

```
In[235]:= Off[rmnsm];  
Remove["Global`*"]  
Off[General::spell1]  
Off[Solve::ifun]
```

1. Introducción de los datos iniciales

Unidades

bi Ancho de los elementos (cm)
t Espesor de los elementos (cm)
v Coeficiente de Poisson
e Modulo de elasticidad en kN/cm²
uni Valor del desplazamiento asociado a cada nodo (cm)
L Longitud del perfil (cm)
n Indice de precisión de los gráficos

■ Notaciones

Se nota por X_{inj} , el valor X_i al mover unitariamente el nudo j .

```
In[239]:= b1 = 1.31;  
b2 = 6.88;  
b3 = 8.97;  
t = 0.167;  
v = 0.3;  
e = 20000;  
un1 = un2 = un3 = un4 = un5 = un6 = 1;  
L = 200;  
n = 100;
```

2. Movimiento de solido rígido de la sección

■ Coordenadas iniciales de los nodos del perfil

```
In[248]:= P1 = {b1, 0};  
P2 = {0, 0};  
P3 = {0, b2};  
P4 = {b3, b2};  
P5 = {b3, 0};  
P6 = {b3 - b1, 0};
```

■ Angulos iniciales entre los elementos del perfil

```
In[254]:= T2 = T3 = T4 = T5 = Pi / 2;  
T02 = T05 = Pi / 2;
```

■ Estado de la seccion antes de la deformación

```
In[256]:= EI = ListPlot[{P1, P2, P3, P4, P5, P6}, PlotStyle -> {AbsolutePointSize[10], RGBColor[1, 0, 0],
  AbsoluteThickness[3]}, Axes -> False, PlotJoined -> True, ImageSize -> 600, AspectRatio -> Automatic];
```



2.1. Movimiento del nudo 3

■ Determinación de la posición del nodo 2 al mover el nodo 3: P2n3'

```
In[257]:= P2n3' = {0, -un3 / b2}
```

```
Out[257]:= {0, -0.145349}
```

■ Determinación de la posición del nodo 3 al mover el nodo 3: P3n3'

```
In[258]:= P3n3' = {un3 / b3, b2 - un3 / b2}
```

```
Out[258]:= {0.111483, 6.73465}
```

■ Determinación de la posición del nodo 4 al mover el nodo 3: P4n3'

```
In[259]:= P4n3' = {b3 + un3 / b3, b2}
```

```
Out[259]:= {9.08148, 6.88}
```

■ Determinación de la posición del nodo 5 al mover el nodo 3: P5n3'

```
In[260]:= P5n3' = P5
```

```
Out[260]:= {8.97, 0}
```

■ Determinación de la posición del nodo 6 al mover el nodo 3: P6n3'

```
In[261]:= N[s1n3 = {X6n3', Y6n3'} /. Solve[{{(X6n3' - P5n3'[[1]])^2 + (Y6n3' - P5n3'[[2]])^2 == b1^2, (P5n3'[[1]] - X6n3') *
  (P4n3'[[1]] - P5n3'[[1]]) + (P5n3'[[2]] - Y6n3') * (P4n3'[[2]] - P5n3'[[2]]) == 0}, {X6n3', Y6n3'}]];
  N[P6n3' = If[s1n3[[2, 1]] <= P5[[1]], s1n3[[2]], s1n3[[1]]]]
```

```
Out[262]:= {7.66017, 0.0212243}
```

■ Determinación de la posición del nodo 1 al mover el nodo 3: P1n3'

```
In[263]:= N[s2n3 = {X1n3', Y1n3'} /. Solve[{{(X1n3' - P2n3'[[1]])^2 + (Y1n3' - P2n3'[[2]])^2 == b1^2, (X1n3' - P2n3'[[1]]) *
(P3n3'[[1]] - P2n3'[[1]]) + (Y1n3' - P2n3'[[2]]) * (P3n3'[[2]] - P2n3'[[2]]) == 0}, {X1n3', Y1n3'}]];
N[P1n3' = If[s2n3[[2, 1]] >= P2n3'[[1]], s2n3[[2]], s2n3[[1]]]]
```

```
Out[264]:= {1.30983, -0.166573}
```

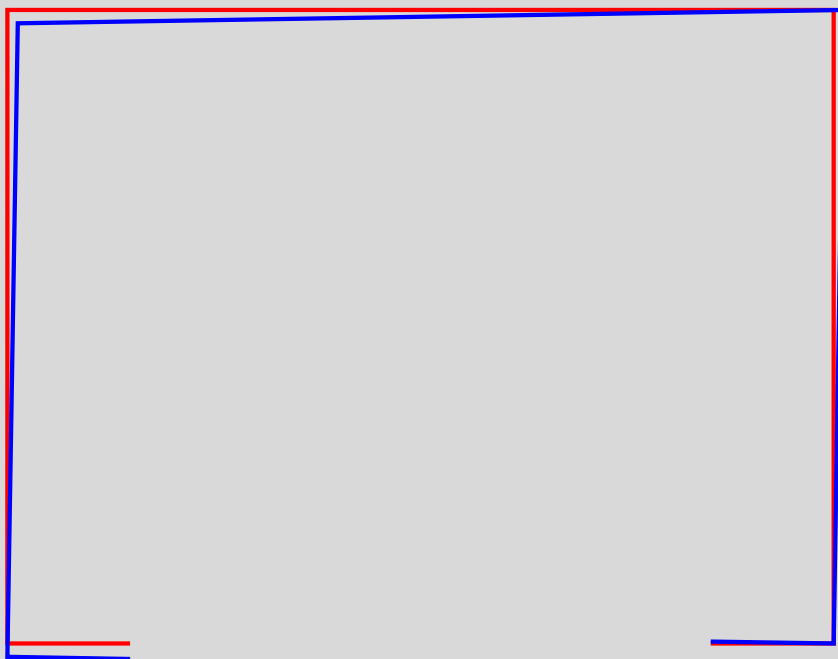
■ Deformada de la sección como un sólido rígido

```
In[265]:= CGn3 = ListPlot[{P1n3', P2n3', P3n3', P4n3', P5n3', P6n3'}, PlotStyle -> {AbsolutePointSize[10],
RGBColor[0, 0, 1], AbsoluteThickness[3]}, Axes -> False, PlotJoined -> True, ImageSize -> 600,
AspectRatio -> Automatic];
```



■ Superposición de la deformada al estado inicial

```
In[266]:= Show[EI, CGn3, ImageSize -> 600, AspectRatio -> Automatic];
```



■ Determinación del ángulo θ_3' al mover el nodo 3: T03n3

```
In[267]:= N[S3n3 = T03n3 /. Solve[((P3n3'[1] - P2n3'[1]) * (P3n3'[1] - P4n3'[1]) + (P3n3'[2] - P2n3'[2]) *
      * (P3n3'[2] - P4n3'[2])) /
      ((P3n3'[1] - P2n3'[1])^2 + (P3n3'[2] - P2n3'[2])^2) *
      ((P3n3'[1] - P4n3'[1])^2 + (P3n3'[2] - P4n3'[2])^2)^(1/2) = Cos[Pi - T03n3], T03n3]];
N[T03n3 = If[S3n3[1] >= 0, S3n3[1], S3n3[2]]];
N[T03n3] * 180 / (Pi)

Out[269]= 88.1433
```

■ Determinación del ángulo θ_4' al mover el nodo 3: T04n3

```
In[270]:= N[S4n3 = T04n3 /. Solve[((P4n3'[1] - P3n3'[1]) * (P4n3'[1] - P5n3'[1]) + (P4n3'[2] - P3n3'[2]) *
      * (P4n3'[2] - P5n3'[2])) /
      ((P4n3'[1] - P3n3'[1])^2 + (P4n3'[2] - P5n3'[2])^2) *
      ((P4n3'[2] - P3n3'[2])^2 + (P4n3'[2] - P5n3'[2])^2)^(1/2) = Cos[Pi - T04n3], T04n3]];
N[T04n3 = If[S4n3[1] >= 0, S4n3[1], S4n3[2]]];
N[T04n3] * 180 / (Pi)

Out[272]= 91.8566
```

■ Cálculo de los momentos hiperestáticos (M32, M43) al mover el nodo 3

```
In[273]:= i = t^3 / (12 * (1 - v^2)^2);
(T = {T3, T4}) // MatrixForm;
(T0n3 = {N[T03n3], N[T04n3]}) // MatrixForm;
(Mn3 = {M32n3, M43n3}) // MatrixForm;
(An3 = (1 / (e * i)) * {
      {-(b2 / 3 + b3 / 3), -b3 / 6},
      {-b3 / 6, -(b3 / 3 + b2 / 3)}
    }) // MatrixForm;
(S5n3 = {M32n3, M43n3} /. Solve[T + An3.Mn3 == T0n3, Mn3]) // MatrixForm

(0.0801816 -0.0801799)
```

