$, whose frequency (which must be as stable as possible) should ideally be tracked by the filter, hence indirectly setting the central frequency of the slave filter.

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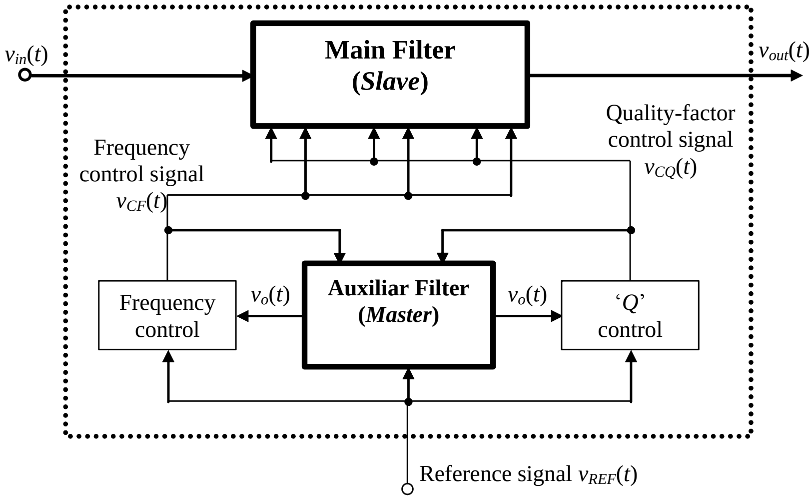

Figure 1. *Block diagram of a tuning system for CTF based on a master-slave strategy.*

The design and implementation of improved controllers for Q and ω_O loops (Figures 2.*a*) requires first the dynamic modeling of the *master* filter considering control signals $v_{CF}(t)$ and $v_{CQ}(t)$ as inputs, and the phase–shift of the filter output signal for the ω_O–control loop, and the amplitude of the filter output signal for the Q–control loop as outputs. Note that both control loops are nonlinear systems with non–DC steady–state.

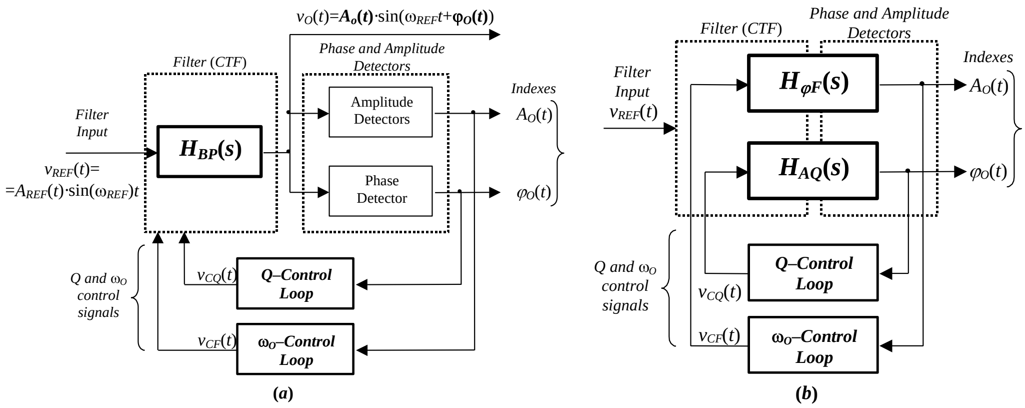

Figure 2. (a) *Basic block diagram of an on–chip tuning system for a CTF.* ***(b)*** *CTF model with amplitude and phase detectors and control loops.*

In order to analytically study the local stability of the tuning system, the first step consists in modeling the filter, considered as the control system plant, yielding the transfer functions of the equivalent small signal circuit. Generally [4], [5], most analog filtering structures, including automatic tuning, consider second order filters with three input signals as a master cell. Namely: The original input signal, $v_{REF}(t)$ in Figures 2.*a* and 2.*b*, and two control inputs, represented as $v_{CF}(t)$ and $v_{CQ}(t)$, that tune, respectively, central frequency (ω_O) and quality factor (Q) of the filter. In addition, this second order *master* filter has two state variables $v_1(t)$, $v_O(t)$, one of which is usually the output signal of the circuit. Therefore, the system can be expressed in a space–state representation form as [2]:

$$\mathbf{x}(t) = \begin{bmatrix} v_O(t) \\ v_1(t) \end{bmatrix} \qquad \mathbf{u}(t) = \begin{bmatrix} v_{REF}(t) \\ v_{CF}(t) \\ v_{CQ}(t) \end{bmatrix} \tag{1}$$

$$\dot{\mathbf{x}}(t) = \mathbf{A}\left(v_{CF}(t), v_{CQ}(t)\right)\cdot \mathbf{x}(t) + \mathbf{B}\left(v_{CF}(t), v_{CQ}(t)\right)\cdot v_{REF}(t)$$

