

3. STRENGTH AGAINST BENDING

A bended metal sheet experiments compression in the superior flange. Due to the big difference between flange width and thickness, the section falls in the classification of a thin wall, or what it is known in Eurocode 3_[8] as section type 4 (those in which local buckling will occur before the attainment, if yield stress in one or more parts of the section is reached). The following sections try to explain, the basic properties and theories that allow the strength's calculation of these metal sheets against bending.

3.1. Introduction

3.1.1. Elastic Local buckling stress of thin plates

As mentioned before, the thin plate on the flange (or web) will buckle before yielding. Our first goal is to calculate the elastic buckling stress of a thin steel plate.

The basis to calculate this local buckling comes from the theory of plates from Von Karman (1932) and the experimental corrections from Winter (1947) (done for the preparation of the ASCI code of 1954).

The theory formulates: “A thin flat rectangular plate subjected to compression forces along its short edges has an elastic critical buckling stress given by”:

$$\sigma_{cr} = \frac{k_{\sigma} \pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \quad (2.1)$$

Where:

-The parameter k_{σ} it is a function of the boundary conditions of the plate, the stress distribution and the aspect ratio of the plate (relationship L/b).

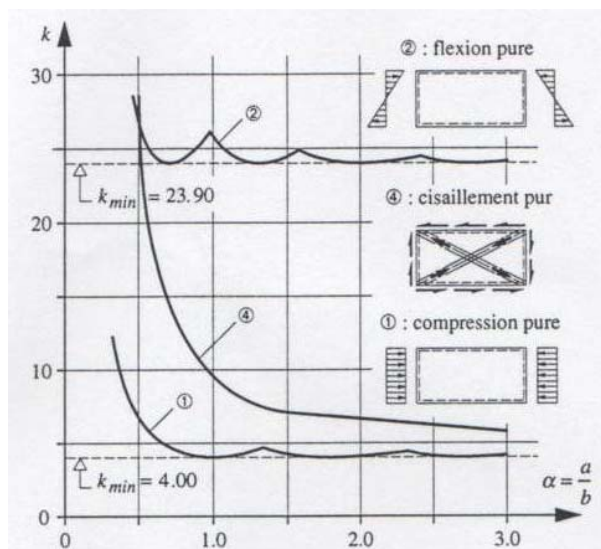


Fig 2.1: Value of k depending on the ratio L/b and stress distribution. Extracted from [12]

- E : Young modulus

- ν : Poisson modulus

A parameter that relates the yield tension of the plate with the critical buckling tension (normalized slenderness) is defined as:

$$\bar{\lambda}_p = (f_y / \sigma_{cr})^{0.5} \quad (2.2)$$

Defined the values $f_y = 235/\varepsilon^2$ (N/mm²), $\nu=0.3$ and $E=210000$ N/mm² [Steel properties]. The normalised slenderness is defined also as:

$$\bar{\lambda}_p = \left(\frac{f_y}{\sigma_{cr}} \right)^{0.5} = \left(\frac{\bar{b}/t}{28.4\varepsilon\sqrt{K_\sigma}} \right) \quad (2.3)$$

Where the slenderness is function of the width and thickness of the plate (b/t), the distribution of stress, the boundary conditions (k_σ) and the steel strength (ε)

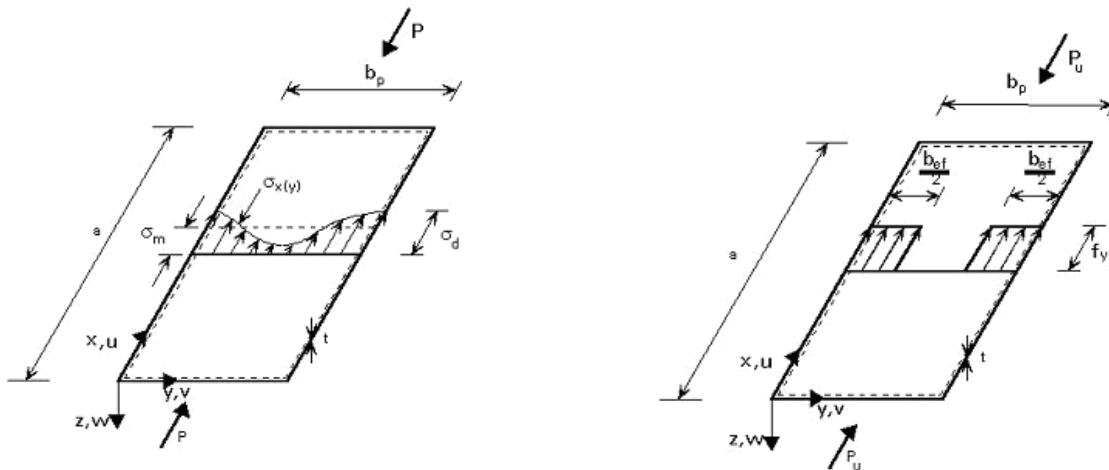


Fig 2.2: On the left side the real distribution of stresses, on the right side the idealised. Extracted from [13]

The real distribution of tensions can be seen on the left side of *fig. 2.2*. A simplification can be made adopting the so called “effective width” method. That consists of substituting the real distribution of stresses on the plate, for a uniform stress tension in two strips, of width b_{eff} , placed on the supported borders of the plate as in the example in *fig 2.2*.