■ Determinación del giro de los ángulos ($\theta_2, \theta_3, \theta_4, \theta_5$) al mover el nodo 3

```
In[279]:= M32n3 = S5n3[[1, 1]];
M43n3 = S5n3[[1, 2]];
theta2n3 = -L / Pi * (1 / 2 * M32n3 / (3 * e * i / b2) + ArcTan[1 / (b3 * b2)]);
theta3n3 = L / Pi * (M32n3 / (3 * e * i / b2) - ArcTan[1 / (b3 * b2)]);
theta4n3 = L / Pi * (M43n3 / (3 * e * i / b3) + 1 / 2 * M32n3 / (3 * e * i / b3) + ArcTan[1 / (b3 * b2)]);
theta5n3 = L / Pi * (1 / 2 * M43n3 / (3 * e * i / b2) - ArcTan[1 / (b3 * b2)]);
(theta3 = {theta2n3, theta2n3, theta3n3, theta4n3, theta5n3, theta5n3}) // MatrixForm

(
  -1.6559
  -1.6559
  0.217358
  0.21741
  -1.65589
  -1.65589
)
```

2.2. Movimiento del nodo 1

■ Determinación de la posición del nodo 3 al mover el nodo 1: P3n1'

```
In[286]:= P3n1' = P3

Out[286]= {0, 6.88}
```

■ Determinación de la posición del nodo 4 al mover el nodo 1: P4n1'

```
In[287]:= P4n1' = P4

Out[287]= {8.97, 6.88}
```

■ Determinación de la posición del nodo 5 al mover el nodo 1: P5n1'

In[288]:= P5n1' = P5

Out[288]:= {8.97, 0}

■ Determinación de la posición del nodo 6 al mover el nodo 1: P6n1'

In[289]:= P6n1' = P6

Out[289]:= {7.66, 0}

■ Determinación de la posición del nodo 2 al mover el nodo 1: P2n1'

In[290]:= P2n1' = {-1 / b1, 0}

Out[290]:= {-0.763359, 0}

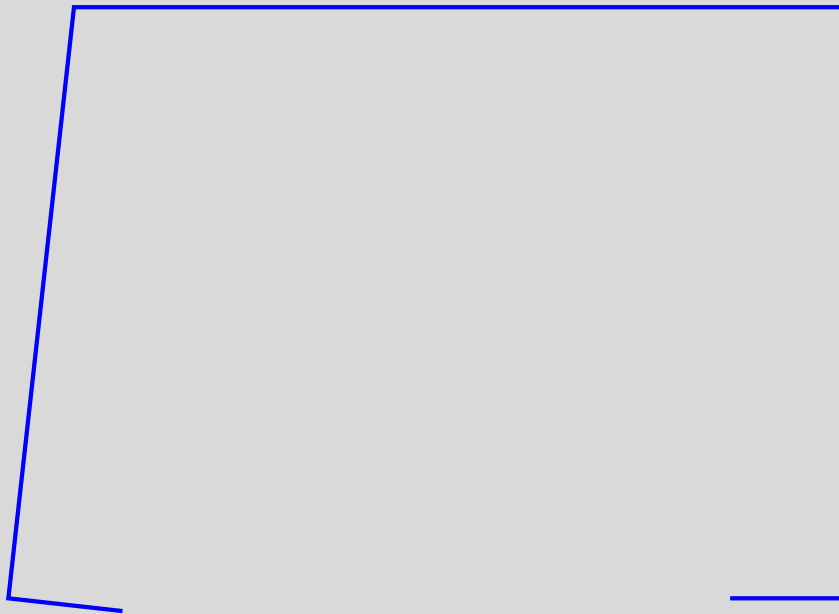
■ Determinación de la posición del nodo 1 al mover el nodo 1: P1n1'

In[291]:= N[s1n1 = {X1n1', Y1n1'} /. Solve[{{(X1n1' - P2n1'[[1]])^2 + (Y1n1' - P2n1'[[2]])^2 == b1^2,
(X1n1' - P2n1'[[1]]) * (P3n1'[[1]] - P2n1'[[1]]) + (Y1n1' - P2n1'[[2]]) * (P3n1'[[2]] - P2n1'[[2]]) == 0},
{X1n1', Y1n1'}];
N[P1n1' = If[s1n1[[2, 1]] >= P2n1'[[1]], s1n1[[2]], s1n1[[1]]]

Out[292]:= {0.538651, -0.144462}

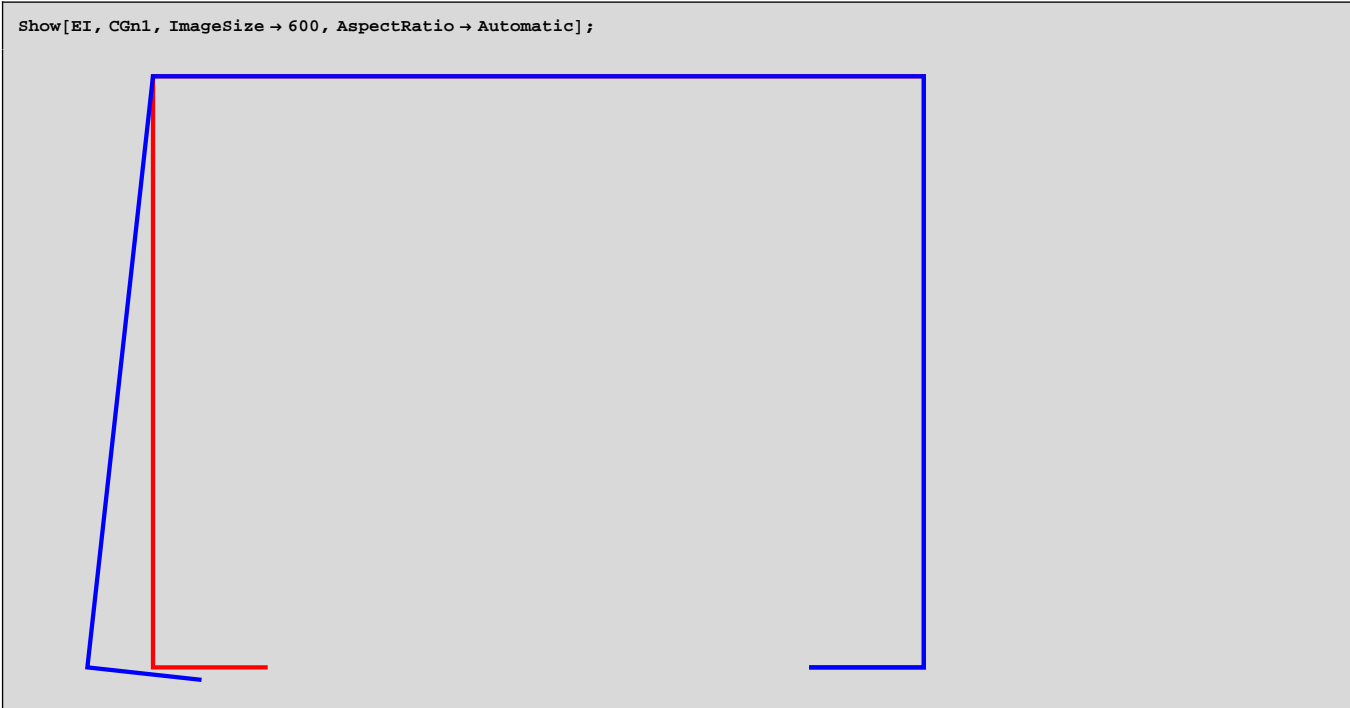
■ Deformada de la sección como un sólido rígido al mover el nodo 1

In[293]:= CGn1 = ListPlot[{P1n1', P2n1', P3n1', P4n1', P5n1', P6n1'}, PlotStyle -> {AbsolutePointSize[10],
RGBColor[0, 0, 1], AbsoluteThickness[3]}, Axes -> False, PlotJoined -> True, ImageSize -> 600,
AspectRatio -> Automatic];



■ Superposición de la deformada con la geometría inicial

```
In[294]:= Show[EI, CGn1, ImageSize -> 600, AspectRatio -> Automatic];
```



■ Determinación del ángulo θ_4 ' al mover el nodo 1: T04n1

```
In[295]:= T04n1 = Pi / 2
```

```
Out[295]:=  $\frac{\pi}{2}$ 
```

■ Determinación del ángulo θ_3 ' al mover el nodo 1: T03n1

```
In[296]:= N[S2n1 = T03n1 /. Solve[ ((P3n1'[1] - P2n1'[1]) * (P3n1'[1] - P4n1'[1]) + (P3n1'[2] - P2n1'[2]) * (P3n1'[2] - P4n1'[2])) /
    ((P3n1'[1] - P2n1'[1])^2 + (P3n1'[2] - P2n1'[2])^2) *
    ((P3n1'[1] - P4n1'[1])^2 + (P3n1'[2] - P4n1'[2])^2)^(1/2) == Cos[Pi - T03n1], T03n1]];
N[T03n1 = If[S2n1[[1]] >= 0, S2n1[[1], S2n1[[2]]]];
N[T03n1] * 180 / (Pi)
```

```
Out[296]:= 83.6687
```

■ Cálculo de los momentos hiperestáticos (M32, M43) al mover el nodo 1

```
In[299]:= (T = {T3, T4}) // MatrixForm;
(T0n1 = {N[T03n1], N[T04n1]}) // MatrixForm;
(Mn1 = {M32n1, M43n1}) // MatrixForm;
(An1 = (1 / (e * i)) * {
    {-(b2 / 3 + b3 / 3), -b3 / 6},
    {-b3 / 6, -(b3 / 3 + b2 / 3)}
}) // MatrixForm;
(S3n1 = {M32n1, M43n1} /. Solve[T + An1.Mn1 == T0n1, Mn1]) // MatrixForm
(0.213118 -0.060305)
```

