4. STRENGTH AGAINST TRANSVERSE LOADING

4.1. Introduction

The basis of calculus for metal sheet in a shear field (class I and II) is presented. The interaction between the transverse loading, and bending mode, is independent except for the fact that and additional support force will have to be added. (According to part 3 of DIN 18807 [19]). This force will be important in the case that the metal sheet lays on the rafters and the span is moderately high. So it will have no importance in our case.

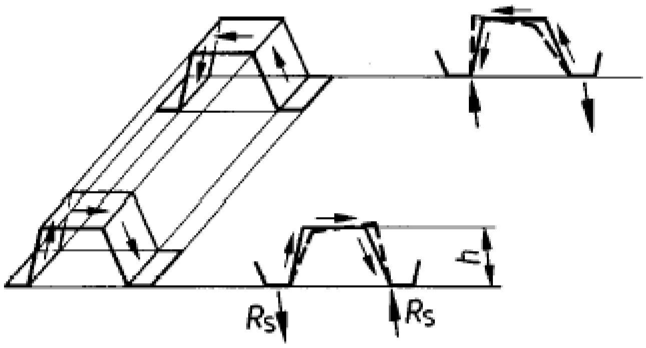


Fig 3.1: Rib in a shear field, additional support reaction. Extracted from [20]

The calculation of the strength of a metal sheet placed in a shear field has two different theoretical approaches:

On one side we have the theory proposed by Schardt/Strehl, which, with some modifications, has been used in the German norm DIN 18807*[15][19][20]*. Which are mainly focused on metal sheets laid on long spans, fastened directly to the rafters on four sides, and with a specific fastening pattern. On the other side we have the theory of Bryan/Davies, used on the British and Scandinavian countries, and which was taken into account in the formulation of the recommendation by the ECCS *[21]*. That is presumingly the theory used for the calculations in the USA and Australia.

Though it may seem a "new" technology, the first uses in the USA of the stressed skin dates from the 1960. From that point until the mid seventies, the calculations where performed based on experimental approvals. The first publications in the USA *[21][22][23][24][26][25]* are from 1975-1980, where the basis of the calculation on the shear cell was developed, and it is still on date. From there on and for the time being the investigation is focused in the capacity of the stressed skin to be effective in case of earthquake.

4.2. Theory Bryan\Davies

4.2.1. Introduction

The theory of Bryan/Davies was published in the same period of time as the one from Schardt/Stehl (in *1972 Bryan publishes: the stressed skin design of steel buildings [27]* and in 1983 the "manual of stressed skin design"[28]

This theory is based in calculations in the elements called "shear cells"(fig. 3.4) which will be explained later.

A) The flexibility of the shear cell depends on three different elements:

The first element is the deformation of the metal sheet itself by two different processes:

$c_{1.1}$ = Profile distortion
$c_{1.2}$ =Shear strain

The second process is the deformation of the fasteners, which force a relative movement of the metal sheet respect to the purlins.

$c_{2.1}$ = deformation of the sheet to purlin fastener in the edge
$c_{2.2}$ = deformation of the seam fasteners
$c_{2.3}$ = deformation in the connection of the sheets with the rafters

And the third process is the deformation of the purlins, evaluated in

c_3 = axial strain in purlins

Adding all this components we will be able to determine a flexibility c of the "shear cell"

B) On a second step all the modes of failure are evaluated, those that are considered ductile, where the sheet tears along a line parallel to the corrugations and those that are considered "not ductile", where this phenomenon is not produced. According to the ECCS [21], this failure mode should be considered a with a 25% higher security reserve. Here there are all listed:

Ductile modus:

- Sheet tearing along a line of seam fasteners
- Sheet tearing along a line of shear connector fasteners

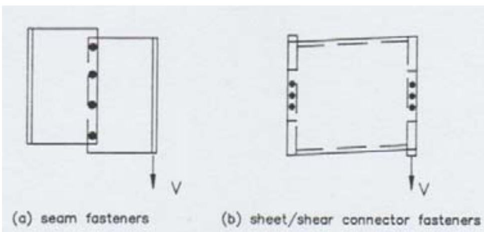


Fig 3.2: ductile failure modus extracted from [21]

Non-ductile modus
- Sheet tearing in the sheet/purlin fasteners
- End collapse of the sheeting profile
- Shear buckling of the sheeting
- Failure of the edge member in tension or compression

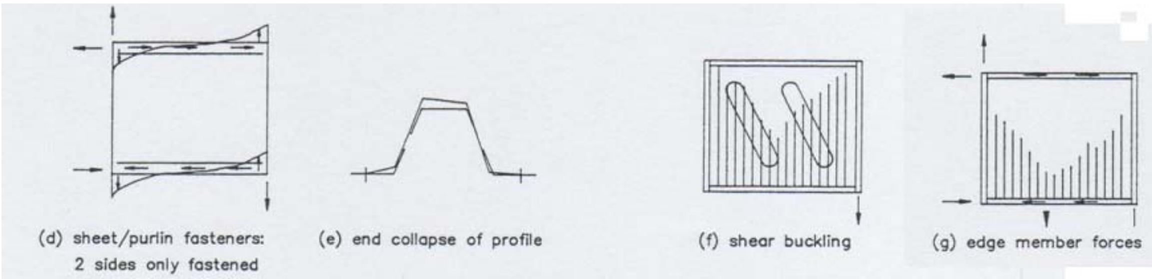


Fig 3.3: Non-ductile failure modus extracted from [21]

To calculate the ultimate state, first the maximum strength of the fastening point and metal sheet is calculated. From there on, the maximum shear load to which the "shear cell" can be loaded is calculated. This method is opposite to the Schardt/Strehl method, where what it is calculated first is the distribution of efforts on the fasteners, and then tested if they fail or not. This procedure allows calculating the distribution at the end of the process and not a priori of the checking.

4.2.2. Basis and theoretical background

The basis for the calculation is the "shear cell element". See fig 3.4, this cell has three basic elements in our case:
- A pinned rectangle formed by two parallel rafters and two parallel edge purlins (laid perpendicular to the previous ones).
- Purlins laid parallel to the edge purlins,
- A metal sheet fastened to the purlins.

An example to this can be found in the figure below.

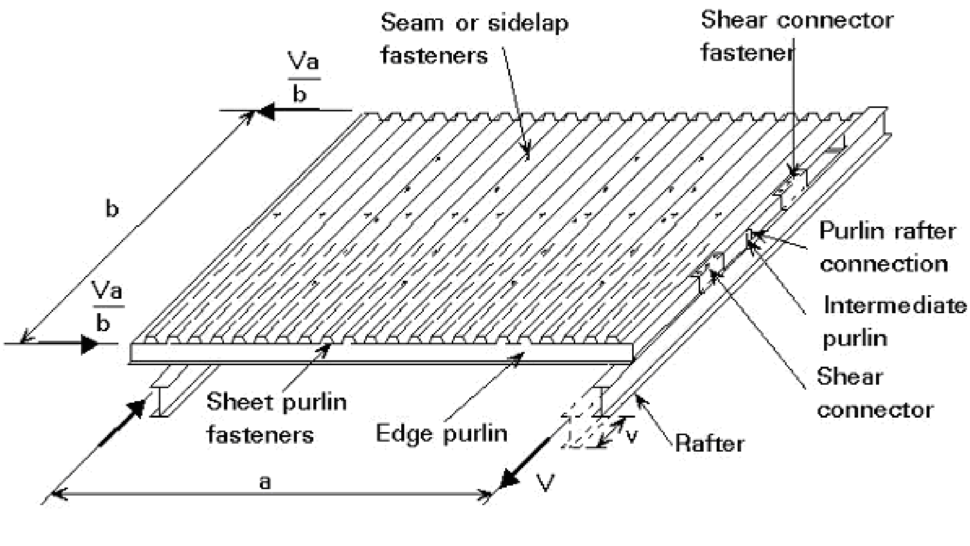


Fig 3.4: Typical shear panels. Extracted from [13]

We can model the figure with the following schema. In this schema it is assumed that the total shear force is equally distributed in each one of the panels:

