TITLE: **ESTIMATION OF FEKETE'S POINTS ON THE UNIT SPHERE**.

AUTHOR: **JOSE MANUEL GESTO BEIROA**.

ADVISORS: **ENRIQUE BENDITO**, **ANDRÉS M. ENCINAS** & **ÁNGELES CARMONA**.

ABSTRACT: this dissertation focuses in the mathematical problem of distributing well a known amount of points over the surface of a sphere. This problem admits a multitude of variants. We will study three of them: arranging n electric punctual charges, equals and of the same sign, on the surface of the sphere so that they are in electrostatic equilibrium; arranging n punctual particles on the surface of the sphere so that it is maximum the product of all its Euclidean distances and arranging n punctual particles on the surface of the sphere so that it is maximum the sum of all its Euclidean distances. The first of these problems is known in the literature as *the problem of J.J. Thomson* and the second as *the problem of Fekete*. The distributions of points that resolve this last problem are called *points of Fekete of order n on the sphere*. It is also frequent to find that those three problems and other similar ones are included under the common name of *problem of Fekete's points*. On the basis of the principles of the Theory of the Potential it is possible to express the three previous problems in terms of minimizing a certain potential energy function, whose expression varies according to the case. The dissertation proposes a new numerical algorithm for locating minimums of these energy functions, restricted to the unit sphere S^2. The algorithm is based on the laws of the Newtonian mechanics and the differential geometry, and it eludes the direct problem of optimization - tremendously hard from the computational point of view - by searching stable static equilibrium positions on the basis of purely dynamic considerations. The known principles of energy conservation that derive from the laws of the classical mechanics guarantee that these stable static positions of equilibrium coincide with local minimums of the corresponding potential energy function. In physical terms, it can be said that the proposed algorithm numerically reproduces the quasi-static path that would follow each particle of a system of n, interacting one another according to forces that can be calculated differentiating a potential function, if these particles were to remain always on S^2 and the medium, that hosts them, would offer a viscous resistance to their movement, which is proportional in module to their velocity vectors and of opposite direction, being infinite the constant of proportionality. In spite of the apparently intricate approach of the algorithm, programming it turns out to be extremely simple, being its computational cost quite low. The algorithm performs $O(n^2)$ operations by iteration and it only needs $3(n-1)$ positions on the memory. Moreover, the algorithm is parallelizable, so that the time of calculation can be drastically reduced if it is possible to dispose of a certain number of computers, all of them connected to and conveniently coordinated by a central computer. Nevertheless, all of the results presented in this dissertation have been obtained with only one personal computer with a processor PENTIUM IV of 2.53GHz. The results provided by the proposed algorithm are compared with the ones obtained by other authors. In particular, the results are compared to the distributions of points given by the vertexes of the *Platonic* and *Archimedean polyhedrons*, to the vertexes of the *geodesic domes*, to the *spiral points* - that are distributions proposed by the group of Saff, Rakhmanov and Zhou as good solutions for the previous problems –, to the solutions calculated by Zhou for values of n between 2 and 200 with the minimization algorithm BFGS and to certain functions extrapolated by Zhou from the potential energy values obtained with n between 2 and 200. As a final conclusion it can be said that, as far as we know, the proposed method can provide good solutions for the three problems mentioned, for values of n much greater than the ones considered up till now and with a reasonable calculating time. In this dissertation, it has been achieved a good distribution for the problem of maximization of the sum distances with a size of 507002, calculated with the previously mentioned processor in approximately five hours. Moreover, the solutions given by the proposed method are always better than the spiral points and the geodesic domes, and for values of n greater that 500 those solutions improve invariably the extrapolation carried out by Zhou for the problems of Thomson and Fekete and they adjust extraordinarily well to the extrapolation that corresponds to the problem of the sum of distances.