■ Determinación del giro de los ángulos ($\theta_2, \theta_3, \theta_4, \theta_5$) al mover el nodo 1

```
In[304]:= M32n1 = S3n1[[1, 1]];
M43n1 = S3n1[[1, 2]];
theta2n1 = L/Pi * (-1/2 * M32n1 / (3 * e * i / b2) - ArcTan[1 / (b1 * b2)]);
theta3n1 = L/Pi * (M32n1 / (3 * e * i / b2) - ArcTan[1 / (b1 * b2)]);
theta4n1 = L/Pi * (-M43n1 / (3 * e * i / b2));
theta5n1 = L/Pi * (1/2 * M43n1 / (3 * e * i / b2));
{theta1 = {theta2n1, theta2n1, theta3n1, theta4n1, theta5n1, theta5n1}} // MatrixForm

{
  -8.6944
  -8.6944
  -3.71539
  0.939258
  -0.469629
  -0.469629
}
```

2.3. Movimiento del nodo 2

■ Determinación de la posición del nodo 4 al mover el nodo 2: P4n2'

```
In[311]:= P4n2' = P4
Out[311]:= {8.97, 6.88}
```

■ Determinación de la posición del nodo 5 al mover el nodo 2: P5n2'

```
In[312]:= P5n2' = P5
Out[312]:= {8.97, 0}
```

■ Determinación de la posición del nodo 6 al mover el nodo 2: P6n2'

```
In[313]:= P6n2' = P6
Out[313]:= {7.66, 0}
```

■ Determinación de la posición del nodo 3 al mover el nodo 2: P3n2'

```
In[314]:= P3n2' = {0, b2 + 1 / b2}
Out[314]:= {0, 7.02535}
```

■ Determinación de la posición del nodo 2 al mover el nodo 2: P2n2'

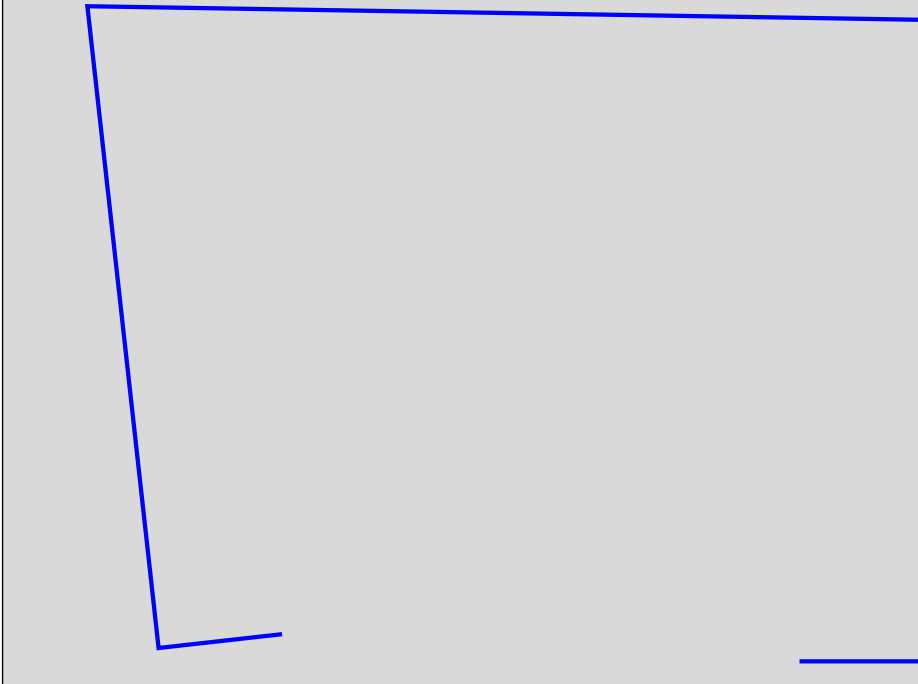
```
In[315]:= P2n2' = {1 / b1, 1 / b2}
Out[315]:= {0.763359, 0.145349}
```

■ Determinación de la posición del nodo 1 al mover el nodo 2: P1n2'

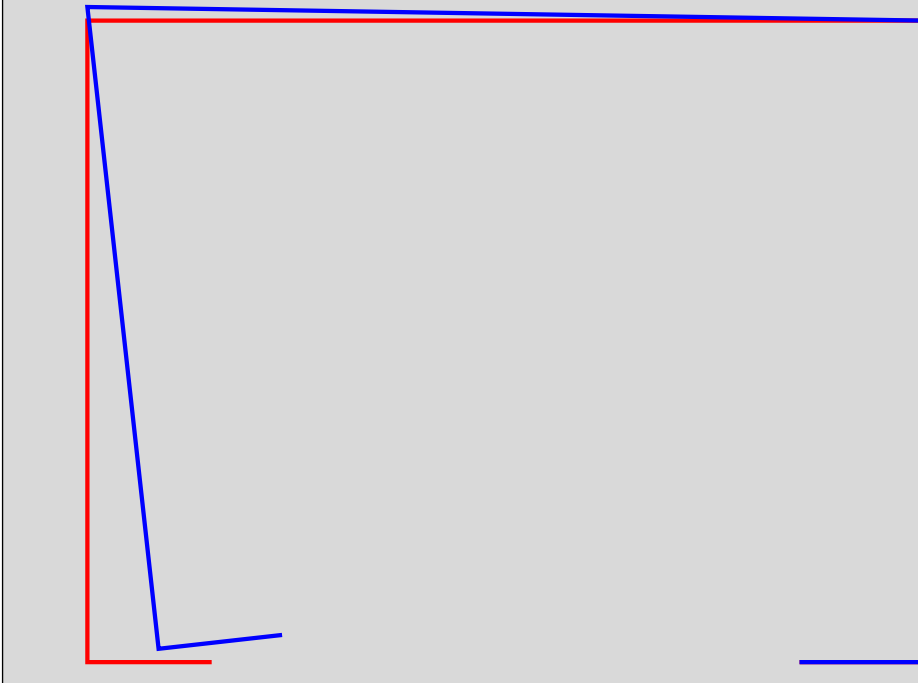
```
In[316]:= N[s1n2 = {X1n2', Y1n2'} /. Solve[{(X1n2' - P2n2'[[1]])^2 + (Y1n2' - P2n2'[[2]])^2 == b1^2,
(X1n2' - P2n2'[[1]]) * (P3n2'[[1]] - P2n2'[[1]]) + (Y1n2' - P2n2'[[2]]) * (P3n2'[[2]] - P2n2'[[2]]) == 0},
{X1n2', Y1n2'}];
N[P1n2' = If[s1n2[[2, 1]] >= P2n2'[[1]], s1n2[[2]], s1n2[[1]]]
Out[317]:= {2.06537, 0.289811}
```

■ Deformada de la sección como un sólido rígido al mover el nodo 2

```
In[318]:= CGn2 = ListPlot[{P1n2', P2n2', P3n2', P4n2', P5n2', P6n2'}, PlotStyle -> {AbsolutePointSize[10],  
RGBColor[0, 0, 1], AbsoluteThickness[3]}, Axes -> False, PlotJoined -> True, ImageSize -> 600,  
AspectRatio -> Automatic];
```

**■ Superposición de la deformada con la geometría inicial**

```
In[319]:= Show[EI, CGn2, ImageSize -> 600, AspectRatio -> Automatic];
```



■ Determinación del ángulo θ_3' al mover el nodo 2: T03n2

```
In[320]:= N[S2n2 = T03n2 /. Solve[((P3n2'[1] - P2n2'[1]) * (P3n2'[1] - P4n2'[1]) + (P3n2'[2] - P2n2'[2]) * (P3n2'[2] - P4n2'[2])) /
  ((P3n2'[1] - P2n2'[1])^2 + (P3n2'[2] - P2n2'[2])^2) *
  ((P3n2'[1] - P4n2'[1])^2 + (P3n2'[2] - P4n2'[2])^2)^(1/2) = Cos[Pi - T03n2], T03n2]];
N[T03n2 = If[S2n2[1] >= 0, S2n2[1], S2n2[2]]];
N[T03n2] * 180 / (Pi)

Out[322]:= 97.2596
```

■ Determinación del ángulo θ_4' al mover el nodo 2: T04n2

```
In[325]:= N[S3n2 = T04n2 /. Solve[((P4n2'[1] - P3n2'[1]) * (P4n2'[1] - P5n2'[1]) + (P4n2'[2] - P3n2'[2]) * (P4n2'[2] - P5n2'[2])) /
  ((P4n2'[1] - P3n2'[1])^2 + (P4n2'[2] - P3n2'[2])^2) *
  ((P4n2'[1] - P5n2'[1])^2 + (P4n2'[2] - P5n2'[2])^2)^(1/2) = Cos[Pi - T04n2], T04n2]];
N[T04n2 = If[S3n2[1] >= 0, S3n2[1], S3n2[2]]];
N[T04n2] * 180 / (Pi)

Out[325]:= 89.0718
```

■ Cálculo de los momentos hiperestáticos (M32, M43) al mover el nodo 2

```
In[326]:= (T = {T3, T4}) // MatrixForm;
(T0n2 = {N[T03n2], N[T04n2]}) // MatrixForm;
(Mn2 = {M32n2, M43n2}) // MatrixForm;
(An2 = (1 / (e * i)) * {
  {-(b2 / 3 + b3 / 3), -b3 / 6},
  {-b3 / 6, -(b3 / 3 + b2 / 3)}
}) // MatrixForm;
(S4n2 = {M32n2, M43n2} /. Solve[T + An2.Mn2 == T0n2, Mn2]) // MatrixForm

(-0.253208 0.100393)
```