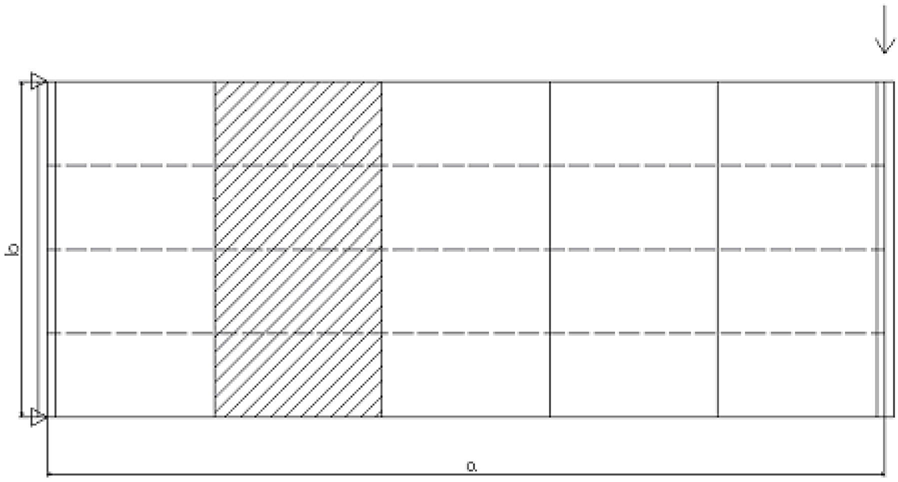


Fig 3.5. Schema of a shear panel (two rafters on the end), purlins laid "horizontal" and the different metal sheets laid "vertically". Example of a sub-panel is lined

The shear force (S) in each one of the sub-panels is going to be:

$$S = V \cdot \frac{a}{b} \cdot \frac{1}{n_{sh}} \quad (3.1)$$

S = shear force in the panel
a,b= distances measured in the fig.1
n_{sh}= number of sheets in each "shear cell"

The e different parts will be analysed the following chapters:

4.2.2.1. Edge purlins

4.2.2.1.1. Flexibility

It is assumed at a first step, that all the horizontal force is distributed exclusively on the two external purlins. The total force shared in the fasteners is:

$$F_{HTotal} = V \cdot \frac{a}{b} \cdot \frac{1}{n_{sh}} \quad (3.2)$$

This force is shared between all the internal fasteners equally and just a half to the two external fasteners (as they are shared with the other panel). Then the "horizontal" force in each individual fastener is:

$$F_{Hi} = V \cdot \frac{a}{b} \cdot \frac{1}{n_{sh}} \cdot \frac{1}{(n_f - 1)} \quad (3.3)$$

n_f: the number of sheet/purlin fasteners in each one of the sub-panels.

The deformation of the fastener is assumed to behave linearly with increasing force. The slip s_p can be defined as the flexibility of the fastener sheet/purlin. The total horizontal displacement of each one of the fasteners in the external purlin is then:

$$\Delta_{Hi} = V \cdot \frac{a}{b} \cdot \frac{1}{n_{sh}} \cdot \frac{1}{(n_f - 1)} \cdot s_p \quad (3.4)$$

The vertical contribution on each one of the sub-panels is then going to be:

$$\Delta_{Vsub-panel} = \frac{a/2n_{sh}}{b} \cdot \Delta_{Hi} \quad (3.5)$$

And added for the whole shear panel is then:

$$c_{2.1} = \Delta_{Vsub-panel} \cdot \frac{n_{sh}}{V} = \frac{2ap}{b^2} \cdot s_p \quad (3.6)$$

Defined p (pitch fasteners in the purlins as):

$$p = \frac{a}{(n_f - 1) \cdot n_{sh}} \quad (3.7)$$

It is assumed in this formulation, that the normal forces are just transmitted by the two more external purlins. This assumption is of course not true, because the internal purlins also transmit part of the normal effort. A linear distribution of forces is assumed in the fasteners of the internal purlins. A parameter α_3 is in this case added to model it [*see annex I, table 3 for values*]

$$c_{2.1} = \frac{2ap}{b^2} \cdot s_p \cdot \alpha_3 \quad (3.8)$$

4.2.2.1.2. Strength

To check the resistance of the sheet/purlins fasteners, the ECCS [21] considers that this is a non ductile modus, then the maximum shear force is the one that breaks the fastener with a 25% of reserve, plus the reserve for the combined shear and prying action by the sheeting, the fastener shear strength is reduced a 40% then:

$$0.6 \cdot F_p = F_{H_i} = V_{ult} \cdot \frac{p}{b} \quad (3.9)$$

$$V_{ult} \leq \frac{0.6 \cdot F_p \cdot b}{p \cdot \alpha_3} \quad (3.10)$$

4.2.2.2. Seam line

4.2.2.2.1. Flexibility

A seam line (line that goes parallel to the corrugations) can be found in each one of the connections between panels. There are two different kinds of fastener in this seam: The fasteners that fasten two sheets together, the seam fasteners, which have a flexibility s_s and a ultimate resistance F_s; and the ones fastened to the purlins (intermediate and external), which have a flexibility s_p and a ultimate resistance of F_p

To be able to calculate the slip of the seam line a distribution of internal forces in the sub-panel is assumed (which has been confirmed to be close enough to the reality) .The following schema gives the distribution in an internal sub-panel:

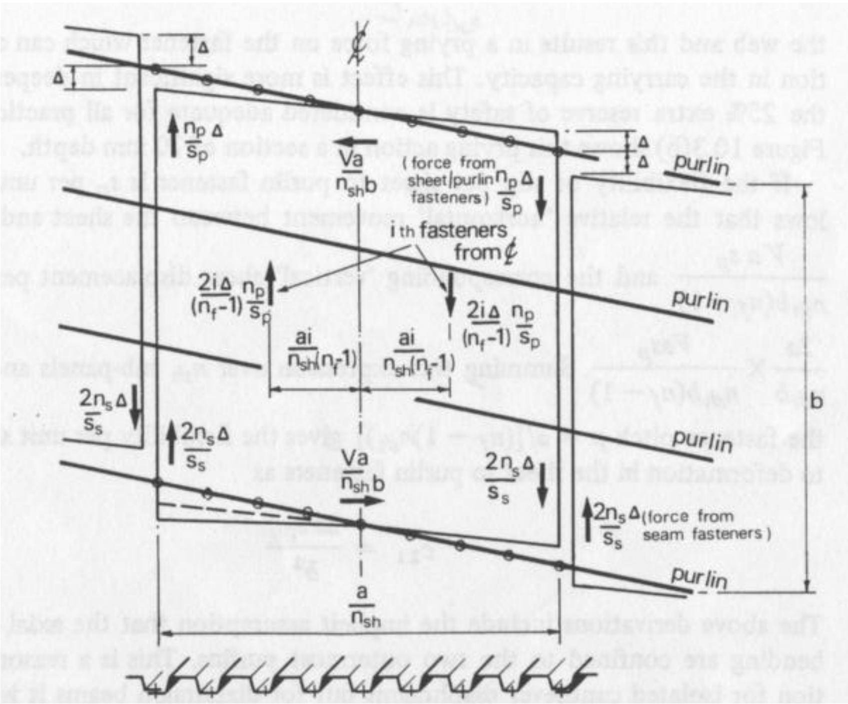


Fig 3.6. Distribution of forces in a panel fastened on the trough. Extracted from [28]

The forces in each one of the different fasteners is calculated as:

<u>*a) Horizontal force in external sheet/purlin fasteners*</u>: As mentioned in the previous point. The total force in the line of fasteners is equal to:

$$F_{HTotal} = V \cdot \frac{a}{b} \cdot \frac{1}{n_{sh}} \quad (3.11)$$

<u>*b) Vertical force in the seam fasteners*</u>: It is assumed that the metal sheet (according to figure 3.6) has a relative displacement to the other metal sheet of 2Δ; that will be the vertical displacement of each one of the fasteners, as the flexibility is known, the force in each one of the fasteners can be calculated as:

$$F_{vs} = \frac{2\Delta}{s_s} \quad (3.12)$$

When propagated for all the n_s seam fasteners in the seam line; the total force on a seam line is:

$$F_{totS} = n_s F_{vs} = \frac{2 n_s \Delta}{s_s} \quad (3.13)$$

<u>*c) Vertical force in the sheet/purlin fasteners*</u>: a linear deformation of the internal fasteners respect to the purlin is assumed. Being the deformation of the most external one equal to Δ. There are two cases depending if the fasteners are an odd or an even number (different distribution of the linear deformation) The force in each *i* fastener will be:

$$F_{ip} = \frac{i}{\frac{n_f - 1}{2}} \frac{\Delta}{s_p} (even) \quad (3.14)$$

$$F_{ip} = \frac{i}{\frac{n_f}{2}} \frac{\Delta}{s_p} (odd) \quad (3.15)$$

And the distance of each one of these fasteners to the central line (symmetry line) is:

$$d_{ip} = \frac{a / 2n_{sh}}{\frac{n_f - 1}{2}} i = \frac{a \cdot i}{n_{sh}(n_f - 1)} (odd) \quad (3.16)$$

$$d_{ip} = \frac{a / 2n_{sh}}{\frac{n_f}{2}} i = \frac{a \cdot i}{n_{sh} n_f} (even) \quad (3.17)$$

