Master in Photonics

MASTER THESIS WORK

COHERENT CONTROL OF LIGHT PROPAGATION IN INTEGRATED OPTICAL WAVEGUIDES

Ricard Menchón Enrich

Supervised by Dr. Verònica Ahufinger (UAB) and Dr. Andreu Llobera (IMB-CNM)

Presented on date 9th September 2009

Registered at
Coherent control of light propagation in integrated optical waveguides

Ricard Menchón Enrich
Grup d’Òptica, Dept. de Física, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Spán
Departament de Micro i Nanosistemes, Institut de Microelectrònica de Barcelona - Centre Nacional de Microelectrònica, E-08193 Bellaterra, Spain
E-mail: ricard.menchon@uab.cat

Abstract. Coherent control and manipulation of light propagation in systems of three coupled rib optical waveguides is addressed. We show that by an appropriate variation of the distance among waveguides, it is possible to transfer a beam of light between the outermost waveguides with very low excitation of the central one, resembling the Stimulated Raman Adiabatic Passage (STIRAP) technique. Results for total internal reflection (TIR) waveguides and for anti-resonant reflecting optical waveguides (ARROW) are presented. In the case of TIR waveguides, we demonstrate that the system can be designed to be used as a spectral filter and as a refractometer. The possibility to use the system as a delay line by chaining a sequence of two STIRAP processes has been explored.

Keywords: Total internal reflection waveguides (TIR), anti-resonant reflecting optical waveguides (ARROW), Stimulated Raman Adiabatic Passage (STIRAP).

1. Introduction: STIRAP and optical waveguides

Silicon photonics has attracted during recent years a lot of attention because of the possibilities it offers to revolutionize computing platforms by manufacturing optical communication devices using traditional CMOS technology [1, 2]. Using light as the carrier of information, new techniques offering full control of light propagation in optical devices built onto microelectronic chips are fully desirable. In this work, we explore the possibilities of applying an analog of the Stimulated Raman Adiabatic passage (STIRAP) technique [3] to obtain a robust and highly efficient transfer of power between rib optical waveguides. Moreover, new STIRAP-based integrated optical devices are proposed, designed and numerically studied: (i) a spectral filter, (ii) a refractometer and (iii) a delay line device.

STIRAP is a robust technique that is routinely used in Λ three-level atomic systems in interaction with two laser pulses to transfer coherently the atomic population between the two ground states (figure 1(a)). The Hamiltonian of the system, considering resonant pulses, in the interaction picture and under the rotating wave approximation, can be written in the following way [3]:

\[
H = \frac{\hbar}{2} \begin{pmatrix}
0 & \Omega_P & 0 \\
\Omega_P & 0 & \Omega_S \\
0 & \Omega_S & 0
\end{pmatrix}
\] (1)
Coherent control of light propagation in integrated optical waveguides

Figure 1. (a) Λ configuration of three internal atomic levels in interaction with two resonant light pulses with Rabi frequencies $\Omega_P$ and $\Omega_S$. Appropriate time sequence for (b) the couplings $\Omega_P$ and $\Omega_S$ and (c) the mixing angle $\Theta$ to perform the STIRAP process, assuming that all the population is initially in state $|1\rangle$.

where $\Omega_P$ and $\Omega_S$ are the Rabi frequencies of the corresponding pulses ($\Omega = \vec{\mu} \cdot \vec{E} / \hbar$ where $\vec{\mu}$ is the electric dipole moment and $\vec{E}$ is the electric field). The eigenstates of the system are:

$$|+\rangle = \frac{1}{\sqrt{2}} (\sin \Theta |1\rangle + |2\rangle + \cos \Theta |3\rangle)$$

$$|D\rangle = \cos \Theta |1\rangle - \sin \Theta |3\rangle$$

$$|\rangle = \frac{1}{\sqrt{2}} (\sin \Theta |1\rangle - |2\rangle + \cos \Theta |3\rangle)$$

where $\tan \Theta \equiv \Omega_P/\Omega_S$, and $|D\rangle$ is the so-called dark state, which does not involve the excited state $|2\rangle$. STIRAP consists of following this energy eigenstate from $|\psi_{in}\rangle = |1\rangle$ to $|\psi_{out}\rangle = |3\rangle$ by means of two partially overlapping pulses as in the counterintuitive sequence (shown in figure 1(b)) adiabatically changing $\Theta$ from 0 to 90° (figure 1(c)). Thus, using the STIRAP technique all the population can be robustly and efficiently transferred from state $|1\rangle$ to state $|3\rangle$ without populating state $|2\rangle$. Adiabaticity means that $|\dot{\Theta}|$ should be smaller than the energy separation between the selected eigenstate and the one energetically closest. For Gaussian couplings and optimal pulse delays, $T$, this condition can be written as $\sqrt{\Omega_P^2 + \Omega_S^2} T > 10$ [3].