■ Determinación del giro de los ángulos ($\theta_2, \theta_3, \theta_4, \theta_5$) al mover el nodo 2

```
In[331]:= M32n2 = S4n2[[1, 1]];
M43n2 = S4n2[[1, 2]];
 $\theta_2n2 = L / \text{Pi} * (-1 / 2 * M32n2 / (3 * e * i / b2) + \text{ArcTan}[1 / (b1 * b2)]);$ 
 $\theta_3n2 = L / \text{Pi} * (M32n2 / (3 * e * i / b2) + \text{ArcTan}[1 / (b1 * b2)]);$ 
 $\theta_4n2 = L / \text{Pi} * (-M43n2 / (3 * e * i / b2));$ 
 $\theta_5n2 = L / \text{Pi} * (1 / 2 * M43n2 / (3 * e * i / b2));$ 
( $\theta n2 = \{\theta_2n2, \theta_3n2, \theta_4n2, \theta_5n2, \theta_5n2\}$ ) // MatrixForm

( 9.00661 )
( 9.00661 )
( 3.09098 )
( -1.56364 )
( 0.781819 )
( 0.781819 )
```

■ Matriz de giro de los nodos al mover los nodos 1, 2 y 3

```
In[338]:= (GA = -Transpose[{ $\theta n1, \theta n2, \theta n3$ }] // MatrixForm

( 8.6944 -9.00661 1.6559 )
( 8.6944 -9.00661 1.6559 )
( 3.71539 -3.09098 -0.217358 )
( -0.939258 1.56364 -0.21741 )
( 0.469629 -0.781819 1.65589 )
( 0.469629 -0.781819 1.65589 )
```

3. Obtención de la deformación de la sección

- Obtención de la función $Viq(s)$ que representa el desplazamiento nodal según el eje local s

```
In[339]:= Viq[1] = {1/b1, 0, 0, 0, 0};
Viq[2] = {-1/b1, 1/b2, 0, 0, 0};
Viq[3] = {0, -1/b2, 1/b3, 0, 0};
Viq[4] = Reverse[-Viq[3]];
Viq[5] = Reverse[-Viq[2]];
Viq[6] = Reverse[-Viq[1]];
V[i_, q_] := Viq[i][[q]];
```

- Definición de funciones necesarias para realizar los cálculos sucesivos

```
In[347]:= b = {b1, b2, b3, b4 = b2, b5 = b1};
s[q_] := Sum[b[[j]], {j, 1, q-1}];
Table[b[[i]], {i, 1, 5}];
Table[s[i], {i, 1, 6}];
xi[q_] := (s - s[q])/b[[q]];
phi1 = 1;
phi2[q_] := 2 * xi[q] - 1;
phi3[q_] := -xi[q] + 3/2 * xi[q]^2 - 1/2 * xi[q]^3;
phi4[q_] := -1/2 * xi[q] + 1/2 * xi[q]^3;
Table[xi[i], {i, 1, 5}];
Kq = e * t^3 / (12 * (1 - nu^2));
```

- Obtención de la función $Wiq(s)$ que representa el desplazamiento nodal según el eje local z

```
In[358]:= W[i_, q_] := Wq[i][[q]] * phi1 + b[[q]] / 2 * phi2[q] * theta[i][[q]] * Pi / L -
b[[q]]^2 / (3 * Kq) * (Mq[i][[2 * q - 1]] * phi3[q] - Mq[i][[2 * q]] * phi4[q]);
```

- Obtención de la función $W1q(s)$ que representa el desplazamiento nodal según el eje local z al mover el nodo 1

$Wq[1] = \{W11, W12, W13, W14, W15\}$

$\theta q[1] = \{\theta11, \theta12, \theta13, \theta14, \theta15\}$

$Mq[1] = \{M12n1, M21n1, M23n1, M32n1, M34n1, M43n1, M45n1, M54n1, M56n1, M65n1\}$

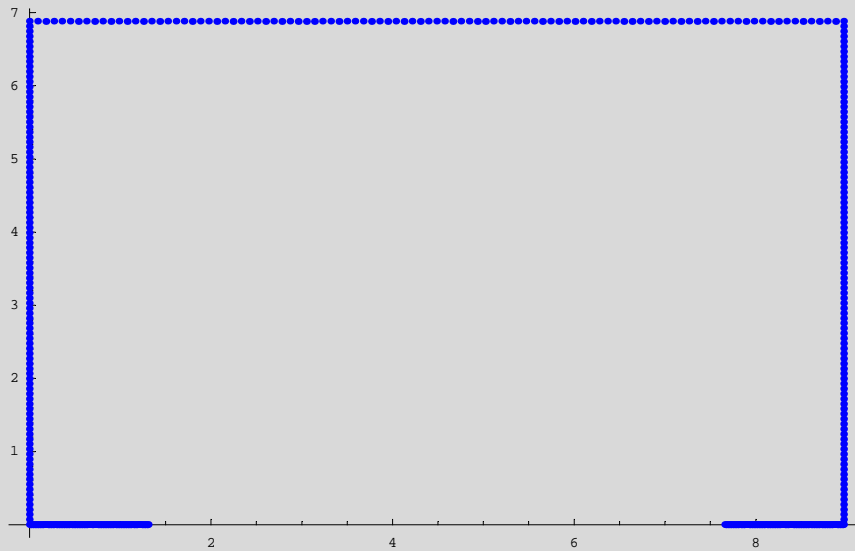
```
In[359]:= Wq[1] = {-(P1n1'[[2]] + P2n1'[[2]])/2, -(P2n1'[[1]] + P3n1'[[1]])/2, 0, 0, 0};
theta[1] = {-L/Pi * ArcTan[1/(b1 * b2)], -L/Pi * ArcTan[1/(b1 * b2)], 0, 0, 0};
Mq[1] = {0, 0, 0, M32n1, -M32n1, M43n1, -M43n1, 0, 0, 0};
```

$P[i, q]$ = coordenadas del elemento q al mover el nudo i

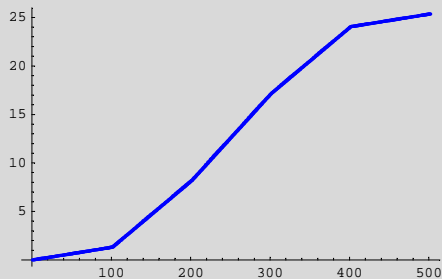
```
In[362]:= psi[1] = Pi;
psi[2] = Pi/2;
psi[3] = 0;
psi[4] = -Pi/2;
psi[5] = -Pi;
(Pass[q_] = {{Cos[psi[q]], -Sin[psi[q]]}, {Sin[psi[q]], Cos[psi[q]]}} // MatrixForm;
Pass[1] // MatrixForm;
Pt[1] = Table[{b1 - k * b1/n, 0}, {k, 0, n}];
Pt[2] = Table[{0, k * b2/n}, {k, 1, n}];
Pt[3] = Table[{k * b3/n, b2}, {k, 1, n}];
Pt[4] = Table[{b3, b2 - k * b2/n}, {k, 1, n}];
Pt[5] = Table[{b3 - k * b1/n, 0}, {k, 1, n}];
Pts = Join[Pt[1], Pt[2], Pt[3], Pt[4], Pt[5]];
Length[Pts];
```

■ Representación discreta de la geometría inicial

```
In[376]:= Fig = ListPlot[Pts, PlotStyle -> {RGBColor[0, 0, 1], AbsoluteThickness[3]}, Axes -> True,
  ImageSize -> 600, PlotJoined -> False];
```