If equilibrium of moments in the lowest point of the sub-panel in the symmetry line (point B)is done, thenthe following expression is obtained (*just the odd case will be explained, is the same procedure for the even*):

<u>*a) Moment due to horizontal forces in the sheet/purlin fasteners*</u>

$$M_{hsp} = -\frac{V \cdot a}{n_{sh} \cdot b} \cdot b = -\frac{Va}{n_{sh}} \quad (3.19)$$

<u>*b) Moment due to the seam fasteners*</u>

$$M_{vs} = 2 \cdot \frac{2 \cdot n_s \cdot \Delta}{s_s} \cdot \frac{a}{2 \cdot n_{sh}} \quad (3.20)$$

<u>*c) Moment due to the internal purlin-sheet fasteners (odd)*</u>

$$M_{isp} = 2 \cdot \frac{2 \cdot i \cdot \Delta}{s_p \cdot (n_f - 1)} \cdot n_p \cdot \sum_{i=1}^{\frac{n_f - 1}{2}} \frac{a \cdot i}{n_{sh} \cdot (n_f - 1)} = \beta_1 \cdot \frac{\Delta \cdot n_p \cdot a}{s_p \cdot n_{sh}} \quad (3.21)$$

$$with\ \beta_1 = \sum_{i=1}^{\frac{n_f - 1}{2}} \left(\frac{2i}{(n_f - 1)} \right)^2 \quad (3.22)$$

The moment equilibrium will be then:

$$\frac{V \cdot a}{n_{sh}} = 2 \cdot \frac{n_s \cdot \Delta}{s_s} \cdot \frac{a}{n_{sh}} + \beta_1 \cdot \frac{n_p \cdot \Delta}{s_p} \cdot \frac{a}{n_{sh}} \quad (3.23)$$

$$\frac{V}{\Delta} = 2 \cdot \frac{n_s}{s_s} + \beta_1 \cdot \frac{n_p}{s_p} \quad (3.24)$$

Adding 2Δ per unit load for each one of the internal seams:

$$c_{2.2} = (n_{sh} - 1) \cdot \frac{2 \cdot \Delta}{V} = 2 \cdot (n_{sh} - 1) \cdot \left(2 \cdot \frac{n_s}{s_s} + \beta_1 \cdot \frac{n_p}{s_p} \right)^{-1} = \frac{2 \cdot s_s \cdot s_p \cdot (n_{sh} - 1)}{2 \cdot n_s \cdot s_p + \beta_1 \cdot n_p \cdot s_s} \quad (3.25)$$

$$c_{2.2} = \frac{2 \cdot s_s \cdot s_p \cdot (n_{sh} - 1)}{2 \cdot n_s \cdot s_p + \beta_1 \cdot n_p \cdot s_s} \quad (3.26)$$

In the case of even number of fasteners then $\beta_1 = \sum_{i=1}^{\frac{n_f - 1}{2}} \left(\frac{2i - 1}{(n_f - 1)} \right)^2$ (3.27)

4.2.2.2.2. <u>Ultimate strength</u>

To calculate the Ultimate strength, the same procedure, as in the calculation of the deformation, is going to be developed. It is assumed in this case that the seam fasteners yield with the design strength (F_s); and failure happens when the sheet to purlin fasteners in the more external line of fasteners reach the design strength (F_p).

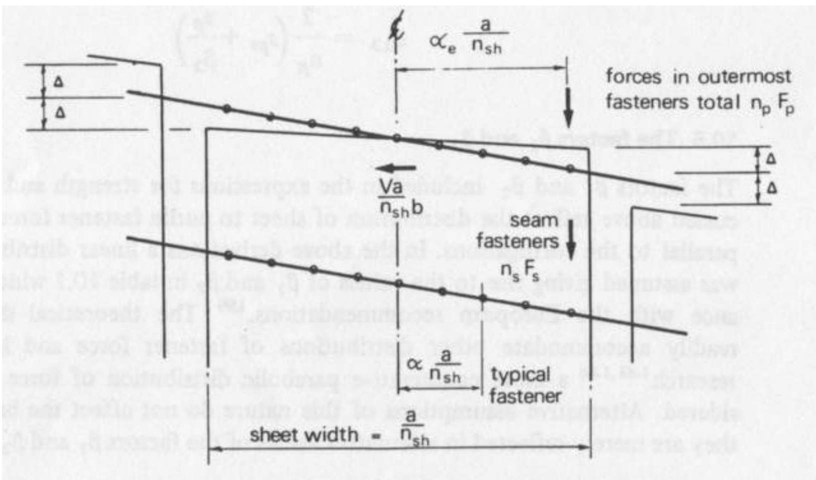


Fig 3.7. Distribution of forces in a panel in ULS. Extracted from [28]

a) Moment due to the horizontal force in the external purlin

$$M_{hsp} = -\frac{V \cdot a}{n_{sh} \cdot b} \cdot b = -\frac{Va}{n_{sh}} \quad (3.28)$$

b) Moment due to the seam fasteners: The force is already knows in this case (it is the design one). The moment is equal to:

$$M_{vs} = 2 \cdot n_s \cdot F_s \cdot \frac{a}{2 \cdot n_{sh}} = \frac{a \cdot n_s \cdot F_s}{n_{sh}} \quad (3.29)$$

c) Moment due to the sheet/purlin fasteners in the internal purlins

$$M_{isp} = \frac{2 \cdot \alpha \cdot \Delta}{s_p} \cdot n_p \cdot \sum \alpha \cdot \frac{a}{n_{sh}} = \frac{2 \cdot a \cdot \Delta \cdot n_p}{s_p \cdot n_{sh}} \cdot \sum \alpha^2 \quad (3.30)$$

as $\beta_1 = 2\sum \alpha^2$ (in even and odd cases) then

$$M_{isp} = \frac{a \cdot \Delta \cdot n_p}{s_p \cdot n_{sh}} \cdot \beta_1 \quad (3.31)$$

The moment equilibrium will be in this case:

$$\frac{Va}{n_{sh}} = \frac{a \cdot n_s \cdot F_s}{n_{sh}} + \frac{2 \cdot a \cdot \Delta \cdot n_p}{s_p \cdot n_{sh}} \cdot \beta_1 \quad (3.32)$$

$$\Delta = \frac{V - n_s \cdot F_s}{\beta_1 \left(n_p / s_p \right)} \quad (3.33)$$

The total force will be when the external line of sheet to purlin fasteners is equal to the design value that is that:

$$\frac{2 \cdot \alpha_e \cdot \Delta}{s_p} \cdot n_p = n_p \cdot F_p \quad (3.34)$$

$$\frac{2 \cdot \alpha_e \cdot \dfrac{V_{ult} - n_s \cdot F_s}{\beta_1 \left(n_p / s_p \right)}}{s_p} \cdot n_p = n_p \cdot F_p \quad (3.35)$$

$$V_{ult} = n_s \cdot F_s + \frac{\beta_1}{\beta_3} \cdot n_p \cdot F_p \quad (3.36)$$

where $\beta_3 = 2 \cdot \alpha_e$

β_3 is defined as the distance to the more external fastener divided by the sheet width.

$$V_{ult} = n_s \cdot F_s + \frac{\beta_1}{\beta_3} \cdot n_p \cdot F_p \quad (3.37)$$

4.2.2.3. End sub-panels

In the end sub-panels two different approaches are assumed depending if there is a diaphragm with direct shear transfer (either fastened to the rafters directly or with the help of shear transfer elements) or with indirect shear transfer (where we will have to make a similar assumption as in the previous case will be done.

4.2.2.3.1. Direct transfer

The ultimate strength of the end sub-panels will be the strength either of the sheet/shear connector's fasteners or of the fasteners fastened directly to the rafters.

The ultimate shear force will be then:

$$V = n_{sc} \cdot F_{sc} \quad (3.38)$$

F_{sc} = Strength of the sheet/shear connectors' fasteners or sheet/rafter fasteners
n_{sh}= number of sheet/shear connector' fasteners or sheet/rafter fasteners

The deformation is then the flexibility of each one of the fasteners divided through the number of elements, and multiplied per two, as there are two rafters in each "shear cell" The flexibility can be defined as:

$$c_{2.3} = 2\frac{s_{sc}}{n_{sc}} \quad (3.39)$$

It is considered that the stiffness at the shear transfer elements is negligible compared to the one in the fasteners

4.2.2.3.2. Indirect shear transfer

4.2.2.3.2.1. Ultimate Strength

In the end sub panels the symmetric deformation of 2Δ on both sides of the metal sheet can not be assumed anymore, as it was done in the internal panels. A deformation of $\Delta+\Delta_e$ on the side fastened to the rafter will be assumed in this case. The shear effort is transmitted to the rafter through the connections first sheet-purlin and then purlin-rafter.