The form of the end distribution depends on the original distribution of stresses in the plate. Eurocode [8] provides a table (table 4.1) where depending on the stress distribution, and the boundary conditions the formulation of the respective effective widths and the value of the buckling factor k_σ can be obtained.

The effective width is defined according to the table 2.1:

Stress distribution (compression positive)				Effective ^p width b_{eff}		
				$\psi = 1:$ $b_{eff} = \rho \bar{b}$ $b_{e1} = 0,5 b_{eff} \quad b_{e2} = 0,5 b_{eff}$		
				$1 > \psi > 0:$ $b_{eff} = \rho \bar{b}$ $b_{e1} = \frac{2}{5 - \psi} b_{eff} \quad b_{e2} = b_{eff} - b_{e1}$		
				$\psi < 0:$ $b_{eff} = \rho b_c = \rho \bar{b} / (1 - \psi)$ $b_{e1} = 0,4 b_{eff} \quad b_{e2} = 0,6 b_{eff}$		
$\psi = \sigma_2 / \sigma_1$	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1	$-1 > \psi > -3$
Buckling factor k_σ	4,0	$8,2 / (1,05 + \psi)$	7,81	$7,81 - 6,29\psi + 9,78\psi^2$	23,9	$5,98 (1 - \psi)^2$

Table 2.1 Value of b_{eff} in Doubly supported compression members. Extracted from [8]

Being ρ defined as:

$$\rho = \left(\frac{(\bar{\lambda}_p) - 0,22}{(\bar{\lambda}_p)^2} \right) \quad (2.4)$$

In the studied case and under the conditions of the metal sheets, the relevant situations are always doubly supported elements (webs and crests). So this is the relevant formulation to calculate the effective widths in the dissertation.

3.1.2. Stiffeners

The metal sheets matter of the study can have stiffeners, either in the webs, in the crest or in both of them. Stiffeners are nothing more than a fold in the surface of the crest or the web. These stiffeners act as a partial support, allowing a wider effective width in the webs or crest along the stiffeners.

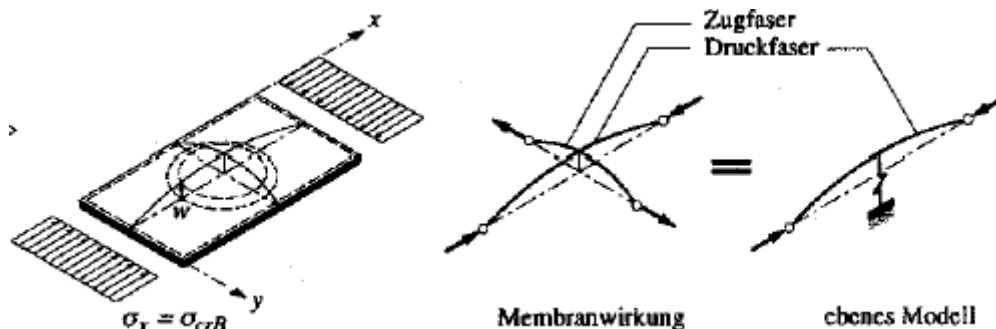


Fig 2.3: Elastic buckling of a plate; and evolution to a model of a beam with a spring. Extracted from [14]

All of these stiffeners have to demonstrate that no collapse of the stiffener either by yielding or instability occurs before the supported element reaches the ultimate state.

“The design of compression elements with edge or intermediate stiffeners should be based on the assumption that the stiffener behaves as a compression member with continuous partial restraint, with a spring stiffener that depends on the boundary conditions and the flexural stiffness of the adjacent plan elements” [8] part 3 (5.5.3.1)