It can be argued that phase and amplitude detectors (Figure 2.*a*) provide a circuit approximation to the baseband demodulation operation, with the objective of extracting from the state phasor $v_O(t)$ (modulated carrier signal), the phase and amplitude slow information that is required to properly tune both frequency and quality factor parameters, respectively (Figure 2.*b*).

From the baseband–equivalent linear equations, the transfer function that relates the amplitude of the state variables to the control voltage that tunes the quality factor $\tilde{v}_{CQ}(t)$, as well as the transfer function that relates the phase–shift of these state variables to the control voltage $\tilde{v}_{CF}(t)$ that tunes the central frequency ω_O of the master (and thus slave) filter can be obtained [2]. These transfer functions are required for studying loop (local) stability and, in turn, design enhanced compensators.

3. Application Example: Discrete of Continuous-Time Filter with Tuning Capability

As an example of a CTF with tuning capability that shows dynamical nonlinear phenomena that cannot be predicted by the small signal design-oriented model, a particular 2nd order continuous-time filter with tuning capability is considered in this work (Figure 3). This circuit structure consists in a 2nd order state–variable filter, in particular, the so–called TQE (transimpedance *Q*–enhancement) filter [6], [7]. In order to perform the automatic tuning of the CTF, resistors must be implemented by means of electronically tunable circuits. In an on chip implementation, there are several available structures in order to perform the tuning capability such as the he cell known as MOS Resistive Circuit (MRC) [8], [9]. However, as a matter of example, and in order to reveal these dynamical nonlinear phenomena (such as the aforementioned bifurcations and chaotic behavior) in automatic tuning of CTFs, analog multipliers are used as electronically tunable cells in order to implement a discrete implementation of the CTF (Figure 4). Notice that, in this case, control voltages $v_{CF}(t)$ and $v_{CQ}(t)$ tune, respectively, the central frequency (ω_O) and the quality factor (Q) of the TQE structure.

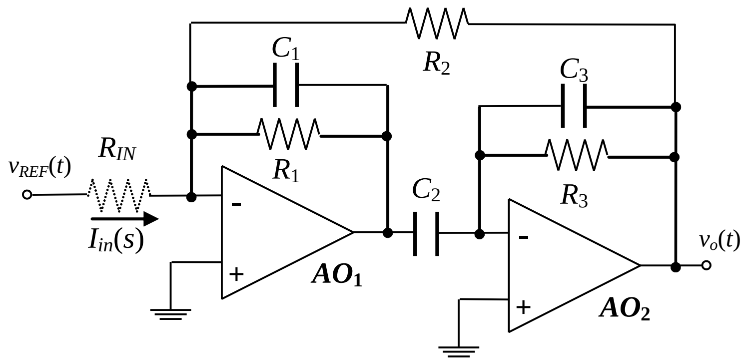

Figure 3. *2nd order band-pass filter with TQE (transimpedance Q–enhancement) topology.*

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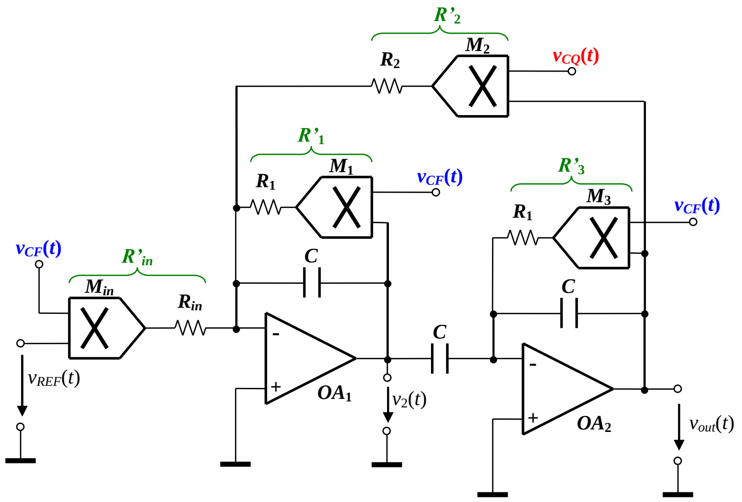