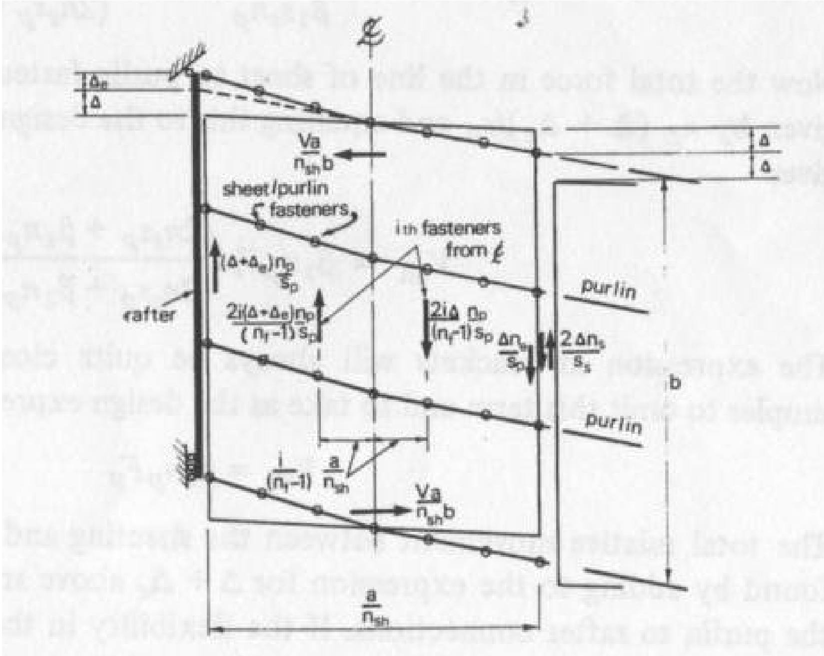


Fig 3.8. Distribution of forces in an end panel in ULS. Extracted from [28]

The demanded equilibrium will be based on vertical forces and displacements, because the equilibrium of momentum can not be satisfied (will be slightly imbalanced); but it proves to be of no consequence compared to the results of tests and the numerical method's approximations.

<u>Balance of "vertical forces"</u>

$$\Delta \cdot \frac{n_p}{s_p} + \sum_{i=1}^{\frac{n_f-1}{2}} \frac{2i\Delta}{(n_f-1)} \cdot \frac{n_p}{s_p} = \sum_{i=1}^{\frac{n_f-1}{2}} \frac{2i(\Delta+\Delta_e)}{(n_f-1)} \cdot \frac{n_p}{s_p} \quad (3.40)$$

$$\text{if we define } \beta_2 = \sum_{i=1}^{\frac{n_f-1}{2}} \frac{2i}{(n_f-1)} (odd) \quad \text{then :}$$

$$\Delta_e = \frac{2 \cdot n_s \cdot s_p}{\beta_2 \cdot s_s \cdot n_p} \Delta \quad (3.41)$$

The total slip between sheet and the end of the purlin is:

$$\Delta_e + \Delta = \left(\frac{2 \cdot n_s \cdot s_p}{\beta_2 \cdot s_s \cdot n_p} + 1 \right) \Delta = \left(\frac{2 \cdot n_s \cdot s_p + \beta_2 \cdot s_s \cdot n_p}{\beta_2 \cdot s_s \cdot n_p} \right) \Delta \quad (3.42)$$

From the previous developments, the relationship between the shear and the slip of an internal seam is:

$$\begin{cases} \dfrac{\Delta_e + \Delta}{V} = \left(\dfrac{2 \cdot n_s \cdot s_p + \beta_2 \cdot s_s \cdot n_p}{\beta_2 \cdot s_s \cdot n_p} \right) \Delta \quad (3.43) \\ \dfrac{V}{\Delta} = 2 \cdot \dfrac{n_s}{s_s} + \beta_1 \cdot \dfrac{n_p}{s_p} \quad (3.44) \end{cases}$$

Combining the two previous expressions we obtain that:

$$\frac{\Delta_e + \Delta}{V} = \left(\frac{2 \cdot n_s \cdot s_p + \beta_2 \cdot s_s \cdot n_p}{\beta_2 \cdot s_s \cdot n_p} \right) \cdot \left(\frac{s_s \cdot s_p}{2 \cdot n_s \cdot s_p + \beta_1 \cdot s_s \cdot n_p} \right) (3.45)$$

The total force in the connection point with the rafter will be the displacement, by the number of fasteners and divided with the flexibility, obtaining that:

$$V_{ult} = n_p \cdot \frac{\Delta_e + \Delta}{s_p} = \beta_2 \cdot n_p \cdot F_p \left(\frac{2 \cdot n_s \cdot s_p + \beta_1 \cdot s_s \cdot n_p}{2 \cdot n_s \cdot s_p + \beta_2 \cdot s_s \cdot n_p} \right) \approx \beta_2 \cdot n_p \cdot F_p \quad (3.46)$$

$$V_{ult} = \beta_2 \cdot n_p \cdot F_p \quad (3.47)$$

The last expression can be used, as the expression in brackets will be close to the unit

4.2.2.3.2.2. Flexibility

To obtain the deformation two factors will have to be taken into account. The first one will be the slip of the sheet relative to the purlin, and the second the relative slip of the purlin connection to the rafter (let's consider this under s_{pr}) This value can be found tabulated in the ECCS [21] recommendation or other publications. It is assumed, as before that the value in brackets will be close to one. The flexibility can be defined as:

$$c_{2.3} = 2 \cdot \left(\frac{s_p}{\beta_2 \cdot n_p} + \frac{s_{pr}}{n_p} \cdot \right) = \frac{2}{n_p} \cdot \left(s_{pr} + \frac{s_p}{\beta_2} \right) (3.48)$$

$$c_{2.3} = \frac{2}{n_p} \cdot \left(s_{pr} + \frac{s_p}{\beta_2} \right) (3.49)$$

4.2.2.4. Flexibility due to profile distortion

When a metal sheet is loaded in shear, tends to deform according to the figure 3.9, due to profile distortion. The first formulation to solve the problem was presented by Bryan, who just considered linear deformation of the elements, which is not according with the real deformation, which has curved lines. Horne and Raslan made further improvements. But the formulation in the ECCS comes from Davies and Lawson who, with energy analysis on the profiles developed the formula next presented. They developed a value K for each sheet (available in the tables) introducing more non-linear displacement terms, which took the form:

$$c_{1.1} = \frac{a \cdot d^{2.5} \cdot \overline{K}}{E \cdot t^{2.5} \cdot b^2} \quad (3.50)$$

a=width of the diaphragm measured normal to the corrugation
b=depth of the diaphragm measured parallel to the corrugation
d=pitch of the corrugation
E=young's modulus
T=net thickness of the sheeting
K= a constant for a given profile

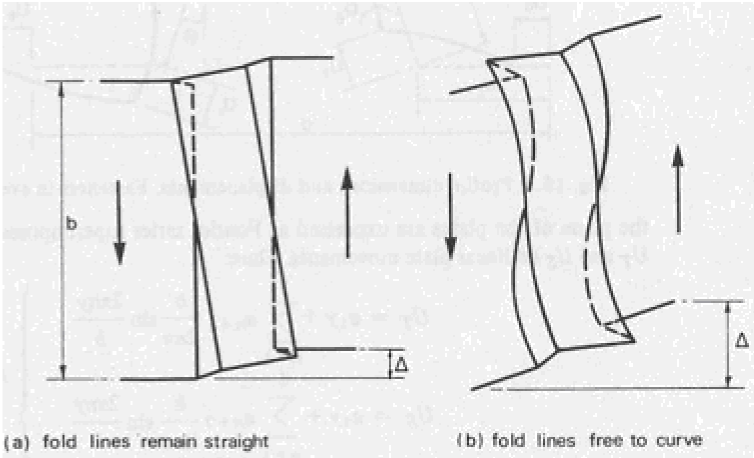


Fig 3.9: Distortion of the corrugation in plan. Extracted from [28]

But some factors external to the sheet itself and which have a great influence are not considered in the previous formulation, and those are the following:

-Intermediate purlins
-Fastening pattern: in every though or alternate

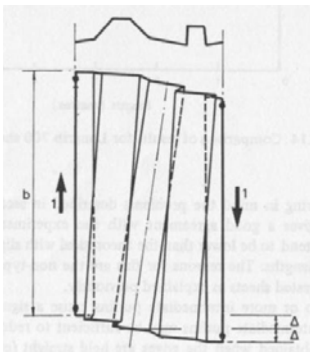


Fig 3.10: Deformation of the metal sheet with alternate fastening pattern. Extracted from [28]

Evolution of the K under this circumstances see fig 3.11. There are just two methods to evaluate this value: FEM or tests. Theoretical evaluations are too complicated to define and present underestimations of the flexibility. The more purlins we have the more independent K turns to be from the length.