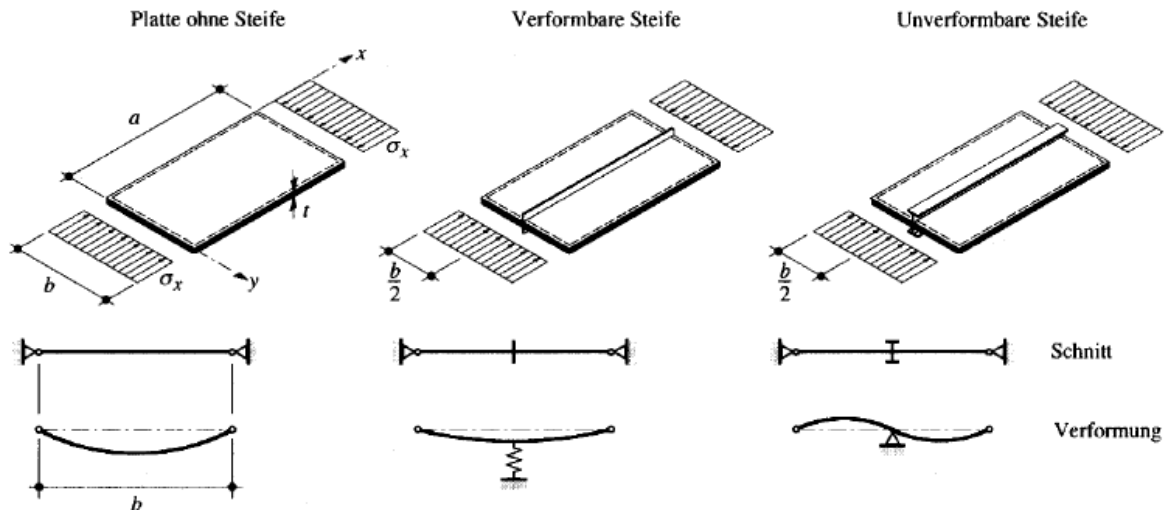


Fig 2.4 Figure that shows a plate without stiffener, with a stiffener acting as a spring or a stiffener considered as a border. Extracted from [14]

3.1.3. Calculation of the stiffness of a stiffener in Eurocode 3

To calculate the stiffness of this support Eurocode considers a double supported beam and the stiffener acting as a spring.

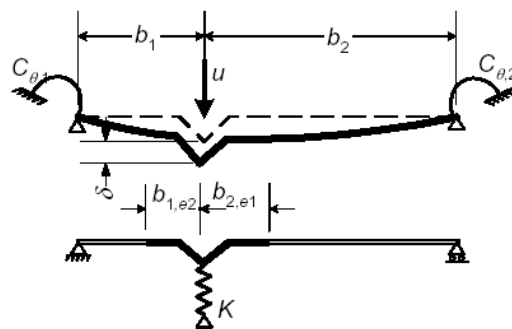


Fig 2.5 Model proposed by Eurocode 3.1 to model a flange (or a web) with stiffener. Extracted from [8]

With the information that the rigidity of the spring is $K=u/\delta$ and depending on the configuration of the spring, the value of the deflection can be easily found. Considering that the ends are pinned ($C=0$), the deflection is: (the deformation (f) of a beam with one last, placed at a distance a and b from the ends)

$$f = \frac{F \cdot a^2 \cdot b^2}{3 \cdot (a + b)} \cdot \frac{1}{E \cdot I} \quad (2.5)$$

In our case

$$\delta = \frac{u \cdot b_1^2 \cdot b_2^2}{3 \cdot (b_1 + b_2)} \cdot \frac{12(1-\nu^2)}{E \cdot t^3} \quad (2.6)$$

From here we can obtain then the rigidity of the spring K

$$K = \frac{3 \cdot (b_1 + b_2)}{b_1^2 \cdot b_2^2} \cdot \frac{Et^3}{12 \cdot (1-\nu^2)} \quad (2.7)$$

if $b_1 = b_2 = b$

$$K = \frac{1}{2 \cdot b^3} \cdot \frac{Et^3}{(1-\nu^2)} \quad (2.8)$$

The Eurocode proposes a method consisting of three Steps to calculate the stiffness of an intermediate stiffener:

1) Assume that the stiffener acts not as a spring but as an end. From there the effective width at each part of the stiffener can be calculated. Considering that the stiffener now divides the plate in two or three sections (practical cases).

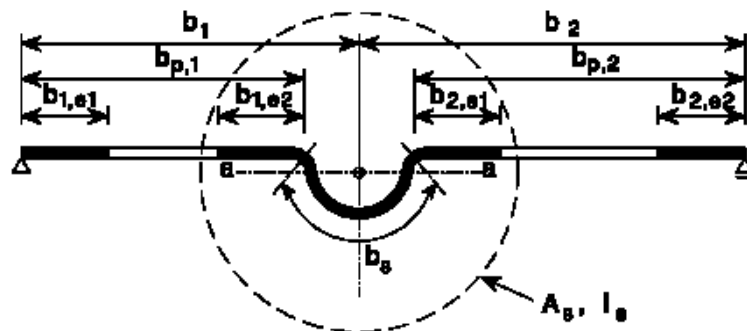


Fig 2.6 Model proposed by Eurocode 3.1 to model a flange (or a web) with stiffener. Extracted from [8]

It is considered that there are two plates of widths $b_{p,1}$ and $b_{p,2}$ both supported on both ends. Each one of the sub-plates is calculated; from there the $b_{1,e1}$ and $b_{1,e2}$ are calculated. The same procedure is repeated for the other two.