The high efficiency of this coherent technique has motivated its application to other research fields such as atomic transport in optical microtraps [4, 5] or light propagation in coupled waveguides [6, 7]. In this work we will focus on the latter. But before going further with the application of the STIRAP technique to light propagation in coupled optical waveguides, it is necessary to present the basic properties of the waveguides we are using. Two different types of optical waveguides are considered: (i) total internal reflection (TIR) waveguides [8, 9] and (ii) anti-resonant reflecting optical waveguides (ARROW) [11, 12, 13]. In TIR waveguides light is confined and propagates inside the core region, which is surrounded by cladding layers with lower refractive index (figure 2(a)). In ARROW waveguides (type A), light is confined in the lowest refractive-index layer using a Fabry-Pérot structure consisting of two layers (first and second cladding), known as the anti-resonant pair, and placed around the core, as can be seen in figure 2(b). For a specific wavelength (the so-called anti-resonant wavelength) and fixed refractive indices, there exist some values for the width of the first and second cladding layers that generate maximum reflection at the core-first cladding layer boundary, being the light guided through the core. TIR waveguides can support one (single-mode waveguide) or more modes (multimode waveguide) depending on the characteristics of their structure (indices and dimensions) and the wavelength of the input light. ARROW waveguides are known as leaky waveguides, it means that all the modes have considerable losses except the fundamental one. Thus, ARROW waveguides are considered as virtual single-mode regardless the thickness of the core layer. To illustrate the basic operation principle we have focused on planar waveguides (figure 2), but in this project we consider three-dimensional waveguides (figure 3), which can be treated effectively as two dimensional by calculating the so-called effective refractive index [9, 10]. The value of these effective indices is based on the refractive indices and the dimensions of the different layers and takes into account which is the layer where the light is confined. In our case, looking at the $x - y$ plane, we perform different cuts
Figure 2. Planar (a) TIR and (b) ARROW (type A) waveguide configurations. In both cases light travels along the \( z \) direction inside the core and is confined in the \( x \) direction.

Figure 3. Three-dimensional rib (a) TIR and (b) ARROW waveguide (ARROW confinement for the \( x \) direction and TIR confinement for the \( y \) direction) configurations.

separating the structure in parts that contain rib and parts without rib, as shown in figure 3. Then, we can calculate the effective refractive indices for the two different regions knowing that light travels inside the SiO\(_x\) (non-stoichiometric silicon oxide) layer. In this way, the three-dimensional structures shown in figure 3 can be seen as the two dimensional ones in figure 2, which are considered infinite in the \( y \) direction with no change in the shape or position of their transverse cross section. With this approximation, the problem has been reduced from three to two dimensions and all partial derivatives of the field with respect to \( y \), appearing in the Maxwell’s equations, can be set to zero [14]. The six resulting Maxwell’s equations can be divided in two decoupled sets, one containing \( H_x, H_z \) and \( E_y \), which form the so called Transverse Magnetic modes with respect to \( y \) (TM\(_y\)), and the other set formed by \( E_x, E_z \) and \( H_y \), which account for the so called Transverse Electric modes with respect to \( y \) (TE\(_y\)) [14, 15].

If two waveguides are placed one close to the other, light can be transferred from one waveguide to the other along the propagation direction. This is because not all the light is confined inside the core layer. For TIR waveguides, the so-called evanescent fields decay exponentially inside of the cladding layers, while for ARROW waveguides, although losses of the fundamental mode are small, they extend out of the core forming the so-called radiative field. Both evanescent and radiative fields can couple different waveguides of a system, being the directional couplers the simplest example [16, 17].

