The evaluation of upper and lower bounds of the plastic limit state o frame structures using the upper bound theorem

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Summary

Traditionally, elasticity has been the constitutive model used in the design of structures. Elasticity has the advantage that it is easy to apply. The stress is proportional to the strains. But such a simple model cannot model reality at the limit state. When the structure is collapsing there are materials, mild steel for example, that allow the structure to resist bigger loads than the ones an elastic analysis allows. This brings us to consider another constitutive model in order to optimize the material in the structure: plasticity.

In plasticity, it is considered that the structure is capable to resist loads and strains after the limit elastic. The structure has bigger strains but it does not collapse at the limit elastic. In this way, the same material can stand bigger loads. This bring us to point out that a plastic analysis has to be combined with an analysis of stability. We have to ensure that the structure collapses under plasticity.

The aim of the thesis is to find the ultimate load of a frame structure under the assumption of perfect rigid plasticity. This is known as the limit state of the structure and forms the basis of modern design procedures and codes of practice. Nowadays, all modern codes of design of structures are based on limit state analysis. Two well known theorems govern this type of analysis, namely the lower bound theorem, for which the problem variables are generalized stresses and the upper bound theorem, for which the unknowns are displacements and permits a similar treatment than traditional elastic analysis.

We will use the upper bound theorem combined with a relaxation technique to obtain both an upper and lower bound of the limit load as well as a positive bound gap per structural element which can direct a refinement algorithm.

It is very important to know when the method which is presented in this paper can be used. This issue is discussed in chapter one. Our aim is to optimize a certain structure. But we have to make sure that the structure verifies the assumptions we are doing.

Chapter two introduces the most important concepts in plasticity. It is important to brush up these concepts for a better understanding of the method that is proposed in future chapters. We start describing the uniaxial stress-strain relations. It is explained the relations between the stress and strain for different kind of materials with different plastic behaviors. The concept of plastic hinge is introduced. The use of the concept of plastic hinge allow us to simplify all the math in this thesis. Finally, we focus on the limit analysis theory. The limit analysis allow us to find the collapse load without referring to the history of the response. This theory is based on the postulate of maximum plastic dissipation, which give us a way to reach the two principal theorems of limit analysis: the upper bound theorem and the lower bound theorem. These theorems are the base of our work.

This work has been structured in a way that allow the reader to understand better the method that is proposed. This method has been developed by Javier Bonet, in chapter three it is presented for the case of a plane stress state. This allow us to give a more general idea of what we do and why we can do it. Chapter four is a particularization of the case explained in chapter three. The method described in chapter three is applied to a continuous beam. In this case it is only considered the contribution of pure plastic bending. In chapter five and six we focus in a more general case. The method is applied to a frame structure. In this case we take into account the combination of plastic bending and tension (or compression).

The aim of this paper is presented in chapters four, five and six. All the theory explained in this chapters is supported by examples.