2) The stiffener is aisled; the critical elastic stress is obtained with the formula

$$\sigma_{cr,s} = \frac{2 \cdot K \sqrt{EI_s}}{A_s} \quad (2.9)$$

Being

K : spring stiffness per unit length

I_s : is the effective second moment of area of the stiffener, taken as that if its effective area A_s about the centroidal axis $a-a$ of its effective cross-section, see figure 5.11

A_s : effective area

Once this critic tension is calculated, the slenderness of the stiffener can be calculated with the formula:

$$\bar{\lambda}_d = \sqrt{\frac{f_{yb}}{\sigma_{cr,s}}} \quad (2.10)$$

From the relative slenderness, the factor χ_d , to obtain the reduced section of the stiffener, can be calculated as:

$$\chi_d = 1 \quad \text{if } \bar{\lambda}_d \leq 0,65 \quad (2.11)$$

$$\chi_d = 1,47 - 0,723\bar{\lambda}_d \quad \text{if } 0,65 < \bar{\lambda}_d < 1,38 \quad (2.12)$$

$$\chi_d = \frac{0,66}{\bar{\lambda}_d} \quad \text{if } \bar{\lambda}_d \geq 1,38 \quad (2.13)$$

Eurocode proposes and iteration process now:

Instead of using f_{yb}/γ_{M0} , use the χ_d multiplied by f_{yb}/γ_{M0} . A different b_{eff} is obtained then from the step one; the critic tension and slenderness can be then calculated again; and the new factor $\chi_{d,1}$; The solution can be iterated until is accurate enough.

The obtained value χ_d , it is going to be smaller than the one in first approximation.

3) Once the χ_d is obtained, to be able to compress the whole section stiffener and non-stiffener to the same tension, the thickness of the stiffener is reduced.

$$A_{s,red} = \chi_d \cdot A_s \cdot \frac{f_{yb} \cdot \gamma_{M0}}{\sigma_{com,red}} \quad (2.14)$$

$$t_{red} = t \cdot A_s / A_{s,red} \quad (2.15)$$

3.1.4. Calculation of the metal sheet properties according to Eurocode

This is the formulation that Eurocode adopts as the theory of buckling for plates. A metal sheet it is nothing more that a set of metal sheets welded together. The effective section must be calculated for the compressed members of the sheet.

Some limitations, mainly on the slenderness of the elements, are required for the use of Eurocode. If a metal sheet does not comply with them, it should be studied with essays.

In normal practice the information is provided by the metal sheets' producers, nevertheless a skim of the most important information in the actual Eurocode is provided for the cases that must be solved in the latter chapters of the dissertation. In some countries, like Germany, there are data tables, that provide all the data required in the norm, where the theoretical values have been validated by the tests of an authorised entity (following the parameters of DIN 18807 part 2 [15])

The next lines are not intended to be a copy of all the information provided in Eurocode 3 part 1.3 [8], if not just an insight of the main points and procedures. For complete information in the topic, please refer to [8] point 5.5.3.4 trapezoidal sheeting profiles with intermediate stiffeners.

Limitations

Check that the metal sheets comply with the limits, and if the round corners can be assumed to be sharp. Other considerations like flange curling or shear lag do have to be considered, if that's the case please refer to Eurocode 3 [8]

$$5t \leq r \leq 0.04tE / f_y \quad (2.16)$$

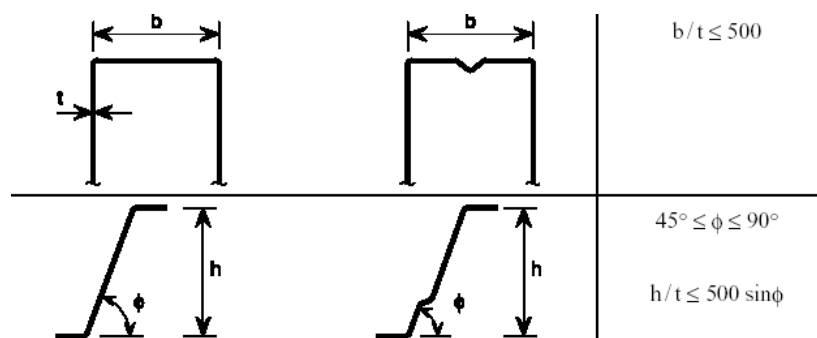


Fig 2.7: Limitations on webs and flanges to use the Eurocode. Extracted from [8]

3.1.5. Calculation of the properties of the metal sheet

Eurocode method to calculate the critical elastic buckling stress of the stiffeners in flanges and webs

1) Calculation of the flanges with intermediate stiffeners

All the metal sheets sold in Spain have a maximum of two stiffeners placed either in the web or the flange. So those will be the only two types considered.

To avoid extra calculations Eurocode provides the following recommendations to get directly a critical tension for one or two stiffeners. In this case no iteration is needed.

These recommendations are of course just valid if the section is in uniform compression. The metal sheets have though “no resistance” on the direction transversal to the ribs so uniform tension should be expected due to bending.

The steps to follow are the following:

1) One stiffener: following the instructions on the diagram the critical tension of the stiffener with the following formulation can be calculated:

$$\sigma_{cr,s} = \frac{4,2 \cdot k_w \cdot E}{A_s} \sqrt{\frac{I_s \cdot t^3}{4 \cdot b_p^2 \cdot (2 \cdot b_p + 3 \cdot b_s)}} \quad (2.17)$$

Being

b_s : is the stiffener width, measured around the perimeter of the stiffener

k_w : is a coefficient that allows for partial rotational restraint of the stiffened flange by the webs or other adjacent elements (for more information refer to Eurocode 3-1 5.53.4.2 (5) and (6))

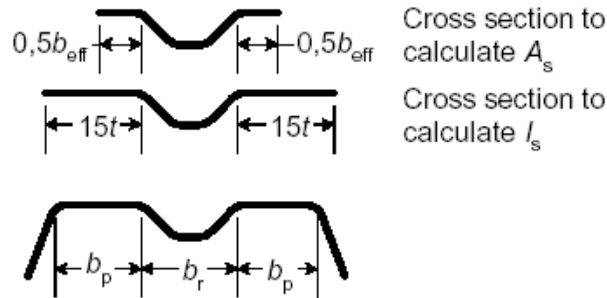


Fig 2.8 Graphic to calculate the A_s and I_s in case there is just one stiffener. Extracted from [8].