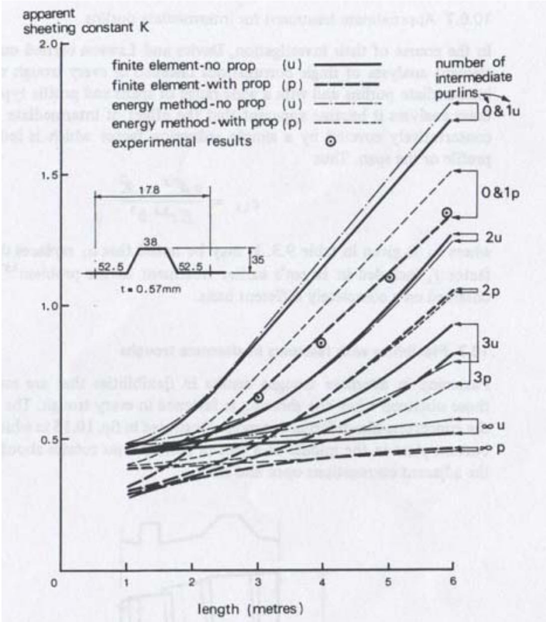


Fig 3.11. Comparison of results for longrib 700 sheeting fastened in every trough
Extracted from [28]

In the ECCS recommendation we can find the values of K. They are divided in two different tables, the table K_1 when the metal sheet is fastened in every through, and values of K_2 if it is fastened in alternate through. These values depend in three parameters that model the form of the metal sheets. The tables can be found in the annex

In the document "Influence of Profile Distortion on the Shear Flexibility of Profiled Steel Sheeting Diaphragms" [29] Duerr and Saal claim that the values of K_2 are wrong and give correct values. The author did not have access to the content of the paper.

Davies and Lawson introduced a reduction factor α_1 to reduce the deformation in case more purlins were used

Influence of the end laps

The fact that the metal sheets are not long enough and have to be overlapped, the flexibility increases at that point, as there is no continuity of the metal sheet. So a second α_4 is introduced to evaluate the number of purlins, number of end laps, and pattern of corrugation.

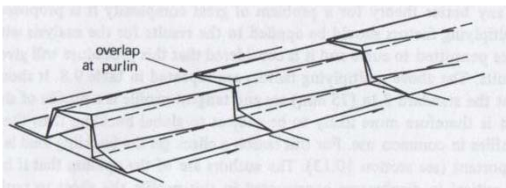


Fig 3.12. Behaviour of end laps
Extracted from [28]

The end formulation for the flexibility can be written as:

$$c_{1.1} = \frac{a \cdot d^{2.5} \cdot \overline{K}}{E \cdot t^{2.5} \cdot b^2} \cdot \alpha_1 \cdot \alpha_4 \quad (3.51)$$

4.2.2.5. **Flexibility due to shear strain**

Given the parameters showed in the schema, the flexibility of this corrugation due to shear strain can be calculated as follows:

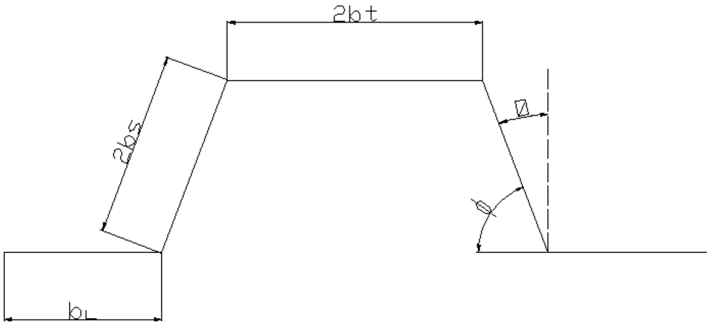


Fig 3.13 Description of the values in the picture

The perimeter of the corrugation is equal to 2 (b_L+2b_s+b_T). The shear strain in each face of the profile will be given by:

$$\gamma = \frac{1}{btG} = \frac{2(1+\nu)}{btE} \quad (3.52)$$

Being t and b the thickness and length respectively

In a single corrugation of width d is then:

$$\gamma = \frac{4(1+\nu)(b_L + 2b_s + b_T)}{btE} \quad (3.53)$$

And in a complete panel of width a is then:

$$\gamma = \frac{4(1+\nu)(b_L + 2b_s + b_T)}{btE} \cdot \frac{a}{d} \quad (3.54)$$

As this value is usually irrelevant to the diaphragm it is usual to approximate the perimeter to that of an equivalent rectangular profile and to take the value as:

$$c_{1.2} = \frac{2a(1+\nu)(1+\frac{2h}{d})}{btE} \quad (3.55)$$

4.2.2.6. Flexibility due to axial strain in the edge members

It is convenient to treat the flexibility due to the edge members as an "equivalent flexibility" because what in reality happens is a bending effect. If the cross section area of the purlin is A, their effective second moment of area can be defined as $Ab^2/2$ and the bending deflection under unit load gives the equivalent shear flexibility as:

$$c_3 = \frac{2 \cdot a^3}{3 \cdot E \cdot A \cdot b^2} \quad (3.56)$$

4.2.2.7. Shear buckling of the metal sheet of the diaphragm

There are two basic modes of buckling:

4.2.2.7.1. Buckling of the flat plate elements of the profile

This mode of buckling,that according to [28] is an unusual mode of failure, can be evaluated with:

$$\tau_{cr} = \frac{k \cdot \pi^2 \cdot E}{12 \cdot (1-\nu^2) \cdot (h/t)^2} \quad (3.57)$$

k=buckling coefficient that can be conservatively taken as 5.35
h/t= breadth to thickness ratio of the plate element

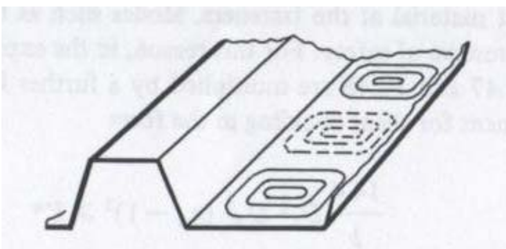


Fig 3.14. Local shear buckling Extracted from [28]

4.2.2.7.2. Overall shear buckling

In this case, one or more buckling waves propagate in all the width of the metal panel and it is not a phenomenon of one corrugation anymore. To analyse this fact the metal panel will be treated as an orthotropic plate with different strengths according to the direction.
The theory to calculate is the one proposed by Easely [30] (Lawson tested that those theories where valid in case every though was fastened)

To calculate this ultimate buckling resistance, Easley considers a buckle wave inclined a value k respect to the axis of major resistance; from there, he obtained a formula depending on these two variables

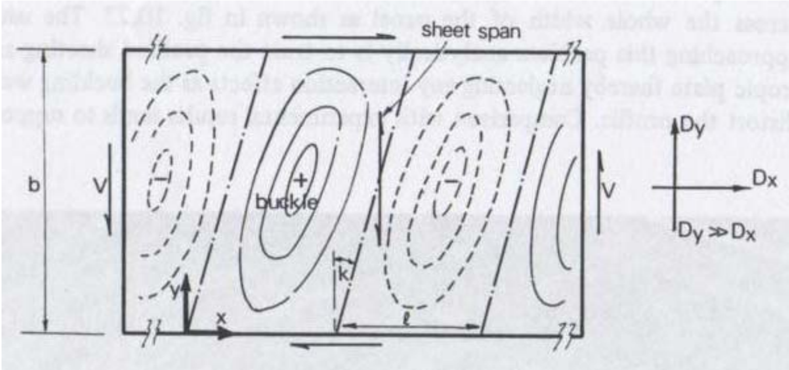


Fig 3.15 Geometry of a buckling wave in an orthotropic plate Extracted from [28]

If the assumption that D_x and D_{xy} can be neglected compared to D_y, the expressions can be defined as:

$$V_{ult} = \frac{36}{b} D_x^{\frac{1}{4}} D_y^{\frac{3}{4}} \quad (3.58)$$

Lawson showed that it is reasonable to apply this expression to each of the (n_p-1) regions of the diaphragm between adjacent purlins, each of depth b/(n_p-1) provided that the fastenings were sufficiently close together to confine the buckling wave to this region. Thus the formula 3.58 can be converted to the 3.59:

$$V_{ult} = \frac{36 \cdot (n_p - 1)^2}{b} D_x^{\frac{1}{4}} D_y^{\frac{3}{4}} \quad (3.59)$$

