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Nomenclature

\[ \sigma \quad \text{Stress} \]
\[ \sigma_y \quad \text{Yield stress} \]
\[ \varepsilon \quad \text{Total strain} \]
\[ \varepsilon_e \quad \text{Elastic strain} \]
\[ \varepsilon_p \quad \text{Plastic strain} \]
\[ \varepsilon' \quad \text{Plastic strain rate} \]
\[ D_{int} \quad \text{Plastic dissipation per unit of volume} \]
\[ M_p \quad \text{Plastic moment} \]
\[ h \quad \text{height} \]
\[ b \quad \text{width} \]
\[ l^e \quad \text{Element length} \]
\[ \varepsilon_p \quad \text{Plastic extension} \]
\[ D_{int} \quad \text{Plastic dissipation} \]
\[ \theta \quad \text{Rotation} \]
\[ W_{ext} \quad \text{External work} \]
\[ \dot{W}_{ext} \quad \text{Unitary external work (per unit of load multiplier)} \]
\[ \mathbf{b} \quad \text{Volume loads vector} \]
\[ \mathbf{t} \quad \text{Surface loads vector} \]
\[ \mathbf{F} \quad \text{Point loads vector} \]
\[ \mathbf{v} \quad \text{Nodal displacements vector} \]
\[ \mathbf{f} \quad \text{External loads vector} \]
\[ \hat{f} \quad \text{Unitary external loads vector} \]
\[ \mu \quad \text{Load multiplier} \]
\[ \mathbf{d} \quad \text{Deformation tensor} \]
\[ \varepsilon(\mathbf{d}) \quad \text{Equivalent strain rate given by Lubliner (1990)} \]
\[ \mathbf{d}' \quad \text{Deformation deviator tensor} \]
\[ \mathbf{u} \quad \text{Collapse mechanism} \]
\[ X \quad \text{Space of motions compatible with boundary conditions} \]
\[ \hat{X} \quad \text{Reduced space of } X \]
\[ X_H \quad \text{Solution space when we consider a mesh the body studied} \]
\[ \hat{X}_H \quad \text{Reduced space of } X_H \]
\[ X_h \quad \text{Reference mesh} \]
\[ \hat{X}_h \quad \text{Reduced space of } X_h \]
\[ \hat{X}_h^* \quad \text{Broken space} \]
\[ \hat{X}_h^* \quad \text{Reduced spate of } \hat{X}_h \]
\[ \mathbf{u}_h \quad \text{Collapse mechanism in the reduce space } \hat{X}_h \]
\[ \mathbf{q} \quad \text{Edge forces vector} \]
\[ \mathbf{p}_H \quad \text{Particular choice of } \mathbf{q} \]
\[ Z_h^* \quad \text{Reduced space of one macroelement} \]
Collapse mechanism of one reduced space of one macroelement

g  Gap

g^e  Gap of one element

η^e  Contributions of one element to the gap

v(x)  Displacement normal to the bar

u(x)  Displacement longitudinal to the bar

(vᵢ, θᵢ)  Degrees of freedom of node i

Nᵢ  Shape function i

N  Shape functions vector

B  Second derivation respect x of N

F_e  External nodal loads

k^e  Stiffness matrix of one element

v^e  Nodal degrees of freedom of one element

K  Stiffness matrix of the structure

Cᵢ  Gauss-Legendre factors

ξᵢ  Gauss-Legendre points

t  Tolerance

t_v  Tolerance respect v

t_u  Tolerance respect u

e_v  Relative error related to v

e_u  Relative error related to u

T  Internal loads vector

u_r  Rigid body displacement vector

(uᵢ, vᵢ, θᵢ)  Degrees of freedom when we consider contribution of normal forces

N_u  Shape functions vector associated with u

N_v  Shape functions vector associated with v

B_u  Second derivation respect x of N_u

B_v  Second derivation respect x of N_v

yᵢ  Coordinate integration point

eᵢ  Plastic extension for y = yᵢ