If we have two stiffeners then:

$$\sigma_{cr,s} = \frac{4,2 \cdot k_w \cdot E}{A_s} \sqrt{\frac{I_s \cdot t^3}{8 \cdot b_1^2 \cdot (3 \cdot b_e - 4 \cdot b_1)}} \quad (2.18)$$

$$b_e = 2b_{p,1} + b_{p,2} + 2b_s \quad (2.19)$$

$$b_1 = b_{p,1} + 0,5b_r \quad (2.20)$$

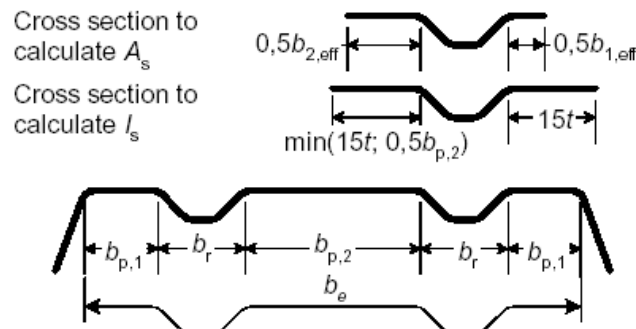


Fig 2.9 Graphic to calculate the A_s and I_s in case there are two stiffeners. Extracted from [8].

2) Calculation of the webs with intermediate stiffeners

For the calculation of the webs depends on if there is just one or two stiffeners; The Eurocode gives a complete and long procedure to calculate the effective width; first a

$$s_{eff,0} = 0.76 \cdot t \cdot \sqrt{\frac{E}{\gamma_{MO} \cdot \sigma_{com,Ed}}} \quad (2.21)$$

Where this sigma is the stress in the compression flange when the cross section strength is reached (has to be calculated from the highest tension allowable on the flanges).

With a complete series of formulation the values of the figure 2.10 can be calculated

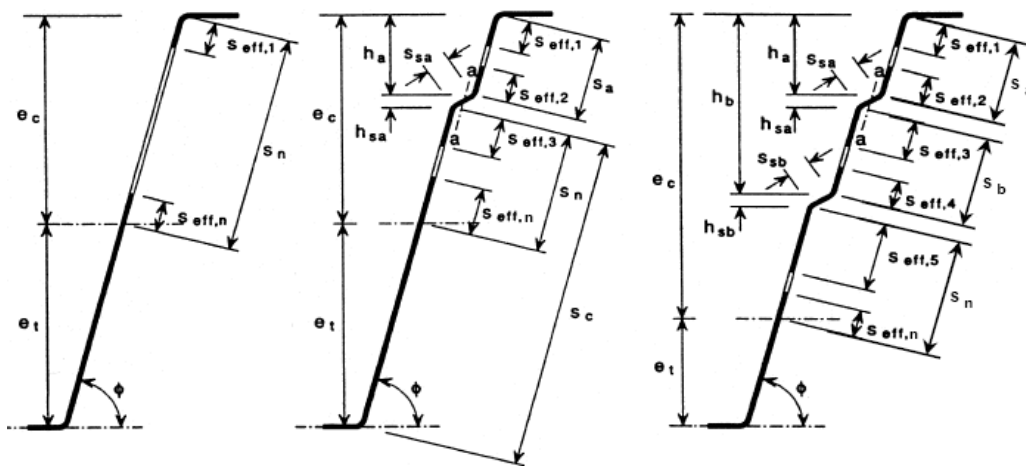


Fig 2.10: Graphic that shows the effective web depending the number of stiffeners. Extracted from [8].

3) Combination in case there are web and flange stiffeners (what's the normal situation)

Once the two critical tensions of the stiffeners on flanges and webs are obtained, a modified critical stress is then “created”, which will substitute the critical stress of the stiffeners.

$$\sigma_{cr,mod} = \frac{\sigma_{cr,s}}{\sqrt[4]{1 + \left[\beta_s \frac{\sigma_{cr,s}}{\sigma_{cr,sa}} \right]^4}} \quad (2.22)$$

4) Calculation of the reduced sections

Now either with the critical stress of the flange stiffeners, the webs stiffeners or the combination of both, the modified section can be calculated . First calculating the parameter χ_d from the flange and/or web, and after the net area of the stiffeners as:

$$A_s = \chi_d \cdot A_s \cdot \frac{f_{yb} / \gamma_{MO}}{\sigma_{com,ser}} \leq A_s \quad (2.23)$$