Being:

$$D_x = \frac{E \cdot t^3}{12\left(1 - \nu^2\right)} \cdot \frac{b_R}{u} \quad (3.60)$$

$$D_y = \frac{E \cdot I_y}{b_R} \quad (3.61)$$

br= wide of one of the ribs
u =length on surface of the same rib
I= moment of inertia of one of the ribs

In the ECCS the security formulation in case there are purlins is the following:

-Check the interaction between the global and the local buckling. When the following inequality is certain then it can be neglected, if not the global buckling stress has to be reduced as expressed in formula 3.63

$$l/t \leq 2.9 \cdot \left(\frac{E}{f_y}\right)^{0.5} \quad (3.62)$$

$$V_{red} = \frac{V_g \cdot V_l}{V_g + V_l} \quad (3.63)$$

-As Lawson saw that a reasonably safe expression could be obtained by the 50% reduction of the value given in 3.59. Finally buckling is a non-ductile modus, so a further 25% reduction is applied, resulting in the formulation 3.64:

$$V_g = \frac{14.4}{b} D_x^{\frac{1}{4}} D_y^{\frac{3}{4}} (\mathrm{n_p} - 1)^2 \quad (3.64)$$

-The local shear buckling can be expressed as follows when the metal sheet is non-stiffened:

$$V_l = 4.83 \cdot E \cdot \left(\frac{t}{l}\right)^2 \cdot \mathrm{bt} \quad (3.65)$$

4.2.2.8. End collapse of the end of the sheeting

In the ECCS recommendations, in order to prevent collapse or gross distortion of the profile at the end of the sheeting, a limitation on the shear force in the shear panel should be observed

Every corrugation fastened at the end of the building

$$V_{ult} \leq \frac{0.9 \cdot t^{1.5} \cdot b \cdot f_y}{d^{0.5}} \quad (3.62)$$

Alternate corrugations fastened at the end of the sheeting

$$V_{ult} \leq \frac{0.3 \cdot t^{1.5} \cdot b \cdot f_y}{d^{0.5}} \quad (3.63)$$

t= net thickness, excluding metallic and other coatings
b =depth of a shear panel in a direction parallel to the corrugations
fy= design yield stress of steel
d=pitch of the corrugations

4.2.2.9. The factors β_1 and β_2

To calculate these factors, a lineal distribution of forces in the purlins has been assumed. Without intervening in the formulas, other distributions can be proposed, like for example a parabolic distribution. It has been observed that the reality adjusts more to one or the other distribution, depending on the relative stiffness between the metal sheet and the purlin. The stiffener the sheet is respect to the purlin, the more it approximates to the parabolic distribution. In [28] it is proposed that:

$$\rho = \frac{\text{stiffness metal sheet}}{\text{stiffness purlin}} = \frac{\left(\frac{a}{n_{sh}}\right)^3 \cdot G_{eff} \cdot d}{EI} \quad (3.64)$$

Where:
d=depth of the diaphragm
EI=minor axis flexural rigidity of purlins
G_{eff}=effective shear modulus of the sheeting

Then:

ρ<8000	Linear
8000≤ρ≤400.000	Parabolic
ρ>400.000	Parabolic with reduced sheet width (see...)

G_{eff} is defined in [23] as:

$$G_{eff} = \frac{E}{\frac{0.144 \cdot d^4 \cdot K}{b^2 \cdot t^2} + 2 \cdot (1+\nu)\left(1 + \frac{2 \cdot h}{d}\right)} \quad (3.65)$$

The parabolic values are the ones in the Table of the ECCS recommendations.

4.3. <u>Theory Schardt/Sthrehl</u>

The first recommendation on the calculation of the metal sheets in shear field load was presented in 1976 in the magazine Stahlbau [31], and was used together with the theories of Bryan/Davies some years after in the publication in 1987 of the DIN 18807, the norm that in Germany regulated the use of metal sheets, which are used as bearing element.

4.3.1. Introduction

4.3.1.1. <u>Assumptions for the calculation</u>

1. The ribs are parallel to each other.
2. The metal sheet is fastened to a four side pinned frame.
3. The number of ribs is big enough (n>10) as to ensure that there is no influence on the middle point due to the lateral fastenings.
4. The length of the sheet should have a minimum relation with the length of the shear field.

The main difference with the postulates of Bryan/Davies is the minimum relation, which will be defined according to each one the metal sheets. The other assumption can be easily fulfilled with any other calculation.

4.3.1.2. <u>Basis of calculations and calculating process.</u>

The metal sheet it is then calculated as a folded plate consisting of three different plates under the circumstances that they can not bear moment M_z (torsion) and M_s (bending) and are subsequently considered zero in the individual sheet.

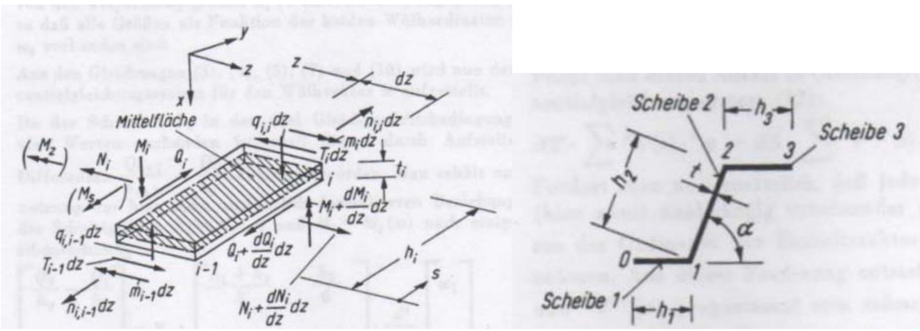


Fig 3.16 Left: tensions on the plate. Right: model of the sheet in three plates Extracted from [31]

With the boundary conditions of symmetry in the points 0 and 3, where there is just vertical displacement and applying the equilibrium conditions [31], the solution of the system is solved thanks to the theory of the elastic bed.

4.3.1.3. **Results: graphics and parameters**

It can be seen that the distribution of the shear flow, tends to constant within the whole length of the metal sheet if the length of the sheet is big enough.
This length will be characterised by the fact that after it, the shear field can be considered with the properties of infinite long

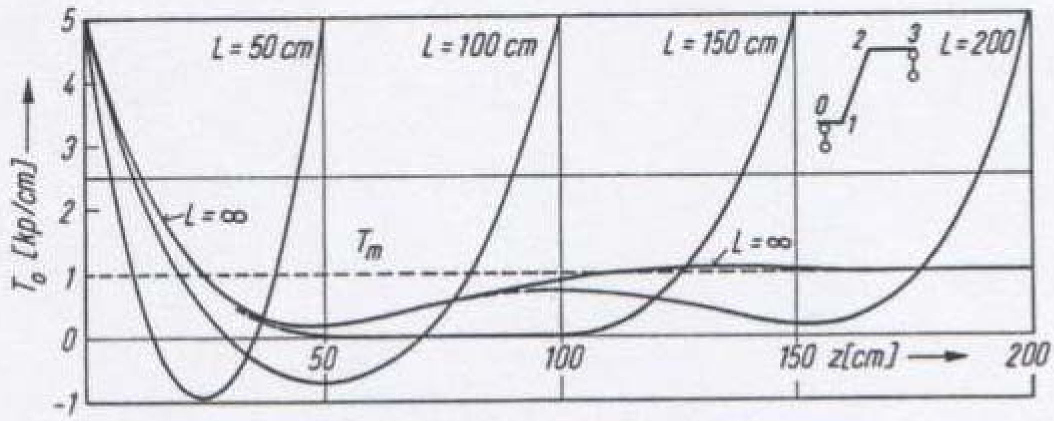


Bild 14. Verlauf des Schubflusses $T_s(z)$ in Abhängigkeit der Schubfeldlänge L für T_m = 1 kp/cm

Fig 3.17 Distribution of the shear effort in the sheet depending on the length. Extracted from [31]

The objective of this theory is to characterize the resistance on shear effort of the metal sheets with 3 parameters easy to check by the designer.

a) The value of the shear modulus: with this value the maximum allowed the shear effort can be set, so that the deformation is limited
b) A value to limit the tension in the connections of the different metal sheets
c) The deformation of the superior flange respect to the inferior flange.