In the present project, systems of three identical single-mode weakly coupled optical waveguides are studied, where the electric field \( \vec{E} \) can be expressed as:

\[
\vec{E}(x, y, z) = \sum_{k=R,C,L} a_k(z) \vec{e}_k(x, y) \exp(-i\beta z)
\]

(5)

where \( a_L, a_C \) and \( a_R \) are the amplitude functions on the left, central and right waveguides, \( \vec{e}_k(x, y) \) is
Coherent control of light propagation in integrated optical waveguides

the field of the fundamental mode of each waveguide and $\beta$ is the propagation constant. The evolution along the propagation distance of the amplitude function of each waveguide is governed by the coupled-mode equations [9, 10]:

$$
i \frac{d}{dz} \begin{pmatrix} a_R(z) \\ a_C(z) \\ a_L(z) \end{pmatrix} = \begin{pmatrix} 0 & \Omega_R(z) & 0 \\ \Omega_R(z) & 0 & \Omega_L(z) \\ 0 & \Omega_L(z) & 0 \end{pmatrix} \begin{pmatrix} a_R(z) \\ a_C(z) \\ a_L(z) \end{pmatrix}$$

(6)

where $\Omega_L$ and $\Omega_R$ are the coupling coefficients between left and central waveguides and central and right waveguides, respectively. It is assumed that left and right waveguides are not directly coupled. $\Omega = \pi/(2L_c)$, where $L_c$ is the coupling length, defined as the distance required to have a maximum transfer of power between two parallel waveguides. As we will see, for both TIR and ARROW waveguides, the coupling coefficients $\Omega$ can be modulated by modifying the distance between waveguides in $x$ direction along the propagation direction $z$.

By comparing (1) and (6), it is evident that there is a close analogy between systems of three coupled waveguides and three internal atomic level systems coupled with two resonant light pulses. Equations (6) are equivalent to the Schrödinger equation using Hamiltonian (1), but with the $z$ direction playing the role of time, the amplitude functions of each guide as the population of each internal atomic level, and the coupling coefficients between waveguides as the Rabi frequencies of the pulses coupling the atomic levels. Hence, for systems of three waveguides there also exists stationary solutions of the system, eigenstates, corresponding to the modes of the whole system, that are equivalent to those seen in (2), (3) and (4). In particular, there is a dark state for the system of waveguides not involving the central waveguide $D(\Theta) = (a_R, a_C, a_L) = (\cos \Theta, 0, -\sin \Theta)$ with $\tan \Theta = \Omega_R/\Omega_L$. Thus, if initially light is injected into the right waveguide and the couplings are set to give $\Theta = 0^\circ$, the state of the system is $D(0^\circ) = (1, 0, 0)$. Similarly to the case of internal atomic levels, engineering the coupling between waveguides (varying the distance among them) such that $\Theta$ changes smoothly from $0^\circ$ to $90^\circ$ (as shown in figure 1(c), but along $z$ direction instead of time), the system follows adiabatically the dark state $D(\Theta)$ and finishes in $D(90^\circ) = (0, 0, 1)$, in other words, with light only in the left waveguide. Notice that $a_C$ is zero during the whole process meaning that light is not confined in the central waveguide.

STIRAP in Ag-Na diffused waveguides has been recently reported [6].

In this project, we start by optimizing the STIRAP process in systems of three coupled rib waveguides following the lines of Longhi et al. in [6]. In section 2, we present two configurations of rib TIR waveguides, one using curved waveguides and another using straight waveguides. Sections 3 and 4 show the applicability of the system as a spectral filter and as a refractometer, respectively. In section 5, we design a configuration chaining two STIRAP sequences opening possibilities for delay line applications. In section 6 we engineer STIRAP for a system of three rib ARROW waveguides. Finally, in section 7 we present the conclusions of our work.

2. STIRAP in rib TIR waveguides

Our purpose is to optimize the STIRAP technique in a system of three coupled rib TIR waveguides taking into account technological and practical limits. Two requirements have to be fulfilled in order to optimize the process: adiabaticity and a counterintuitive sequence of the couplings, meaning that if light is injected in the right waveguide, the coupling has to be first stronger between left and central waveguides and later on and with a certain overlap for central and right. It can be achieved by changing appropriately the distance between the waveguides. From the adiabaticity condition for STIRAP between internal atomic levels [3], one can infer that there are two significant aspects to consider: the bigger the magnitude of the couplings and the longer the overlap between the couplings the more adiabatic is the process. Note that the overlap length between couplings can be modified either by changing the length of the couplings or changing the delay (now in space) between them. The exact optimization is done using numerical simulations.