Once all the reduced section has been calculated; the following information from the metal sheet is available. (Assumed that with the previous schema one is able to define the following properties)

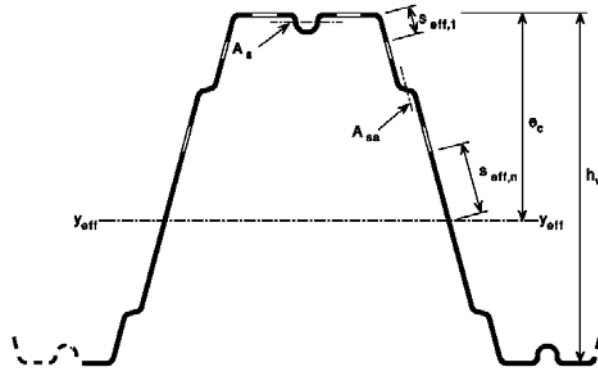


Fig 2.11: Effective section of a metal sheet in bending. Extracted from [8].

A	Area of the section
A_{eff}	Effective area of the section
---	Weight of the section
I_{eff}	Second moment of area
e_c	Shift of centroidal axis

Table 2.2 Mechanical parameters of the metal sheet

3.2. Ultimate limit state

3.2.1. Elastic calculation

The metal sheets can just bear efforts on the direction of the ribs, because the strength on the perpendicular direction is much inferior. The assumption can be made: the metal sheet behaves as an elastic beam.

This assumption is not real, because the distribution of tensions in the section is not completely lineal, but it is a feasible approximation and the results of the calculations lay on the safe side.

Next points show how to calculate the following parameters: The maximum moment ($M_{c,Rd}$), the maximum shear resistance ($V_{d,Rd}$) and the maximum reaction on the support ($R_{w,Rd}$):

Maximum Elastic Moment

Yielding at the compressed flange

The point of maximum compressive tension is in the upper flange then the maximum moment is:

$$M_{c,Rd} = \frac{f_{ya} \cdot I_{eff}}{\gamma_{M0} \cdot e_c} \quad (2.24)$$

Yielding at the tension flange

In this case the plastic reserves of the metal sheet can be used, as there is no buckling of the metal sheet in the tension zone, until the maximum compressed effort is reached.

Shear strength

This shear resistance should be calculated as:

$$V_{b,Rd} = \frac{\frac{h_w}{\sin \phi} \cdot t \cdot f_v}{\gamma_{M0}} \quad (2.25)$$

This shear resistance is taking into account: the shear buckling stress of the webs and the type of support that the sheet has with respect to the support.

f_{bv} is shear the strength considering buckling according to table 2.3 this buckling stress is a reduction of the f_{yb} depending on the slenderness λ_w

Relative web slenderness	Web without stiffening at the support	Web with stiffening at the support ¹⁾
$\bar{\lambda}_w \geq 0,83$	$0,58 f_{yb}$	$0,58 f_{yb}$
$0,83 < \bar{\lambda}_w < 1,40$	$0,48 f_{yb} / \bar{\lambda}_w$	$0,48 f_{yb} / \bar{\lambda}_w$
$\bar{\lambda}_w \geq 1,40$	$0,67 f_{yb} / \bar{\lambda}_w^2$	$0,48 f_{yb} / \bar{\lambda}_w$

¹⁾ Stiffening at the support, such as cleats, arranged to prevent distortion of the web and designed to resist the support reaction.

Table 2.3 Shear buckling stress f_{bv} . Extracted from [8]

λ_w is function of the longitude of the stiffened web, the thickness of the web and the number and value of the stiffeners on the web.

$$\bar{\lambda}_w = 0.346 \frac{s_w}{t} \sqrt{\frac{f_{yb}}{E}} \quad \text{for webs without longitudinal stiffeners (2.26)}$$

$$\bar{\lambda}_w = 0.346 \frac{s_d}{t} \sqrt{\frac{5.34 f_{yb}}{k_\tau E}} \geq 0.346 \frac{s_w}{t} \sqrt{\frac{f_{yb}}{E}} \quad \text{for webs with longitudinal stiffeners (2.28)}$$

$$\text{with } k_\tau = 5.34 + \frac{2.1}{t} \left(\frac{\sum I_s}{s_d} \right)^{1/3} \quad (2.29)$$

h_w = web height between the midlines of the flanges

ψ = slope of the web relative to the flanges

I_s = the second moment of area of the individual longitudinal stiffener as defined in 5.5.3.4.3(7)_[8], about the axis a-a as indicated in figure 2.12

s_d = total developed slat height of the web, as in figure 2.12

s_p = slant height of the largest plane element in the web

s_w = slant height of the web, between the midpoints of the corners; these points are the median points of the corners

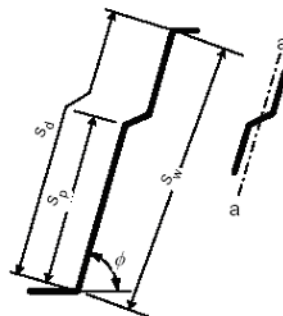


Fig 2.12 Longitudinally stiffened web. Extracted from [8]