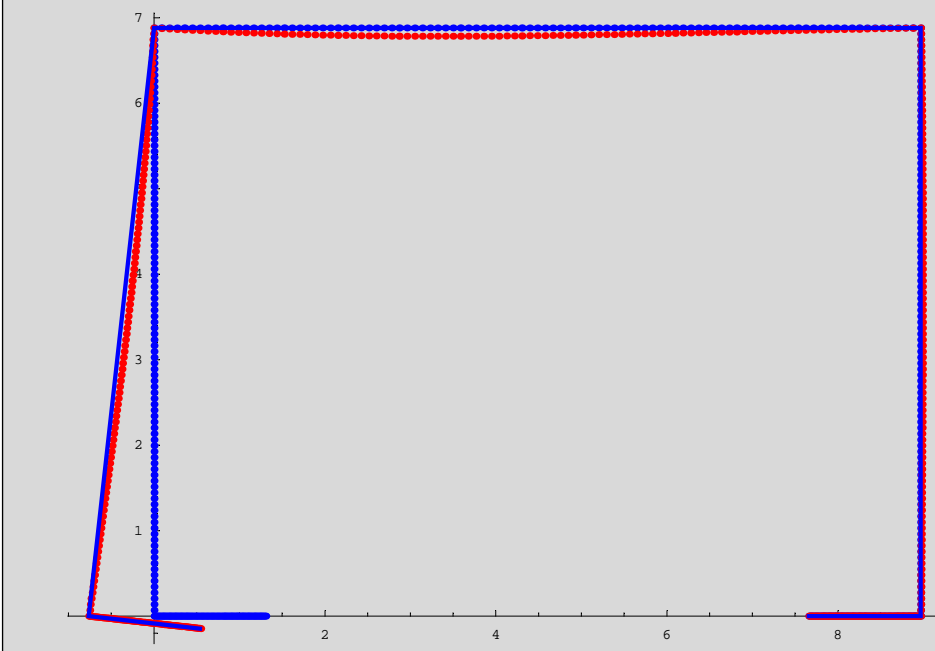


```
In[377]:= CoordPoint[p_] := Pts[[p]]
CoordPoint[1];
ss[p_] =
  If[1 <= p <= n+1, (p-1)*b1/n,
  If[n+2 <= p <= 2*n+1, b1 + (p-n-1)*b2/n,
  If[2*n+2 <= p <= 3*n+1, b1+b2 + (p-2*n-1)*b3/n,
  If[3*n+2 <= p <= 4*n+1, b1+b2+b3 + (p-3*n-1)*b2/n,
  If[4*n+2 <= p <= 5*n+1, b1+b2+b3+b2 + (p-4*n-1)*b1/n, "ssp pas compris"]]]];
tab1 = Table[ss[i], {i, 1, 5*n+1}];
ListPlot[tab1, PlotStyle -> {RGBColor[0, 0, 1], AbsoluteThickness[3]}, Axes -> True,
  ImageSize -> 300, PlotJoined -> False];
Length[tab1];
El[p_] =
  If[1 <= p <= n+1, 1,
  If[n+2 <= p <= 2*n+1, 2,
  If[2*n+2 <= p <= 3*n+1, 3,
  If[3*n+2 <= p <= 4*n+1, 4,
  If[4*n+2 <= p <= 5*n+1, 5, "segment non trouvé"]]]];
CoordPointdef[i_, p_] := (CoordPoint[p] + Pass[El[p]].{V[i, El[p]], ReplaceAll[W[i, El[p]], s -> ss[p]]});
L1 = ListPlot[Table[CoordPointdef[1, p], {p, 1, 5*n+1}], PlotStyle -> {RGBColor[1, 0, 0], AbsoluteThickness[3]},
  Axes -> True, ImageSize -> 1, AspectRatio -> Automatic, PlotJoined -> False];
```



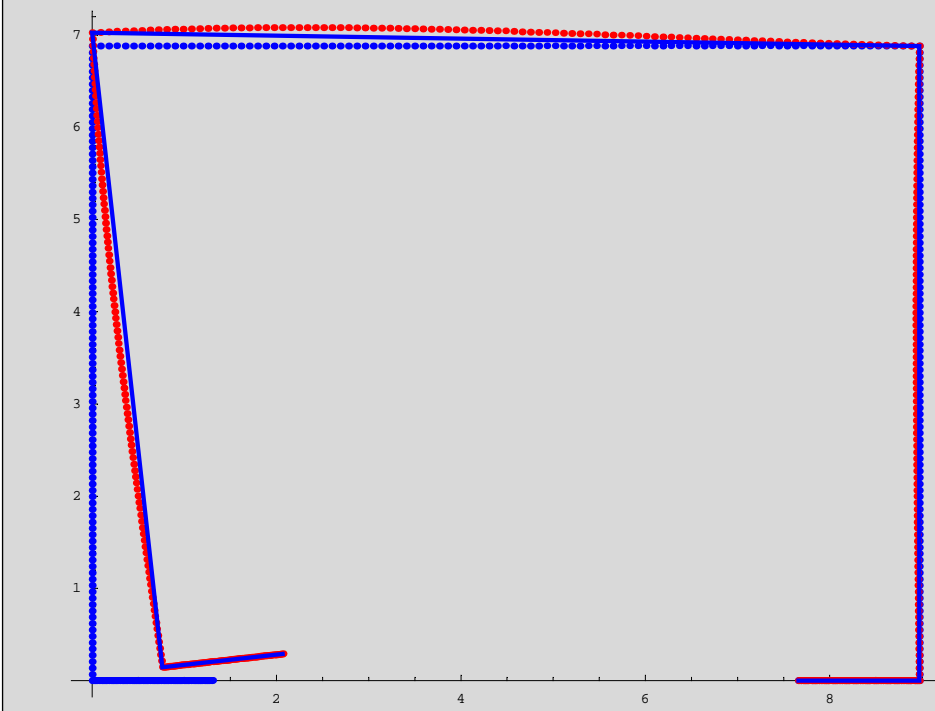
■ Deformada de la sección al mover el nodo 1

```
In[386]:= Show[Fig, L1, CGn1, ImageSize -> 600, AspectRatio -> Automatic];
```



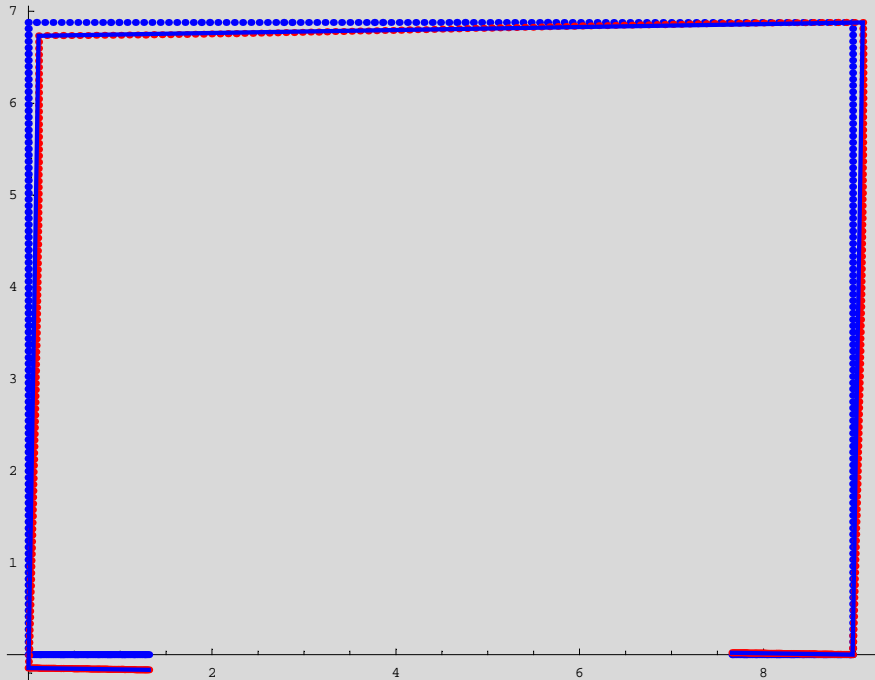
■ Deformada de la sección al mover el nodo 2

```
In[387]:= Wq[2] = {-(P1n2'[ [2]] + P2n2'[ [2]]) / 2, -(P2n2'[ [1]] + P3n2'[ [1]]) / 2, (P3n2'[ [2]] - P4n2'[ [2]]) / 2, 0, 0};
N[Wq[2]];
θq[2] = {L / Pi * ArcTan[1 / (b1 * b2)], L / Pi * ArcTan[1 / (b1 * b2)], -L / Pi * ArcTan[1 / (b3 * b2)], 0, 0};
Mq[2] = {0, 0, 0, M32n2, -M32n2, M43n2, -M43n2, 0, 0, 0};
L2 = ListPlot[Table[CoordPointdef[2, p], {p, 1, 5 * n + 1}], PlotStyle -> {RGBColor[1, 0, 0], AbsoluteThickness[3]},
  Axes -> True, ImageSize -> 1, AspectRatio -> Automatic, PlotJoined -> False];
Show[Fig, L2, CGn2, ImageSize -> 600, AspectRatio -> Automatic];
```



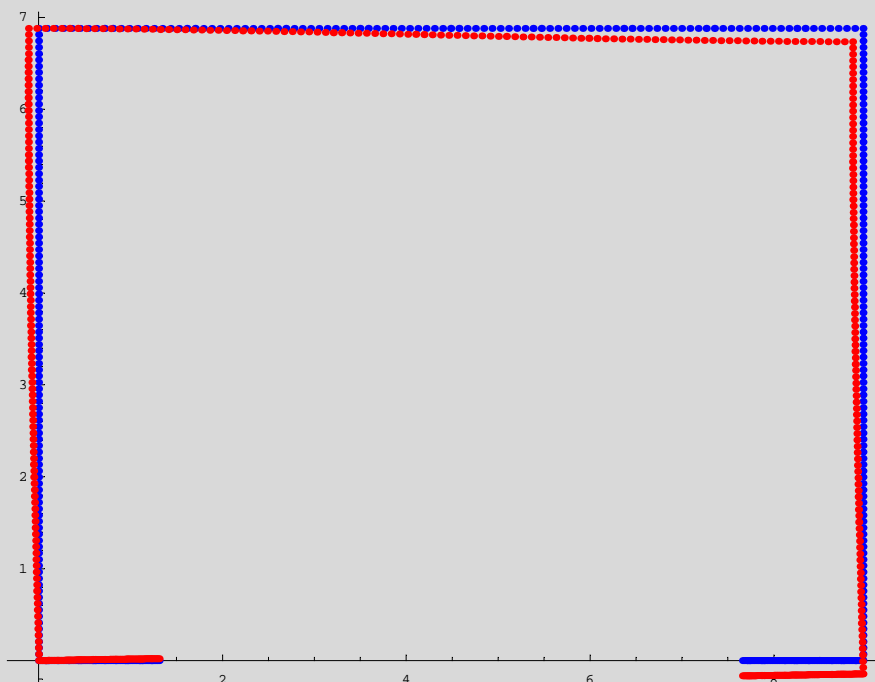
■ Deformada de la sección al mover el nodo 3

```
In[393]:= Wq[3] = {-(P1n3'[[2]] + P2n3'[[2]])/2, -(P3n3'[[1]] + P2n3'[[1]])/2, -(P4n3'[[2]] - P3n3'[[2]])/2,
(P4n3'[[1]] - P5n3'[[1]])/2, -(P6n3'[[2]] - P5n3'[[2]])/2};
θq[3] = {-L/Pi * ArcTan[1/(b2 * b3)], -L/Pi * ArcTan[1/(b2 * b3)], L/Pi * ArcTan[1/(b2 * b3)], -L/Pi *
ArcTan[1/(b2 * b3)], -L/Pi * ArcTan[1/(b2 * b3)]};
Mq[3] = {0, 0, 0, M32n3, -M32n3, M43n3, -M43n3, 0, 0, 0};
L3 = ListPlot[Table[CoordPointdef[3, p], {p, 1, 5 * n + 1}], PlotStyle -> {RGBColor[1, 0, 0], AbsoluteThickness[3]},
Axes -> True, ImageSize -> 1, AspectRatio -> Automatic, PlotJoined -> False];
Show[Fig, L3, CGn3, ImageSize -> 600, AspectRatio -> Automatic];
```