Regarding the magnitude of the coupling, we start by fixing the $x - y$ transverse section of our identical single-mode waveguides. Since TIR waveguides are coupled via evanescent field, which decays
Figure 4. Simulations of STIRAP in TIR waveguides working at 633 nm of wavelength. (a) Top view of the curved structure. The minimum distance between waveguides is $a_0 = 8 \mu m$, the radius of the curved waveguides is $3.5 \, m$ and the z distance between the center of the curved waveguides is $\delta = 0.47 \, cm$. (b) Top view of the straight structure. Normalized power integrated over the width of the rib of each waveguide along the z propagation distance (c) for the curved structure and (d) for the straight structure.

exponentially moving away transversally from the waveguide, we have to work with weakly confined waveguides to have a long tail of evanescent field. Then, we surround the core of the waveguide by cladding layers of very similar refractive index material. Moreover, the height of the rib is chosen to be small enough within the technological limits to increase the magnitude of the coupling. For the chosen dimensions and details about the structure see figure 3(a).

Calculating the effective refractive indices for the rib and the lateral zones we can compute two-dimensional simulations of light propagation through the system of three coupled waveguides. Two designs have been studied. The first one consists of two curved lateral waveguides and one straight central waveguide. The lateral waveguides are placed as close as possible to the central one, although avoiding coupling between the outermost waveguides, in order to maximize the coupling to ensure adiabaticity. Furthermore, the radius of the curved waveguides is chosen to be large enough to have a big overlap between the couplings. Finally, numerical simulations are used to choose the optimal longitudinal distance $\delta$ between the centers of the curved waveguides. The second design consists of three straight waveguides, the central one tilted a certain angle. To fulfill adiabaticity, as for the curved structure, the lateral waveguides are placed as close as possible to the central one and the length of the system is set to be large enough to have long enough couplings.

Details of the final optimized structures are shown in figure 4. We can observe that in both cases the STIRAP technique is well implemented, obtaining complete transfer of light from the right to the left waveguide with almost no intensity in the central waveguide. This behaviour remains even if the parameters are not very close from the optimal ones confirming the robustness of the technique, an important feature that usual directional couplers do not exhibit. As expected, the structure formed by curved waveguides presents more losses due to the propagation itself (total internal reflection is not exactly fulfilled when light propagates through the curved waveguides). On the other hand, curved structures offer more degrees of freedom in the design in order to improve the efficiency of the process.
3. STIRAP in TIR waveguides as a spectrum filter

In this section we demonstrate the use of the system of three coupled waveguides described above as a spectral filter. The idea is based on the fact that the coupling length $L_c$ varies inversely with the wavelength of the excitation light [9]. Therefore, the coupling coefficient is proportional to the wavelength, favoring the fulfillment of the adiabaticity condition for large wavelengths. Then, we expect that, for low wavelengths, the STIRAP process do not satisfy the adiabaticity condition and light transfer is not complete, whereas for high wavelengths, the adiabaticity condition is fulfilled and the STIRAP sequence is correctly performed and almost all the light ends up in the left waveguide. Hence, it is possible to design a frequency filter based on the properties of the STIRAP technique.

The profile of the waveguides chosen as well as the configuration of the system of waveguides is the same that the one used in the previous section for the curved case at 633 nm, although the total length has been increased in the simulation in order to observe longer evolutions of the field along the $z$ direction. Note that the effective refractive indices smoothly depend on the wavelength.

Simulations are presented in figure 5. We can distinguish a region where the STIRAP works properly (light injected in the right waveguide at $z = 0$ is transferred to the left waveguide at the end of the process ($z$ large) and there is no light in the other waveguides), approximately from 550 nm to 1000 nm. There exists also a region where the technique does not efficiently work, from 200 nm to 550 nm, proving that the system acts as a high-pass filter measuring the output of the left waveguide. For higher wavelengths (900 − 1000 nm) the STIRAP efficiency decreases due to direct coupling between the lateral waveguides. From the results we can see that the device can also work as a low-pass filter for wavelengths between 200 and 300 nm measuring the output of the central waveguide. This behaviour corresponds to a simple coupling between the right and the central waveguide, when the STIRAP process does not work properly. We realize that this system could work as a demultiplexer: if we inject two different $\lambda$ in the right waveguide, one between 200 and 300 nm and another between 550 nm and 1000 nm, each one would be ejected through a different waveguide. Finally, note that losses are clearly observed again during the propagation of light along the curved waveguides.
Coherent control of light propagation in integrated optical waveguides