Support strength

Crippling is a phenomenon associated with local loading of high intensity perpendicular to the plane of the web. It is more evident in the case of concentrated loading or at intermediate

supports of continuous beams. There is no post-critical strength. (Semi empirical expressions in the codes)

Conditions to be satisfied

$$\begin{aligned} r/t &\leq 10 \\ h_w/t &\leq 200\sin\Phi \\ 45^\circ &\leq \Phi \leq 90^\circ \end{aligned}$$

Where:

h_w = web height between the midlines of the flanges

r = internal radius of the corners

Φ = slope of the web relative to the flanges

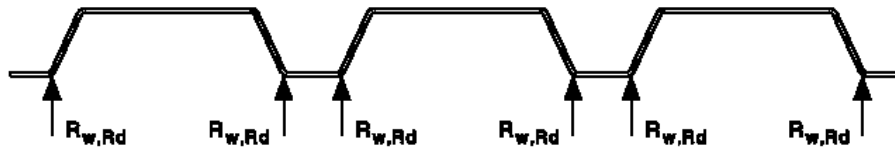


Fig 2.13 Distribution of $R_{w,Rd}$ in the metal sheet. Extracted from [8]

$R_{w,Rd}$ is defined for non stiffened webs as:

$$R_{w,Rd} = \frac{\alpha}{\gamma_{M1}} \cdot t^2 \sqrt{f_{yb} \cdot E} \cdot (1 - 0.1\sqrt{r/t}) \left[0.5 + \sqrt{0.02l_a/t} \right] \cdot (2.4 + (\phi/90)^2) \quad (2.30)$$

l_a is the effective bearing length for the relevant category

Category 1	$l_a=10$ mm
Category 2	$l_a=s_s$ mm if $\beta_v \leq 0.2$ $l_a=10$ mm if $\beta_v \geq 0.3$ Interpolate if $0.2 < \beta_v < 0.3$

With $\beta_v = \frac{|V_{Ed,1}| - |V_{Ed,2}|}{|V_{Ed,1}| + |V_{Ed,2}|} \quad (2.31)$

α is the coefficient for the relevant category

Category 1	$\alpha=0.075$
Category 2	$\alpha=0.15$

This category mentioned depends on the position of the load according to the following table:

	Category 1 - local load applied with $e \leq 1,5 h_w$ clear from the nearest support;
	Category 1 - local load applied with $c \leq 1,5 h_w$ clear from a free end;
	Category 1 - reaction at end support with $c \leq 1,5 h_w$ clear from a free end.
	Category 2 - local load applied with $e > 1,5 h_w$ clear from the nearest support;
	Category 2 - local load applied with $c > 1,5 h_w$ clear from a free end;
	Category 2 - reaction at end support with $c > 1,5 h_w$ clear from a free end.
	Category 2 - reaction at internal support.

Table 2.4 Local loads and supports. Extracted from [8]

A parameter $\kappa_{a,s}$ is defined in case there are stiffened webs. This value multiplies the previous value obtained, and it is defined as follows:

$$\kappa_{a,s} = 1.45 - 0.05 \frac{e_{\max}}{t} \leq 0.95 + 35000 \frac{t^2 \cdot e_{\min}}{b_d^2 \cdot s_p} \quad (2.32)$$

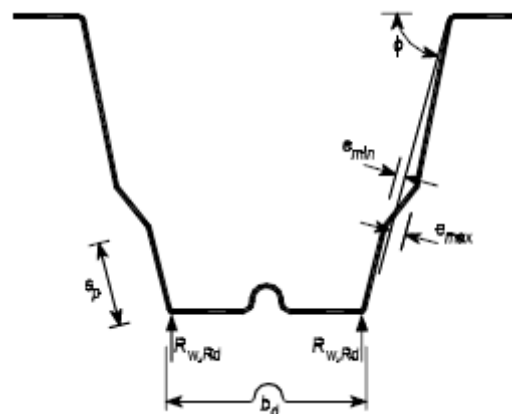


Fig 2.14 stiffened webs. Extracted from [8]

3.2.2. Plastic calculation

It was seen, on the big amount of tests done for the approvals, that after the maximum elastic moment was reached in the middle support, a plastic hinge was formed (due to the yield of the tensioned part), the moment in the support was reduced to a smaller moment (residual moment) and that redistribution of moments was established in the beam.

This is because in the thin walled structures the graphic of force tension is as follows:

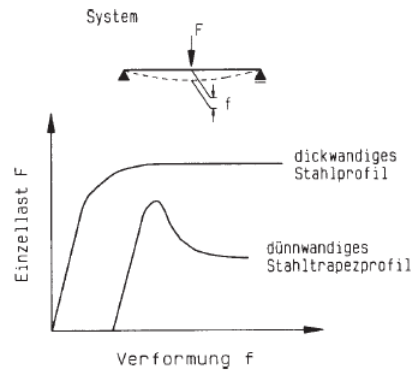


Fig 2.15 Deformation of a metal sheet with increasing load. Extracted from [16]

The experiments demonstrated that this resilient moment was dependant on the length of the metal sheet and following the schema in figure 2.16. There is still no formulation to calculate this minimum lengths theoretically, and the results are provided in the approvals of the metal That means that in case a calculation wants to be made when there is no data available, has to be considered that the rest moment left on the support is 0 or a elastic calculation of the profile.