Figure 4. *2nd order analog multiplier-based band-pass filter with TQE topology.*

On the one hand, equivalent resistances R'_1=R'_3=R'_{in} (all equal) that set the central frequency □$_O$ of the structure are given by:

$$R'_1 = R'_3 = R'_{IN} = \frac{R_1}{K_M V_{CF}} \ , \qquad (2)$$

where constant K_M is the same for all analog multipliers (1/10 for commercial model AD633). Control voltage $v_{CF}(t)$ that tunes the central frequency (ω_O) is applied to analog multipliers M_1, M_3 y M_{in}. Therefore, the central frequency □$_O$ of the 2nd order topology is given by:

$$\omega_O = \frac{1}{R'_1 C} = \frac{K_M}{R_1 C} V_{CF} \qquad (3)$$

On the other, the equivalent resistance R'_2 that fixes the quality factor Q of the structure is:

$$R'_2 = \frac{R_2}{K_M V_{CQ}} \qquad (4)$$

Considering that Q is:

$$Q = \frac{1}{\left(2 - \frac{R'_1}{R'_2}\right)} \Rightarrow Q = \frac{1}{2 - \frac{R'_1}{R'_2}} = \frac{1}{2 - \frac{R_1 V_{CQ}}{R_2 V_{CF}}} \qquad (5)$$

3.1. *Q*-Control Loop

The control loop that tunes Q is shown in Figure 5. The input of the CTF filter (reference voltage $v_{REF}(t)$ in Figure 4) is applied to the analog multiplier M_1 by means of an amplifier with a gain proportional to the desired Q. On the other hand, the output of the CTF filter $v_o(t)$ is applied to the analog multiplier M_2. The difference of signals provided by these multipliers is applied to an integral control (the loop controller OA_3), in order to obtain the control voltage $v_{CQ}(t)$ that tunes the quality factor (Q) of the TQE structure. Basically, this control signal that tunes Q is:

$$v_{CQ}(t) = K_Q \int_{-\infty}^{t} \left(Q^2 V_{REF}^2 - V_O^2\right) dt \qquad (6)$$

Thus, the control loop makes that, when the difference of the squared output amplitude (V_O^2) is equal to the squared input amplitude (V_{REF}^2) multiplied by Q^2, the quality factor (Q) is tuned [10].

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Notice that a common choice for controlling the quality factor with a zero static error and a high loop bandwidth is by using a compensator with a single pole at the origin [11]. However, it has been shown in [10] that significant improvements on system response can be achieved if a more advanced controller is used. In this paper the same controller proposed in [10] is used. The transfer function of this controller (operational amplifier OA_3) is:

$$H_{CQ}(s) = \frac{K_i(s/\omega_z + 1)}{s(s/\omega_p + 1)}, \tag{7}$$

where parameter K_i is the controller gain, and ω_z and ω_p are an additional zero and pole, respectively.

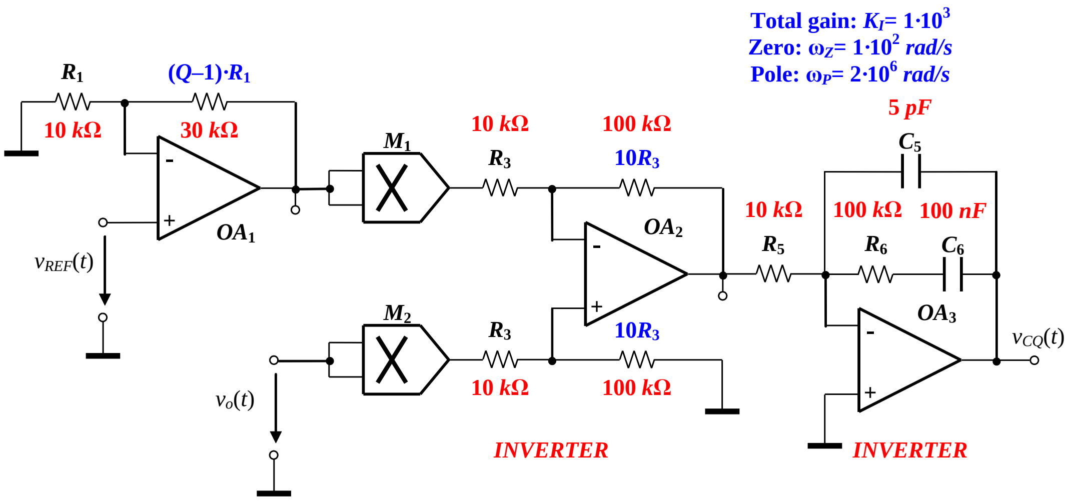

Figure 5. *Q-control loop that tunes the quality factor of the CTF. The controller (operational amplifier OA_3) acts as a pole-zero compensator.*