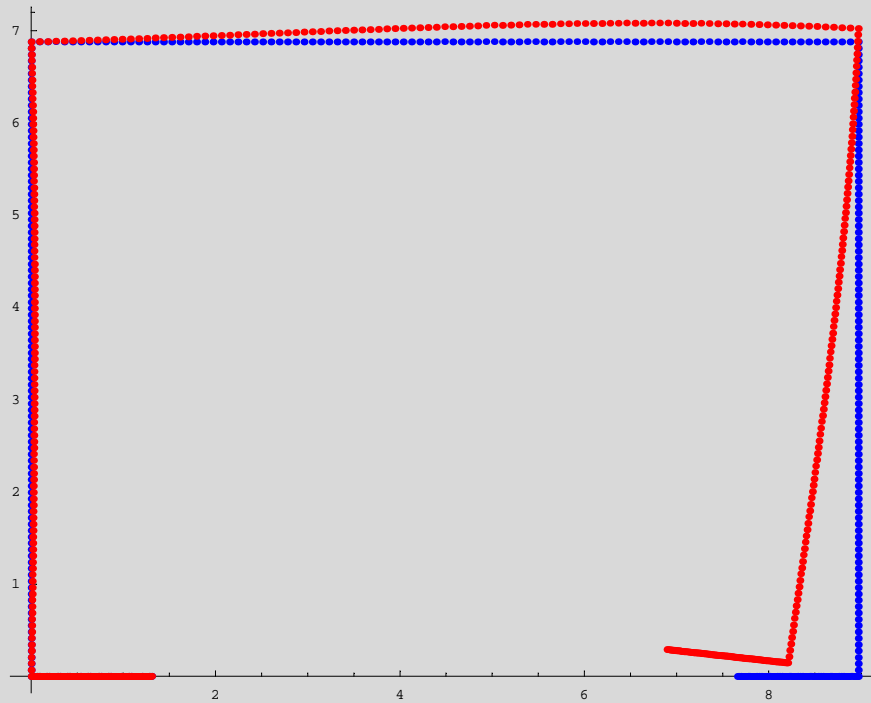
■ Deformada de la sección al mover el nodo 4 (se utilizan las propiedades de simetría del perfil)

```
In[398]:= Wq[4] = Reverse[Wq[3]];
θq[4] = Reverse[-θq[3]];
Mq[4] = Reverse[-Mq[3]];
L4 = ListPlot[Table[CoordPointdef[4, p], {p, 1, 5 * n + 1}], PlotStyle -> {RGBColor[1, 0, 0],
AbsoluteThickness[3]}, Axes -> True, ImageSize -> 1, AspectRatio -> Automatic, PlotJoined -> False];
Show[Fig, L4, ImageSize -> 600, AspectRatio -> Automatic];
```



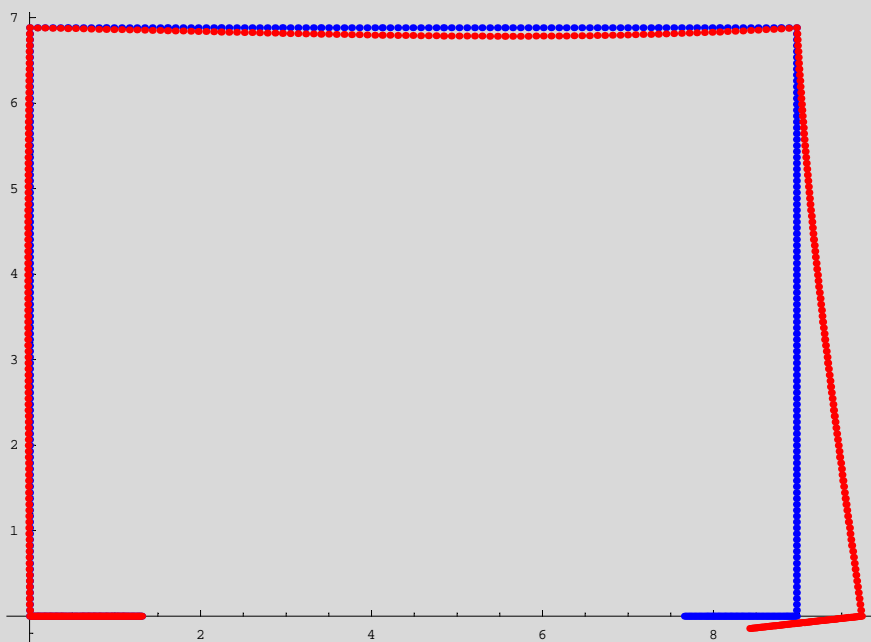
■ Deformada de la sección al mover el nodo 5 (se utilizan las propiedades de simetría del perfil)

```
In[403]:= Wq[5] = Reverse[Wq[2]];
          θq[5] = Reverse[-θq[2]];
          Mq[5] = Reverse[-Mq[2]];
          L5 = ListPlot[Table[CoordPointdef[5, p], {p, 1, 5 * n + 1}], PlotStyle -> {RGBColor[1, 0, 0],
            AbsoluteThickness[3]}, Axes -> True, ImageSize -> 1, AspectRatio -> Automatic, PlotJoined -> False];
          Show[Fig, L5, ImageSize -> 600, AspectRatio -> Automatic];
```



■ Deformada de la sección al mover el nodo 6 (se utilizan las propiedades de simetría del perfil)

```
In[408]:= Wq[6] = Reverse[Wq[1]];
          θq[6] = Reverse[-θq[1]];
          Mq[6] = Reverse[-Mq[1]];
          L6 = ListPlot[Table[CoordPointdef[6, p], {p, 1, 5 * n + 1}], PlotStyle -> {RGBColor[1, 0, 0],
            AbsoluteThickness[3]}, Axes -> True, ImageSize -> 1, AspectRatio -> Automatic, PlotJoined -> False];
          Show[Fig, L6, ImageSize -> 600, AspectRatio -> Automatic];
```



4. Determinación de las matrices de la T.V.G. no diagonalizadas

■ Propiedades del material utilizado

```
In[413]:= {e, t, v}
Out[413]:= {20000, 0.167, 0.3}
```

■ Calculo de la matriz Cik1

```
In[414]:= (Cik1 = t * {{b1 / 3, b1 / 6, 0, 0, 0, 0}, {b1 / 6, b2 / 3 + b1 / 3, b2 / 6, 0, 0, 0},
  {0, b2 / 6, b2 / 3 + b3 / 3, b3 / 6, 0, 0}, {0, 0, b3 / 6, b3 / 3 + b4 / 3, b4 / 6, 0},
  {0, 0, 0, b4 / 6, b4 / 3 + b5 / 3, b5 / 6}, {0, 0, 0, 0, b5 / 6, b5 / 3}} // MatrixForm
( 0.0729233  0.0364617  0          0          0          0
  0.0364617  0.45591    0.191493  0          0          0
  0          0.191493  0.882317  0.249665  0          0
  0          0          0.249665  0.882317  0.191493  0
  0          0          0          0.191493  0.45591    0.0364617
  0          0          0          0          0.0364617  0.0729233 )
```

■ Calculo de la matriz Cik2

```
In[415]:= Cik2[i_, j_] := (t^3 / 12) Sum[ (Integrate[W[i, q] * W[j, q] ds, {s, 0, 1}]), {q, 1, 5}];
(Cik2 = Table[Table[Cik2[i, j], {j, 1, 6}], {i, 1, 6}]) // MatrixForm
( 0.000451757  -0.000467416  -0.0000184256  0.0000541549  -0.000056436  0.0000363651
  -0.000467416  0.00052002   -0.0000234042  -0.0000598423  0.0000870786  -0.000056436
  -0.0000184256  -0.0000234042  0.0000755028  -0.0000279856  -0.0000598423  0.0000541549
  0.0000541549  -0.0000598423  -0.0000279856  0.0000755028  -0.0000234042  -0.0000184256
  -0.000056436  0.0000870786  -0.0000598423  -0.0000234042  0.00052002   -0.000467416
  0.0000363651  -0.000056436  0.0000541549  -0.0000184256  -0.000467416  0.000451757 )
```