This was used to define the DIN 18807[20]; in this norm the shear effort comes limited by three shear flow tensions T_1, T_2 and T_3 defined as follows:

(a) T_1:
Test that the maximum normal tension in the upper corners of the metal sheet is not bigger than the yield tension; the distribution of normal tensions is on the fig 3.18:

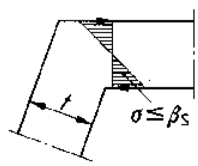


Fig 3.18: tension on the upper corner. Extracted from [32]

(b) T_2
The deformation of the upper flange respect to the inferior flange should not be bigger than a 20th of the height of the profile

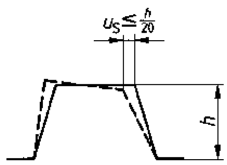


Fig 3.19: deformation of the upper flange. Extracted from [32]

(c) T_3
The third one limits the deformation, of the sheet. Inside the shear field the total metal sheet's angle respect to the initial position should be smaller than 1/750

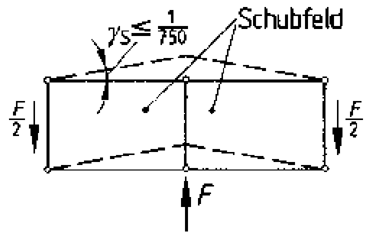


Fig 3.19: deformation of the shear field. Extracted from [30]

4.3.2. Resume

The problem that presents this formulation is:

- It is more a serviceability limit than an ultimate stress calculation. There is in reality no parameter (as in the theory Bryan/Davies) that mention the buckling of the metal sheet
- As commented in [33] the problem of this formulation relies in the fact; that the theory says that if the value is exceeded then there should be a plastic hinge created in the angle, but this could not be proved in any experimental analysis

- The second limitation makes sense in some special cases, mainly in those in which the insulation is glued to the surface of the profile, but it is not especially relevant to the fastened with screws or other methods. The third limitation on the other hand makes a lot of sense.

The easy part is that this model, in respect to the one proposed by Bryan/Davies, it is much easier to calculate the rigidity of the metal sheet as the rigidity of the metal sheet will be just dependant on one factor, the metal sheets and the length of it in the direction of the ribs. Which values had been taken in experimental test on the metal sheets. But for some circumstances that will be seen further, that will not be of a much use in the roof disposition that is studied in this dissertation. As the values of G are associated with certain fastening patters that do not coincide with the ones studied in the dissertation

4.3.3. Rigidity of the shear field

The producer should provide the proper values, otherwise the possible approximation is to use the formulation (3.65) of this dissertation to get a value of G. This theory is though thought for metal sheets that span from rafter to rafter, that is direct shear connections, and in each trough. So the values K_1 and K_2 will be not valid in the case of "indirect shear connections", or fastenings in alternate trough. Even if the values of G are known, the methods to calculate the shear efforts in the sheets is quite different as the one proposed by Bryan/Davies and is focused as said for certain fastening conditions and spans.

We can define as Elements:

t_N: thickness of the metal sheet

Min L_s: this is the minimum length to apply the following calculations, if the length (always calculated on the direction of the rib) is lower than this, then the following correction should be made in the parameter K_2; what will give a higher K_2, which will reduce the stiffness:

$$K_2^{new} = \frac{MinL_s}{L_s} \cdot K_2 \ (3.66)$$

K_1, K_2: those are the values that will be considered to calculate the rigidity

The G shear rigidity can be calculated as:

$$G = \frac{10^4}{K_1 + \frac{K_2}{L_s}} \ (3.67)$$

And the angle will be then:

$$\gamma = \frac{T}{G_s} \ (3.68)$$

All these values are also depend on the fastening pattern of the elements; these two different patterns are either:
a) Normal: means that each nest is fastened with one fastener as it can be seen on the picture:

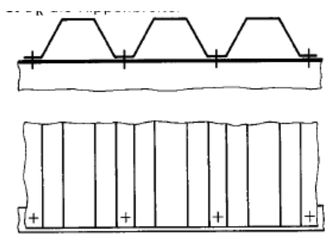


Fig 3.20 Normal fastening pattern. Extracted from [19]

b) Special: it is either two fasteners per rib (each one close to the web) or a square or circle that cover the whole flange it is used. In this case the shear effort will be higher, at it is considered that the whole of the flange is fastened to the base material.

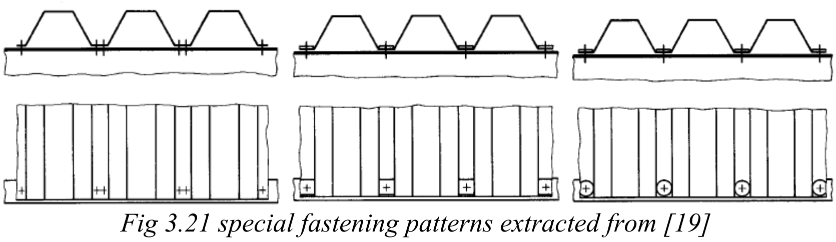


Fig 3.21 special fastening patterns extracted from [19]

This theory though that valid and easy (at least on the calculation of the deformation) will be of restricted use in a practical case with purlins.

4.4. Fasteners

An important part for the calculation of the stressed skin effect is the fasteners used for them. The valid fastener elements and its properties will be specified in this part. In the figure we can see the main kinds: screws (being the self-screwed the most popular), there are also available normal screws, fired pins and blind rivets

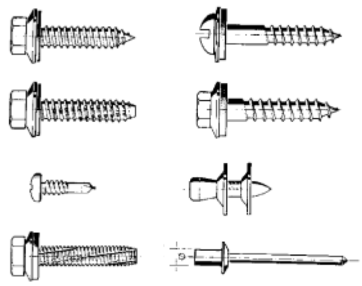


Fig 3.22: Different fasteners: screws, pins and rivets. Extracted from [32]

According to ECCS the sheet/member fasteners "should carry shear force without reliance on friction or bending of the fasteners themselves. The fasteners should be of a type which will not wok loose in service and which will neither pull out nor fail in shear before causing tearing of the sheet (self tapping, self-drilling screws, shot pins, bolts or welding).

In this dissertation just the self-drilling screws of diameter 4.4mm, 4.8mm, 5.5mm and 6.3mm will be presented. For other type or diameters please apply to the general formulations. The following formulation presented in Eurocode is valid for thickness between 0.5 and 1.5 millimetre

4.4.1. Eurocode formulation

According to Eurocode the following measures for a screw apply:

		Screw $3.0 \leq d \leq 8.0$ mm
Bearing failure $F_{b,Rd}$	$t=t_1$	$F_{b,Rd} = \frac{3.2 \cdot f_u \sqrt{t^3 \cdot d}}{\gamma_{M2}} < \frac{2.1 \cdot t \cdot d \cdot f_u}{\gamma_{M2}}$ (3.67)
	$t_1 \geq 2.5t$	$F_{b,Rd} = \frac{2.1 \cdot t \cdot d \cdot f_u}{\gamma_{M2}}$ (3.68)
	$t < t_1 < 2.5t$	Linear interpolation previous ones
Pull trough failure Fp,Rd	Static loading	$F_{p,Rd} = \frac{t \cdot d_w \cdot f_u}{\gamma_{M2}}$ (3.69)
	Loading with wind	$F_{p,Rd} = 0.5 \frac{t \cdot d_w \cdot f_u}{\gamma_{M2}}$ (3.70)
Pull-out failure Fo,Rd	$T_1 \geq 0.9$ mm $0.5 \leq t \leq 1.5$mm	$F_{p,Rd} = \frac{0.65 \cdot t_1 \cdot d \cdot f_{u1}}{\gamma_{M2}}$ (3.71)

Tensile failure of screw $F_{t,Rd}$		From tables provided by the manufacturer
Respect to shear failure of the screw $F_{v,Rd}$		From tables provided by the manufacturer

Table 3.1. Resistance of the fasteners according to ENV 1993-1-3

f_u is characteristic ultimate stress(Mpa)
t is the thickness of thinner sheet in the connection, mm, excluding zinc
t_1 is thickness of thicker sheet in connection, mm
d is diameter of the fastener, (thread diameter), mm
d_w is the diameter of the washer, mm
γ_{M2}=1.25 is the partial factor

On the following table the reference values presented for hardened carbon steel

	Screw diameter (mm)			
	4.8	5.5	6.3	8.0
$F_{t,Rd}$ (KN)	4.99	6.91	9.41	15.6
$F_{v,Rd}$ (KN)	4.16	5.76	7.84	13.0

Table 3.2. Ultimate resistance of a Hardened carbon steel screw due to tensile failure and shear failure of the screw Extracted from [6]

4.4.2. ECCS formulation

In the ECCS there is just an indication for the shear behaviour of the screws, doing a differentiation, according to the role of the fasteners in the global stability, that is, seam, sheet/purlin or sheet/shear connector fastener. For the calculation of the screws in tension we have to take the Eurocode reference values. The following formulas already include a 1.1 safety factor (instead of the 1,25 recommended due to the large number of fasteners in a panel).