3.2. ω_O-Control Loop

On the other hand, the control loop that tunes ω_O is shown in Figure 6. The output of the CTF filter $v_o(t)$ is applied to a *tuned* integrator (thanks to M_1) in order to provide a 90º-phase shift (operational amplifier OA_1). This 90º-phase-shifted signal is then squared in order to "lose" the amplitude information and keep only phase information [10]. This squared waveform is multiplied with reference voltage $v_{REF}(t)$ in order to obtain an error signal proportional to the phase shift ϕ_ε between both signals (output and input signals to the CTF). This error is applied to an integral controller (OA_2) to obtain the final control voltage $v_{CF}(t)$ that tunes the central frequency (ω_O) of the TQE structure. As a consequence, this control signal that tunes ω_O is, basically:

$$v_{CF}(t) = V_{CF0} - K_F \int_{-\infty}^{t} \left[\sin(\phi_\varepsilon(\omega))\right] \cdot dt \tag{8}$$

Therefore, when the phase shift φ_ε between both signals (output and input signals to the CTF) is zero, the central frequency (ω_O) is tuned [10] by the control loop.

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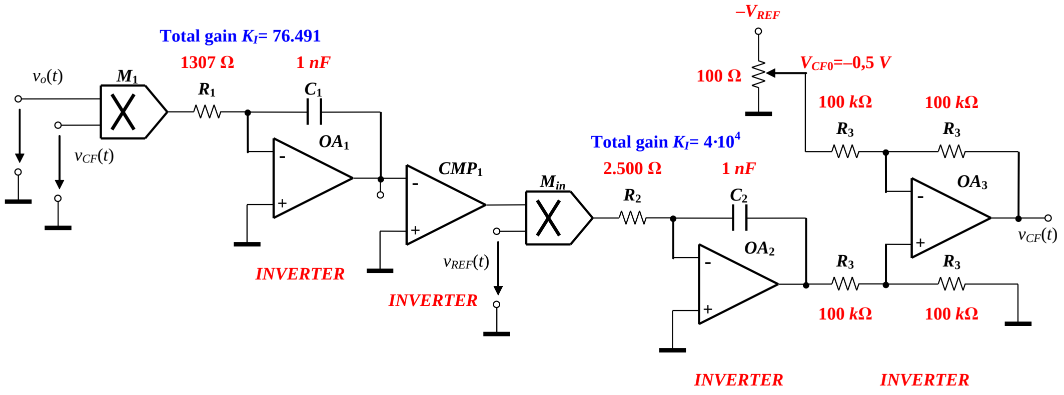
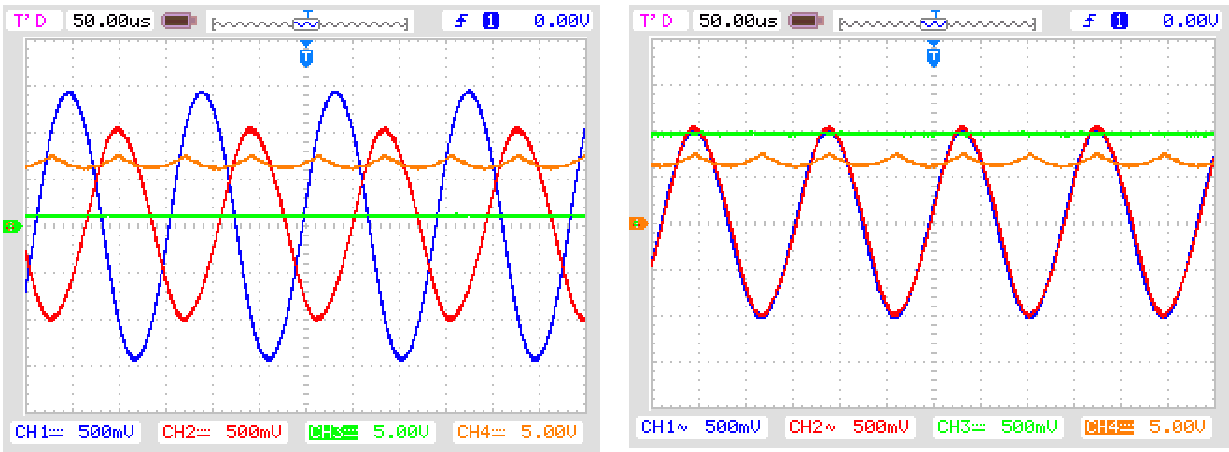