4. STIRAP in TIR waveguides as a refractometer

As discussed in section 2, the magnitude of the coupling between TIR waveguides strongly depends on the index of refraction difference between the core and the cladding. As the STIRAP required adiabaticity condition is related to the coupling, measuring the output on the left waveguide after the STIRAP sequence, it is possible to estimate the refractive index of the material surrounding the core, that is, the system acts as a refractometer.

The design of the TIR waveguide $x - y$ transverse section has been changed with respect to the previous cases discussed in sections 2 and 3 (figure 3). Here, the SiO$_2$ layers that surround the core are etched so as a liquid of unknown refractive index can fill the emptied space. The refractive index of the core is kept to 1.48. Regarding the configuration of the waveguide system, we also consider here the curved waveguide system but the minimum distance between waveguides and the distance $\delta$ have been modified (figure 6(d),(e)).

Simulations varying the refractive index of both above and below layers from 1 to 1.4 (because is the range of refractive indices of most liquids) have been done at $\lambda = 633$ nm. From figure 6 it can be seen that the efficiency of the STIRAP process grows gradually, relating directly the output of the left waveguide with the refractive index of the cladding layers. Therefore, calibrating properly an intensity measuring tool placed at the left waveguide, it is possible to know the refractive index of the liquid filling the cladding. Note that the output at large $z$ decreases due to losses on propagation inside curved waveguides once the STIRAP process has been performed.

5. Double STIRAP in TIR waveguides

In this section we study the implementation of a double STIRAP process, which means the chaining of two single STIRAP ones in three TIR waveguides. In this way, applying twice the STIRAP technique,
Figure 7. Top view simulations of double STIRAP sequences in the three rib TIR waveguide system. (a) Curved structure with the left waveguide with Gaussian shape centered at $z = 1500 \mu m$ and with a width of $4100 \mu m$ and a circular right waveguide with radius of 16 m and $a_0 = 7 \mu m$. (b) Straight structure.

Light is transmitted to the outermost waveguide and comes back to the initial one without almost any excitation of the central waveguide. Since light ends up inside the initial waveguide but does not have followed a straight line during the process, we envisage the use of such system as a delay line [18, 19] device by chaining several double STIRAP processes.

The waveguide $x - y$ transverse section chosen is shown in figure 3(a). As in the case of single STIRAP we study two configurations: a system formed by curved waveguides and another formed by straight waveguides. The straight case consists exactly on the chaining of two of the single straight STIRAP systems viewed in section 2. The curved double STIRAP is made up of a Gaussian-shaped left waveguide, a straight central waveguide and a right circular waveguide.

Results are shown in figure 7. Although for both structures a high transfer of power is achieved, for the curved case, there is more intensity in the central waveguide compared to the straight case. It could be improved making the system longer, but technological limitations prevent us to implement devices longer than 3 cm.

6. STIRAP in ARROW waveguides

As discussed in section 1, light confinement in ARROW waveguides is based on the presence of anti-resonant pairs forming a Fabry-Pérot type structures [11, 12, 13] and coupling between waveguides is due to the radiative field. Contrarily to the TIR waveguides where the coupling length $L_c$ monotonically increases with the distance between waveguides, in ARROW waveguides $L_c$ oscillates as we move away from the core allowing remote coupling between waveguides at distances much larger than it can be achieved by usual TIR waveguides [20, 21].