■ Calculo de la matriz Cik=Cik1+Cik2

```
In[417]:= (Cik = Cik1 + Cik2) // MatrixForm
( 0.0733751    0.0359943    -0.0000184256  0.0000541549  -0.000056436  0.0000363651
  0.0359943    0.45643        0.19147        -0.0000598423  0.0000870786  -0.000056436
  -0.0000184256  0.19147        0.882392      0.249637      -0.0000598423  0.0000541549
  0.0000541549  -0.0000598423  0.249637      0.882392      0.19147        -0.0000184256
  -0.000056436  0.0000870786  -0.0000598423  0.19147      0.45643        0.0359943
  0.0000363651  -0.000056436  0.0000541549  -0.0000184256  0.0359943    0.0733751 )
```

■ Calculo de la matriz Bik

```
In[452]:= (Bik = Table[Table[e * t^3 / (12 * (1 - v^2)) Sum[ (Integrate[partial_s partial_s W[i, q] * partial_s partial_s W[j, q] ds, {s, 0, 1}]), {q, 1, 5}], {j, 1, 6}], {i, 1, 6}]) // MatrixForm
( 0.0258789  -0.0307471  0.00973645  -0.00973625  0.0121908  -0.00732283
  -0.0307471  0.0370428  -0.0125915  0.0125913  -0.0184862  0.0121908
  0.00973645  -0.0125915  0.00571034  -0.00571034  0.0125913  -0.00973625
  -0.00973625  0.0125913  -0.00571034  0.00571034  -0.0125915  0.00973645
  0.0121908  -0.0184862  0.0125913  -0.0125915  0.0370428  -0.0307471
  -0.00732283  0.0121908  -0.00973625  0.00973645  -0.0307471  0.0258789 )
```

■ Cálculo de la matriz Dik

```
In[419]:= (Dik = Table[Table[
  t^3 / 3 Sum[ (integrate[S[q+1]
    partial_s W[i, q] * partial_s W[j, q] ds)
  + (2*v*e*t^3 / (12*(1-v^2))) / (e / (2*(1+v))) Sum[ (integrate[S[q+1]
    (W[i, q] * partial_s W[j, q] + W[j, q] * partial_s W[i, q]) ds)
  , {j, 1, 6}], {i, 1, 6}]) // MatrixForm
(0.000186797 -0.000183646 0.0000172132 -0.0000249832 0.0000125961 -7.97658*10^-6
-0.000183646 0.000182883 -0.0000174129 0.000025183 -0.0000196031 0.0000125961
0.0000172132 -0.0000174129 1.58025*10^-6 -1.58042*10^-6 0.000025183 -0.0000249832
-0.0000249832 0.000025183 -1.58042*10^-6 1.58025*10^-6 -0.0000174129 0.0000172132
0.0000125961 -0.0000196031 0.000025183 -0.0000174129 0.000182883 -0.000183646
-7.97658*10^-6 0.0000125961 -0.0000249832 0.0000172132 -0.000183646 0.000186797)
```

5. Determinación de la matriz de cambio de base

■ Determinación de los vectores propios del sistema (Bik-λ.Cic)

```
In[420]:= (Vp = Eigenvectors[{Bik, Cik}]) // MatrixForm
(0.666562 -0.212525 0.102609 -0.102609 0.212525 -0.666562
-0.687421 0.163373 -0.0276029 -0.0276029 0.163373 -0.687421
0.604825 0.425653 -0.0931041 0.122583 -0.36498 -0.544152
-0.271715 -0.426273 -0.337623 0.169721 -0.465727 -0.620295
-0.531607 -0.321513 0.621641 -0.139084 -0.410142 -0.200063
-0.0980455 0.143252 -0.0502517 -0.960313 -0.0311421 0.210168)
```

■ Coordenadas sectoriales (calculadas con el programa CUFMS)

```
In[421]:= sect1 = {1, 1, 1, 1, 1, 1};
sect2 = {31.75, 44.85, 44.85, -44.85, -44.85, -31.75} / 10;
sect3 = {43.017, 43.017, -25.783, -25.783, 43.017, 43.017} / 10;
sect4 = {-2884.71, -1528.65, 1557.02, -1557.02, 1528.65, 2884.71} / 100;
```

■ Obtención de la matriz de cambio de base

```
In[425]:= (Vp1 = {Take[Vp, {1}][[1]] / Norm[Take[Vp, {1}]] * 1 / Vp[[1, 1]], Take[Vp, {2}][[1]] /
  Norm[Take[Vp, {2}]] / Vp[[2, 1]], sect1, sect2, sect3, sect4}) // MatrixForm
(1. -0.318838 0.153937 -0.153937 0.318838 -1.
1. -0.237661 0.0401542 0.0401542 -0.237661 1.
1 1 1 1 1 1
3.175 4.485 4.485 -4.485 -4.485 -3.175
4.3017 4.3017 -2.5783 -2.5783 4.3017 4.3017
-28.8471 -15.2865 15.5702 -15.5702 15.2865 28.8471)
```

6. Determinación de las matrices de la T.V.G. diagonalizadas

■ Cálculo de la matriz Cik diagonalizada

```
In[426]:= Vp1.Cik.Transpose[Vp1] // MatrixForm
(0.185823 -5.34613*10^-17 -3.05311*10^-16 0.000649702 -1.74302*10^-15 0.00666356
-5.56331*10^-17 0.160579 -3.08426*10^-7 2.1515*10^-15 -0.0000112263 -1.04608*10^-14
-2.848*10^-16 -3.08426*10^-7 4.23345 -4.44572*10^-15 0.000015866 3.19139*10^-14
0.000649702 2.18488*10^-15 -4.71845*10^-15 62.7535 -1.72419*10^-14 0.036906
-1.78012*10^-15 -0.0000112263 0.000015866 -1.74759*10^-14 28.8296 8.28877*10^-14
0.00666356 -1.07104*10^-14 3.10862*10^-14 0.036906 9.09446*10^-14 523.504)
```

■ Cálculo de algunas propiedades del perfil que se obtiene en la matriz Cik diagonalizada

Cálculo del área del perfil:

$$\text{In[427]}:= \text{Area} = t * \sum_{q=1}^5 b[[q]]$$

Out[427]= 4.23345

Calculo de la inercia a flexion segun el eje debil:

$$\begin{aligned} \text{In[428]}:= \text{Zg} &= \text{Zg} /. \text{Solve}[\text{Zg} * (2 * t * b1 + t * b3 + 2 * t * (b2 - 2 * t)) = 2 * b1 * t * t / 2 + 2 * (b2 - 2 * t) * b2 / 2 * t + \\ & t * b3 * (b2 - t / 2), \text{Zg}][[1]]; \\ \text{IdebilExacta} &= (2 * (b1 * t^3 / 12 + (\text{Zg} - t / 2)^2 * b1 * t) + 2 * (t * (b2 - 2 * t)^3 / 12 + (b2 / 2 - \text{Zg})^2 * \\ & (b2 - 2 * t) * t) + b3 * t^3 / 12 + (b2 - t / 2 - \text{Zg})^2 * b3 * t) * e; \\ \text{Idebilaprox} &= (2 * (b1 * t^3 / 12 + (\text{Zg})^2 * b1 * t) + 2 * (t * (b2)^3 / 12 + (b2 / 2 - \text{Zg})^2 * (b2) * t) + \\ & b3 * t^3 / 12 + (b2 - \text{Zg})^2 * b3 * t) \end{aligned}$$

Out[430]= 28.8296

Calculo de la inercia a flexion segun el eje fuerte:

$$\begin{aligned} \text{In[431]}:= \text{Yg} &= \text{Yg} /. \text{Solve}[\text{Yg} * (2 * t * b1 + t * b3 + 2 * t * (b2 - 2 * t)) = b1 * t * b1 / 2 + t * (b2 - 2 * t) * t / 2 + b3 * t * b3 / 2 + \\ & t * (b2 - 2 * t) * (b3 - t / 2) + b1 * t * (b3 - b1 / 2), \text{Yg}][[1]]; \\ \text{Ifortaprox} &= (t * b3^3 / 12 + 2 * (b2 * t^3 / 12 + b2 * t * (b3 / 2)^2 + t * b1^3 / 12 + b1 * t * (b3 / 2 - b1 / 2)^2)) \end{aligned}$$

Out[432]= 62.7535

■ Calculo de la matriz Bik diagonalizada

$$\text{In[433]}:= \text{Vp1.Bik.Transpose[Vp1]} // \text{MatrixForm}$$

$$\begin{pmatrix} 0.14993 & -5.36375 \times 10^{-17} & -3.22659 \times 10^{-16} & 0.000524206 & -1.50735 \times 10^{-15} & 0.00537643 \\ -5.32191 \times 10^{-17} & 0.0568489 & -1.0919 \times 10^{-7} & 6.96037 \times 10^{-16} & -3.97438 \times 10^{-6} & -3.51347 \times 10^{-15} \\ -3.15195 \times 10^{-16} & -1.0919 \times 10^{-7} & 2.10711 \times 10^{-13} & 6.46961 \times 10^{-16} & 7.63561 \times 10^{-12} & 9.1326 \times 10^{-15} \\ 0.000524206 & 7.07761 \times 10^{-16} & 6.46957 \times 10^{-16} & 1.8328 \times 10^{-6} & 6.39136 \times 10^{-15} & 0.0000187978 \\ -1.47253 \times 10^{-15} & -3.97438 \times 10^{-6} & 7.63561 \times 10^{-12} & 6.39138 \times 10^{-15} & 2.77854 \times 10^{-10} & 1.97399 \times 10^{-14} \\ 0.00537643 & -3.54187 \times 10^{-15} & 9.13289 \times 10^{-15} & 0.0000187978 & 1.97406 \times 10^{-14} & 0.000192797 \end{pmatrix}$$