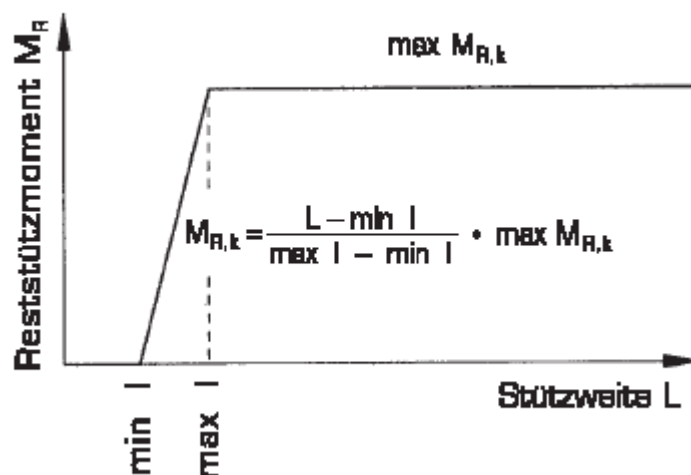


Fig 2.16: Resilient moment depending on the length of the span of the metal sheet. Extracted from [16]

Either if we choose plastic or elastic we require the following calculations:

1. Maximum moment in the field
2. Maximum moment in the support
3. Reaction in the support
4. Reaction in the end support
5. Combination of moment and reaction in the middle support

In our case as the length between purlins is going to be small, this effect will not bring us special advantages, and is not considered in the following chapters, as the span and profiles used, will be not limited by this bending condition.

Combined bending moment and local load or support at reaction

Cross sections subject to the combined action of a bending moment M_{Ed} and a transverse force due to a local load or support reaction F_{Ed} should satisfy the following:

$$M_{Ed} / M_{c,Rd} \leq 1 \quad (2.33)$$

$$F_{Ed} / R_{w,Rd} \leq 1 \quad (2.34)$$

$$\frac{M_{Ed}}{M_{c,Rd}} + \frac{F_{Ed}}{R_{w,Rd}} \leq 1.25 \quad (2.35)$$

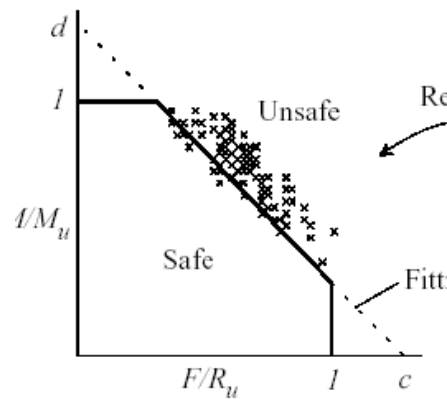
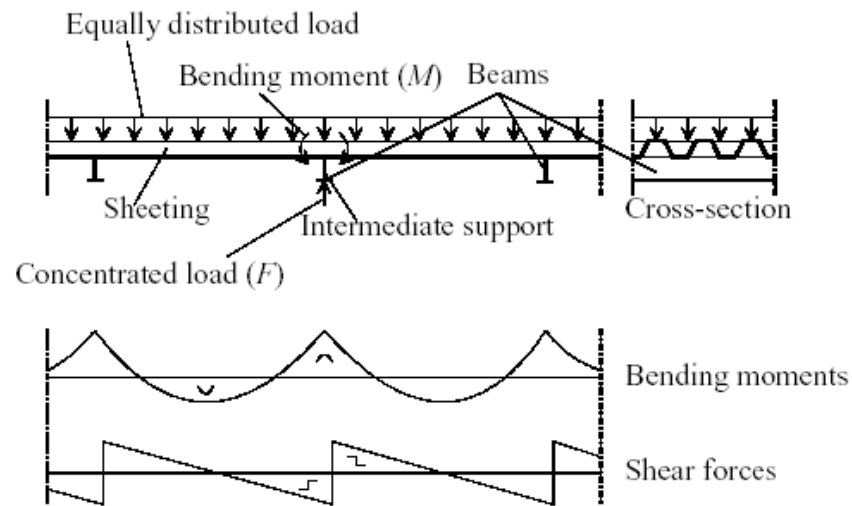


Fig 2.17. Graphic that shows the relation of acceptable combinations bending and reaction. Extracted from [18]



Figs 2.18: Usual bending load distribution of a metal sheet. Extracted from: [18]

3.3. Serviceability limit state

Eurocode proposes to calculate the inertia to calculate the deformation as an interpolation of the effective inertia and the real inertia depending on the stress in serviceability stress

$$I_{fic} = I_{gr} - \frac{\sigma_{gr}}{\sigma} (I_{gr} - I(\sigma)_{eff}) \quad (2.36)$$