For our study, since we are only interested in the lateral ARROW confinement (in $x$ direction), the vertical confinement (in $y$ direction) is still based on TIR (figure 3(b)). In order to satisfy the anti-resonant condition it is important to choose the optimal width $d_1$ of the lateral anti-resonant structures (LAS), which is given by [11, 12, 13]:

$$d_1 = \frac{(2g + 1)\lambda}{4n_{eff1}} \left[ 1 - \left( \frac{n_{effc}}{n_{eff1}} \right)^2 + \left( \frac{m + 1}{2n_{eff1}d_c} \right)^2 \right]^{-1/2}$$ (7)

where $n_{eff1}$ and $n_{effc}$ are the effective refractive indices of the first cladding LAS structure and the core, respectively. Note that $n_{eff1}$ depends on the high $h$ of the LAS structure. We choose the first anti-resonant condition $g = 0$ and the fundamental mode $m = 0$. In this situation, applying (7) to the first and second cladding layers, in the latter, $n_{eff1}$ should be replaced by $n_{eff2}$, $n_{effc}$ by $n_{eff1}$ and $d_c$ by $d_1$, it can be seen that the width of the second cladding layer, since it has the same value for the refractive index as the core, is given by $d_2 = d_c/2$. 
Coherent control of light propagation in integrated optical waveguides

Figure 8. (a) Coupling length, $L_c$, as a function of the separation between two ARROW waveguides $d_0$. Top view simulations of STIRAP in a system straight optical ARROW waveguides at $\lambda = 633$ nm using (b) one anti-resonant pair and (d) two anti-resonant pairs. (c) Definition of the distance $d_0$ between two waveguides.

Once the $x$–$y$ transverse section is defined and the effective refractive indices of the core and the LAS regions are calculated, two-dimensional simulations for the propagation of light in two parallel ARROW waveguides with one anti-resonant pair are implemented to characterize the dependence of the coupling length $L_c$ as a function of the separation between waveguides $d_0$ (figure 8(a) and (c)). From the results shown in figure 8(a), which show the oscillatory behaviour for remote coupling, we engineer the system of three waveguides to implement the STIRAP sequence. In particular, we chose $d_0$ to vary from $d_{0\text{max}} = 12.5$ $\mu$m and $d_{0\text{min}} = 18.7$ $\mu$m, corresponding to maximum and minimum coupling, respectively (figure 8(a)).

After optimization of the length of the system in order to fulfill the adiabaticity condition, simulations show that it is possible to implement successfully the STIRAP technique in ARROW waveguides (figure 8(b)). Note that waveguides are surrounded by absorbent silicon to reduce undesired reflections. Second anti-resonant pairs can be added to the system providing better confinement and, then, reducing the magnitude of the coupling [20]. This second anti-resonant pairs are added to the previous ARROW waveguide system (i) in the external part of the lateral waveguides to decrease the losses towards the absorbing surrounding silicon and (ii) in the initial part of the right and the final part of the left waveguides to increase the difference of couplings favoring the STIRAP process. Final power in the left waveguide is increased using the second anti-resonant pairs (figure 8(d)).

7. Conclusions

In the present project we have shown that it is possible to coherently control and manipulate light propagation in systems of three coupled rib optical waveguides using an analogous of the STIRAP
Coherent control of light propagation in integrated optical waveguides

Following the work done by Longhi et al. [6] we demonstrated the feasibility of performing STIRAP in systems of rib TIR waveguides coupled by evanescent fields. Furthermore, we have shown that the system exhibits a wealth of new promising applications as simultaneous low-pass high-pass all-photonic filter or a refractometer. Acting as a filter, we have shown that high-pass or low-pass filtering is achieved depending on the waveguide used for the measurement. Whereas as a refractometer, we have illustrated that indices varying from 1 to 1.4 can be measured. Double-STIRAP configurations have also been designed opening possibilities for application of the system as a delay line. Finally, we have extended the STIRAP technique to rib optical ARROW waveguides, which allow remote coupling due to radiation losses.

All the parameters in the simulations have been chosen taking into account the existing technological limitations since our short term purpose is to fabricate and characterize the studied devices at the Institut de Microelecrònica de Barcelona - Centre Nacional de Microelecrònica.

Acknowledgments

I am indebted to my advisors Dr. Verónica Ahufinger and Dr. Andreu Llobera for their help and encouragement throughout the course of this work. I would also like to thank Dr. Jordi Mompart and Albert Benseny for fruitful discussions.

References

[15] Simulations of the effective refractive indices and modes of the waveguides are done with the software FIMMWAVE (Photon Design Ltd. UK) (using EIM and FEM methods). Two-dimensional simulations of light propagation are implemented by the software FIMMPROP (Photon Design Ltd. UK), using TE polarized light (effective index solver (complex)).