■ Calculo de la matriz Dik diagonalizada

$$\text{In[434]}:= \text{Vp1.Dik.Transpose[Vp1]} // \text{MatrixForm}$$

$$\begin{pmatrix} 0.000715488 & 3.18828 \times 10^{-17} & -1.46977 \times 10^{-17} & -0.000218153 & -7.10108 \times 10^{-17} & -0.00529587 \\ 3.18384 \times 10^{-17} & 0.000537149 & -6.68808 \times 10^{-11} & -7.10689 \times 10^{-17} & 0.000132325 & 2.28139 \times 10^{-16} \\ -1.47725 \times 10^{-17} & -6.68808 \times 10^{-11} & 5.37324 \times 10^{-17} & -4.86768 \times 10^{-17} & -2.54162 \times 10^{-10} & 2.85445 \times 10^{-16} \\ -0.000218153 & -7.10412 \times 10^{-17} & -4.86739 \times 10^{-17} & -1.53411 \times 10^{-6} & -2.28355 \times 10^{-16} & -0.0000285054 \\ -7.13539 \times 10^{-17} & 0.000132325 & -2.54162 \times 10^{-10} & -2.28389 \times 10^{-16} & -1.85023 \times 10^{-8} & 1.53706 \times 10^{-15} \\ -0.00529587 & 2.2744 \times 10^{-16} & 2.85145 \times 10^{-16} & -0.0000285054 & 1.53542 \times 10^{-15} & 0.0389223 \end{pmatrix}$$

7. Determinación de los desplazamientos nodales

■ Calculo de la matriz $[\bar{v}]$

$$\begin{aligned} \text{In[435]}:= \text{Vdesp1} &= \{\{\text{P1n1}' - \text{P1}\}[[1,1]], \{\text{P2n1}' - \text{P2}\}[[1,1]], \{\text{P3n1}' - \text{P3}\}[[1,1]], \{\text{P4n1}' - \text{P4}\}[[1,1]], \{\text{P5n1}' - \text{P5}\}[[1,1]], \{\text{P6n1}' - \text{P6}\}[[1,1]]\}; \\ \text{Vdesp2} &= \{\{\text{P1n2}' - \text{P1}\}[[1,1]], \{\text{P2n2}' - \text{P2}\}[[1,1]], \{\text{P3n2}' - \text{P3}\}[[1,1]], \{\text{P4n2}' - \text{P4}\}[[1,1]], \{\text{P5n2}' - \text{P5}\}[[1,1]], \{\text{P6n2}' - \text{P6}\}[[1,1]]\}; \\ \text{Vdesp3} &= \{\{\text{P1n3}' - \text{P1}\}[[1,1]], \{\text{P2n3}' - \text{P2}\}[[1,1]], \{\text{P3n3}' - \text{P3}\}[[1,1]], \{\text{P4n3}' - \text{P4}\}[[1,1]], \{\text{P5n3}' - \text{P5}\}[[1,1]], \{\text{P6n3}' - \text{P6}\}[[1,1]]\}; \\ \text{Vdesp4} &= -\text{Reverse}[\text{Vdesp3}]; \\ \text{Vdesp5} &= -\text{Reverse}[\text{Vdesp2}]; \\ \text{Vdesp6} &= -\text{Reverse}[\text{Vdesp1}]; \\ \text{DespX} &= \text{Transpose}[\{\text{Vdesp1}, \text{Vdesp2}, \text{Vdesp3}, \text{Vdesp4}, \text{Vdesp5}, \text{Vdesp6}\}] // \text{MatrixForm} \end{aligned}$$

$$\begin{pmatrix} -0.771349 & 0.755369 & -0.000171947 & -0.000171947 & 0. & 0. \\ -0.763359 & 0.763359 & 0 & 0. & 0. & 0. \\ 0 & 0 & 0.111483 & -0.111483 & 0. & 0. \\ 0. & 0. & 0.111483 & -0.111483 & 0 & 0 \\ 0. & 0. & 0. & 0 & -0.763359 & 0.763359 \\ 0. & 0. & 0.000171947 & 0.000171947 & -0.755369 & 0.771349 \end{pmatrix}$$

■ Cálculo de la matriz $[\bar{w}]$

```
In[442]:=
Wdesp1={ {P1n1'-P1}[[1,2]], {P2n1'-P2}[[1,2]], {P3n1'-P3}[[1,2]], {P4n1'-P4}[[1,2]], {P5n1'-P5}[[1,2]], {P6n1'-P6}[[1,2]]};
Wdesp2={ {P1n2'-P1}[[1,2]], {P2n2'-P2}[[1,2]], {P3n2'-P3}[[1,2]], {P4n2'-P4}[[1,2]], {P5n2'-P5}[[1,2]], {P6n2'-P6}[[1,2]]};
Wdesp3={ {P1n3'-P1}[[1,2]], {P2n3'-P2}[[1,2]], {P3n3'-P3}[[1,2]], {P4n3'-P4}[[1,2]], {P5n3'-P5}[[1,2]], {P6n3'-P6}[[1,2]]};
Wdesp4=Reverse[Wdesp3];
Wdesp5=Reverse[Wdesp2];
Wdesp6=Reverse[Wdesp1];
(DespY=Transpose[{Wdesp1,Wdesp2,Wdesp3,Wdesp4,Wdesp5,Wdesp6}])//MatrixForm
```

$$\begin{pmatrix} -0.144462 & 0.289811 & -0.166573 & 0.0212243 & 0 & 0 \\ 0 & 0.145349 & -0.145349 & 0 & 0 & 0 \\ 0. & 0.145349 & -0.145349 & 0. & 0. & 0. \\ 0. & 0. & 0. & -0.145349 & 0.145349 & 0. \\ 0 & 0 & 0 & -0.145349 & 0.145349 & 0 \\ 0 & 0 & 0.0212243 & -0.166573 & 0.289811 & -0.144462 \end{pmatrix}$$

■ Cálculo de la matriz de vectores propios $[\bar{u}_i]$

```
In[449]:=
(Vp1desp=Transpose[{sect1,-sect2,sect3, sect4,-Take[Vp,{1}][[1]]/Norm[Take[Vp,{1}]]*1/Vp[[1,1]],
-Take[Vp,{2}][[1]]/Norm[Take[Vp,{2}]]/Vp[[2,1]]}])//MatrixForm
```

$$\begin{pmatrix} 1 & -3.175 & 4.3017 & -28.8471 & -1. & -1. \\ 1 & -4.485 & 4.3017 & -15.2865 & 0.318838 & 0.237661 \\ 1 & -4.485 & -2.5783 & 15.5702 & -0.153937 & -0.0401542 \\ 1 & 4.485 & -2.5783 & -15.5702 & 0.153937 & -0.0401542 \\ 1 & 4.485 & 4.3017 & 15.2865 & -0.318838 & 0.237661 \\ 1 & 3.175 & 4.3017 & 28.8471 & 1. & -1. \end{pmatrix}$$

■ Cálculo de la matriz de desplazamiento horizontal nodal para cada modo i $[\bar{v}_i]$

```
In[450]:=
DespX.Vp1desp//MatrixForm
```

$$\begin{pmatrix} -0.0163234 & -0.938798 & -0.0678526 & 10.7042 & 1.01219 & 0.950884 \\ 0. & -1. & 0. & 10.3516 & 1.00675 & 0.94478 \\ -2.35922 \times 10^{-16} & -1. & 6.08292 \times 10^{-16} & 3.47162 & -0.0343227 & 4.82123 \times 10^{-16} \\ 2.35922 \times 10^{-16} & -1. & -6.08292 \times 10^{-16} & 3.47162 & -0.0343227 & 4.63176 \times 10^{-16} \\ 0. & -1. & 0. & 10.3516 & 1.00675 & -0.94478 \\ 0.0163234 & -0.938798 & 0.0678526 & 10.7042 & 1.01219 & -0.950884 \end{pmatrix}$$

■ Cálculo de la matriz de desplazamiento vertical nodal para cada modo i $[\bar{w}_i]$

```
In[451]:=
DespY.Vp1desp//MatrixForm
```

$$\begin{pmatrix} -1.73472 \times 10^{-17} & 0.00113631 & 1. & -3.18692 & 0.265774 & 0.219176 \\ 0. & 0. & 1. & -4.48499 & 0.0687173 & 0.0403802 \\ 0. & 0. & 1. & -4.48499 & 0.0687173 & 0.0403802 \\ 0. & 0. & 1. & 4.48499 & -0.0687173 & 0.0403802 \\ 0. & 0. & 1. & 4.48499 & -0.0687173 & 0.0403802 \\ -1.73472 \times 10^{-17} & -0.00113631 & 1. & 3.18692 & -0.265774 & 0.219176 \end{pmatrix}$$