Figure 6. *ω_O-control loop that tunes the central frequency of the CTF.*

4. Experimental Results

In order to corroborate and validate the chaotic behavior, simulations and experimental trials have been carried out with CTF shown in Figure 4, operating with its two control loops (Figures 5 and 6). In Figure 7.*a* circuit signals can be observed when the system is not tuned. In this case, with an input signal (CH1) with amplitude of 3 *V*, and frequency of 7 *kHz*, and a *Q* set point of 1, the output signal (CH2) is not in phase with the reference input. However, if the filter is tuned (Figure 7.*b*), input and output signal signals of the filter are *in phase*, and the output amplitude of the output signal is a value that agrees with the desired *Q* factor (*Q*=1 in this case).



(*a*) **(*b*)**

Figure 7. *Experimental waveforms when the filter is not tuned* **(*a*)** *and tuned* **(*b*)**.

Channel 1: Input signal v_{REF}(t); *Channel 2: Output signal* v_o(t) ; *Channel 3:* Q*-control voltage* v_{CQ}(t); *and Channel 4:* ω_O*-control voltage* v_{CF}(t).

Finally, if the input (reference frequency) of the master filter decreases (Figure 8) below 4 *kHz*, the filter shows chaotic behavior. The phase-space diagram of the tuned CTF that shows the intermediate signal (v_2) and the output signal (v_{out}) with chaotic behavior is also shown in Figure 8.

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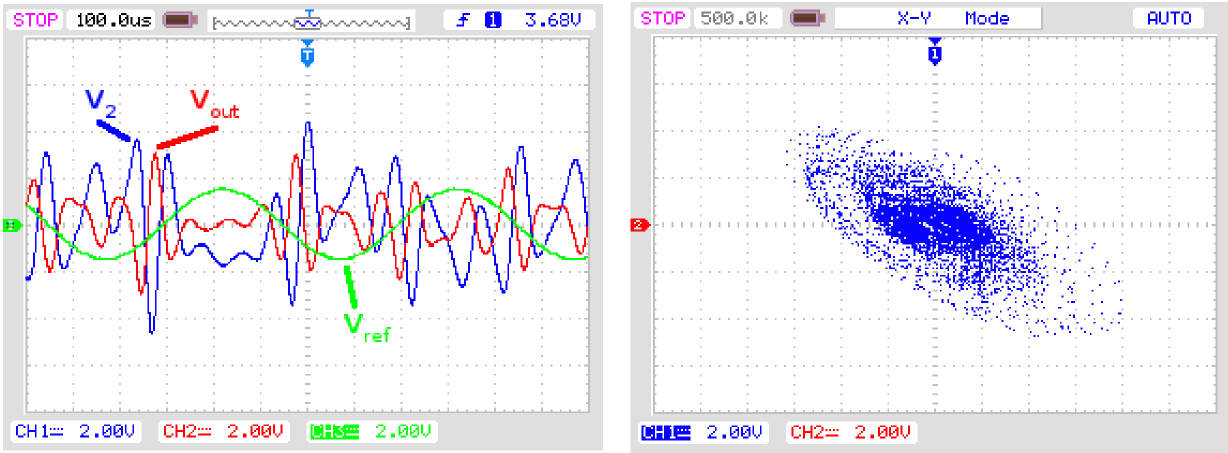

(*a*) **(*b*)**

Figure 8. **(*a*)** *Experimental voltage waveforms when the filter shows chaotic behavior.*

(*b*) *Phase-space diagram of the tuned CTF that shows the input voltage* (v_{ref})*, the intermediate voltage* (v_2) *and the output voltage* (v_{out}) *with chaotic behavior.*

5. Conclusions

In this paper, we have studied the dynamic behavior of a continuous-time filter with automatic tuning loops, which is a nonlinear feedback system that may present nonlinear phenomena. The small signal model, which is usually considered for designing the controllers, fails to predict the real behavior of the system.

However, this paper shows and reveals that nonlinear phenomena such as bifurcations and chaotic behavior may be found in this kind of tuned systems.

Finally, it is important to highlight that results obtained in this work can provide some help to advance in the observation and study of the aforementioned dynamical nonlinear phenomena, avoiding unstable behavior in automatic tuned filters.

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