The following assumptions are made:
The base material in the sheet/sheet shear is 2,5 times thicker than the metal sheet. And for the seams that are equal. The slip values are based on test results and are approximate.

Sheet/purlin and sheet/shear connectors	Diameter mm	Design shear strength (KN)	Slip (mm/KN)
Screws (collar head	5.5	$1.9 \cdot f_u \cdot d_n \cdot t \leq 6.5KN$ (3.72)	0.15
	6.3 mm	$1.9 \cdot f_u \cdot d_n \cdot t \leq 8KN$ (3.73)	0.15

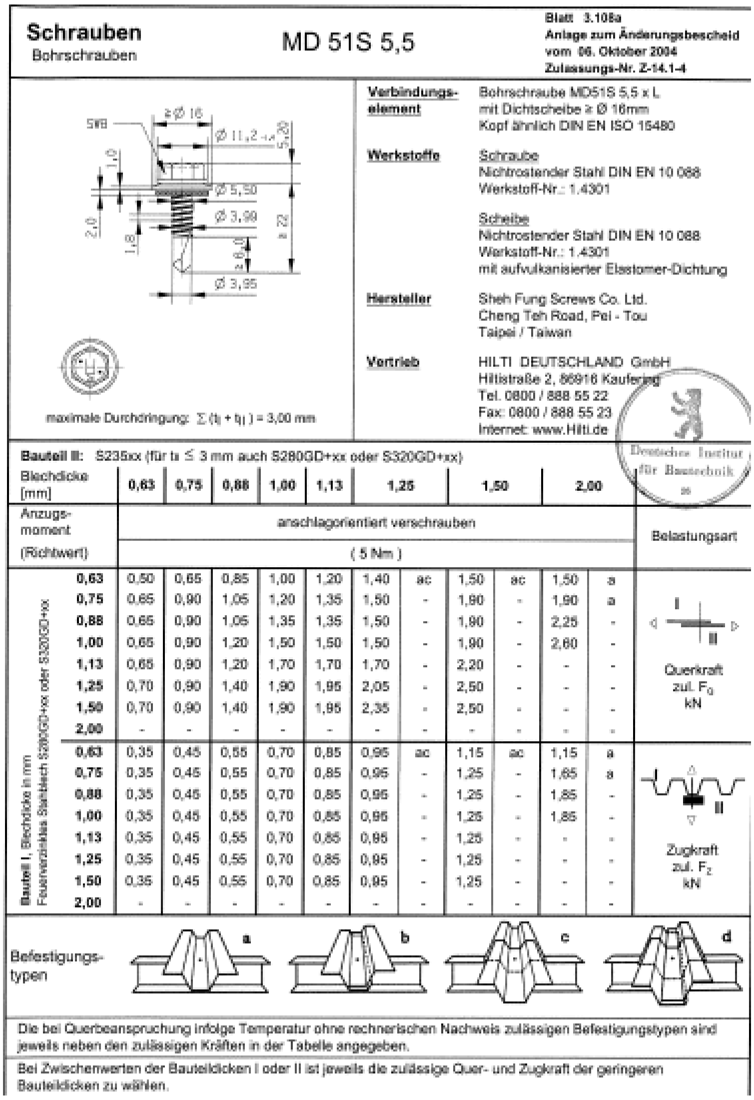
Seam fasteners	Diameter	Design shear strength (KN)	Slip (mm/KN)
Screws	4.1-4.8	$2.9 \cdot f_u \cdot d_n \cdot t\sqrt{\frac{t}{d_n \cdot}} \leq 3.8KN$ (3.74)	0.25

Table 3.3 Values proposed by the ECCS recommendation

f_u is characteristic ultimate stress (Mpa)
t is the thickness of the sheet
d_n is the nominal diameter of the fastener (mm)

4.4.3. DIN 18807

In the DIN norm the reference values of the fasteners are obtained by the specification of the manufacturer. Given the diameter of the fastener, base material thickness and sheet thickness and strength the shear and tension values of the fastener can be obtained.



Schrauben
Bohrschrauben

MD 51S 5,5

Blatt 3.108a
Anlage zum Änderungsbescheid
vom 06. Oktober 2004
Zulassungs-Nr. Z-14.1-4

Verbindungs-element: Bohrschraube MD51S 5,5 x L mit Dichtscheibe ≥ Ø 16mm Kopf ähnlich DIN EN ISO 15480

Werkstoffe: Schraube: Nichtrostender Stahl DIN EN 10 088, Werkstoff-Nr.: 1.4301
Scheibe: Nichtrostender Stahl DIN EN 10 088, Werkstoff-Nr.: 1.4301, mit aufvulkanisierter Elastomer-Dichtung

Hersteller: Sheh Fung Screws Co. Ltd., Cheng Teh Road, Pei - Tou, Taipei / Taiwan

Vertrieb: HILTI DEUTSCHLAND GmbH, Hiltistraße 2, 86916 Kaufering, Tel. 0800 / 888 55 22, Fax: 0800 / 888 55 23, Internet: www.Hilti.de

maximale Durchdringung: $\Sigma (t_I + t_{II}) = 3{,}00$ mm

Bauteil II: S235xx (für t_{II} ≤ 3 mm auch S280GD+xx oder S320GD+xx)

Blechdicke [mm]	0,63	0,75	0,88	1,00	1,13	1,25		1,50		2,00		
Anzugs-moment (Richtwert)	anschlagorientiert verschrauben (5 Nm)											Belastungsart
Bauteil I, Blechdicke in mm, Feuerverzinktes Stahlblech S280GD+xx oder S320GD+xx												
0,63	0,50	0,65	0,85	1,00	1,20	1,40	ac	1,50	ac	1,50	a	Querkraft zul. F_Q kN
0,75	0,65	0,90	1,05	1,20	1,35	1,50	-	1,90	-	1,90	a	
0,88	0,65	0,90	1,05	1,35	1,35	1,50	-	1,90	-	2,25	-	
1,00	0,65	0,90	1,20	1,50	1,50	1,50	-	1,90	-	2,60	-	
1,13	0,65	0,90	1,20	1,70	1,70	1,70	-	2,20	-	-	-	
1,25	0,70	0,90	1,40	1,90	1,95	2,05	-	2,50	-	-	-	
1,50	0,70	0,90	1,40	1,90	1,95	2,35	-	2,50	-	-	-	
2,00	-	-	-	-	-	-	-	-	-	-	-	
0,63	0,35	0,45	0,55	0,70	0,85	0,95	ac	1,15	ac	1,15	a	Zugkraft zul. F_Z kN
0,75	0,35	0,45	0,55	0,70	0,85	0,95	-	1,25	-	1,65	a	
0,88	0,35	0,45	0,55	0,70	0,85	0,95	-	1,25	-	1,85	-	
1,00	0,35	0,45	0,55	0,70	0,85	0,95	-	1,25	-	1,85	-	
1,13	0,35	0,45	0,55	0,70	0,85	0,95	-	1,25	-	-	-	
1,25	0,35	0,45	0,55	0,70	0,85	0,95	-	1,25	-	-	-	
1,50	0,35	0,45	0,55	0,70	0,85	0,95	-	1,25	-	-	-	
2,00	-	-	-	-	-	-	-	-	-	-	-	

Befestigungs-typen: a, b, c, d

Die bei Querbeanspruchung infolge Temperatur ohne rechnerischen Nachweis zulässigen Befestigungstypen sind jeweils neben den zulässigen Kräften in der Tabelle angegeben.

Bei Zwischenwerten der Bauteildicken I oder II ist jeweils die zulässige Quer- und Zugkraft der geringeren Bauteildicken zu wählen.

Fig 3.23: Example of a screw specification. Extracted from [34]

4.4.4. Distribution of fasteners

In order to distribute the fasteners in the different positions of the metal sheets the following recommendations are given in the SBI147, these are more restrictive than the ones recommended in the DIN 18807, and are clearly presented. Shallow profiles are those that have the following requirements. A deep profile is the one, which height (h) is bigger than 50 millimetres and the rib wide (d) is bigger than 200 millimetres. The shallow profile is the one that does not fulfil both dispositions.

	Shallow profile	**Deep profile**
Side overlap	C/c 500 mm	C/c 500 mm
Sheet to edge beam	C/c 500 mm	C/c 500 mm
End reinforcement	C/c 300 mm	C/c 300 mm
Free end of sheet	One fastener in each through	One fastener in each through
Intermediate support	One fastener in every other through	One fastener in each through
Intermediate support at end overlap	One fastener in each through	One fastener in each through

Table 3.3 recommendations of fastenings from SBI174

4.5. <u>Example</u>

In annex 1 there is an example developed on how to perform a shear cells calculation in a concrete case. The results of all these calculations are in the table 1 of annex II.