CHAPTER 5
NON-LINEAR FEM MODELLING

5.1. INTRODUCTION

The scope of the non-linear finite element analysis has been to model the shear direct loading through push-off tests, subjected to different boundary conditions. That is to model the experiments carried out whose results have been presented in chapter 3. This analysis has been carried out using the Non-Linear module of the software DIANA.

5.2. METHODOLOGY USED: DISCRETE CRACKING

In push-off tests on SFRC, a discrete crack is formed when it is being loaded and this load exceeds produces a tensile stress greater than the tensile strength of concrete. As it has been mentioned in point 2.6, exists a certain controversy in concrete the use of a discrete crack model or an smeared one. However, the fracture occasioned between the edges of the two notches in push-off tests can be considered as a geometric discontinuity, so seems to be most conveniently the modelling trough a discrete crack. With purpose, the crack is modelled using structural interface elements. These elements relate the forces acting on the interface to the relative displacements of the two sides of the interface. Moreover, the using of this methodology seems to represent better the physical phenomenon than the smeared modelling.

The discretization of the geometry has been done dividing the specimen in two zones, a linear elastic one and a non-linear one. The linear elastic zone has
been modelled using three-node triangular and four-node quadrangular isoparametric plane stress elements. The use of the triangular elements responds to the needed of a refinement of the mesh as we are approaching the crack. In the rest of the specimen, four-nodes quadrangular elements have been used. The crack has been modelled by the using of interface elements, specifically, an interface element between two lines in a two-dimensional configuration. Figure 5.1 shows the finite element mesh.

![Finite element mesh](image)

**Figure 5.1:** Finite element mesh used in this master thesis.

The integration schemes have been 1 point either in three-node triangular or four-node quadrangular isoparametric plane stress elements, through a Gauss integration. In interface elements a three point Lobatto integration scheme has been used so that we have the results of the integration in the nodes of the mesh (there is no interpolation of the results in the interface elements).

As it has been mentioned previously, we cannot consider shear behaviour as an uncoupled one from tensile behaviour. This is the reason why a crack dilatancy model has been used: with this model mode-I and mode-II are coupled in order to describe the development stage of the macro-crack.

The characteristics of the materials used in 1.0% SFRC modelling (linear elastic and non-linear) are shown in Table 5.1.
In the above material 1 is linear elastic and material 2 corresponds to the interface elements. Note that forces are expressed in [N], length in [mm] and mass in [kg]. DSTIF is the stiffness until the material remains elastic. DILVAL command involves the cube compressive strength, the tensile strength and maximum aggregate size. MODE1 indicates a linear tension softening criterion in the development stage of the crack and MO1VAL is the fracture energy. Lastly, value of 1 in MODE2 command indicates constant shear modulus after cracking and MO2VAL is the shear modulus in the development stage of the crack. Note that the values of $E$, $f_t$ and $G_F$ are the values obtained from the inverse analysis in 1_pilot_1 test specimen, while the value of the cube compressive strength has been obtained as a result of the compressive test of the cylinders cast.

Linear tension softening has been used, so only $f_t$ and $G_F$ are required to defined it completely. Even though a bi-linear stress-crack opening relationship has been used in this thesis, using a linear one does not introduce so much error as it can be seen in Figure 5.2:

**Figure 5.2:** Comparison between the bi-linear stress crack opening relationship obtained from the inverse analysis and the linear stress crack-opening relationship used in the modelling.
In order to be as realistic as possible, loading has been carried out through displacement control. Nodes located under the loading bar, have been loaded through displacement control. Loading has been done through vertical displacements steps of 0.0225 mm.

Parameter CRDILA indicates the dilatancy curve used for the model. In this analysis a Rough Crack Model defined by Bazant and Gambarova has been used [14]. It seems to be the more suitable for modelling a push-off test. However, we must take into account that all dilatancy models supplied by DIANA, are a result of an empirical work with plain concrete, so, they only consider that shear stress is transferred across the interface by crack surface asperities. Moreover, they consider that there is an increase of the compressive stress when the model is activated, and it does not start when the maximum tensile stress produced equals the tensile strength of the material. The crack dilatancy model is activated when the crack is considered as open, it means, when its normal relative displacement $\Delta u_n$ has become greater than the ultimate magnitude of the normal relative displacement $\Delta u_{n,ult}$ of a softening model. And it is here when doubts arise about the use of a crack dilatancy model. As it has been mentioned in previous considerations about fracture energy, there is a big difference between plain concrete and SFRC in terms of an extraordinary increase in the ductility of the concrete, that allows it to have traction strains over 1500% greater in SFRC than in plain concrete. This fact produces that, as the model is thought to plain concrete and the fracture energy given corresponds to SFRC, the crack dilatancy model is never activated. That is the reason why, as it will be seen in point 5.3.3. there is no softening curve in shear stress vs. vertical and horizontal displacement, but there is a hardening curve.

5.3. RESULTS

Push-off tests modelling can be resumed analysing the shear and the compressive stresses in the interface and comparing the shear stress in the interface vs. displacements in the specimen.

5.3.1. Shear Stresses

The shear distribution between the edges of the two notches of the specimen follow a curve like it is shown in Figures 5.3 and 5.4. Note that in Figure 5.3, it is plot the shear stress distribution from the top of the interface to the middle of it, while Figure 4.4 shows the rest of the distribution, it means, from the middle to the bottom notch.
Due to the impossibility of plotting the shear stress distribution along the interface elements, expressed in global coordinates, plots above show the shear stress distribution along the contiguous plane-stress elements. The value of the shear stress in each element corresponds to the average of the element.

Figure 5.5 shows the shear stresses in the specimen for a load step of 0.209 mm. It cannot be noticed from the plot but the specimen has not cracked yet.
Figure 5.5: Shear stress in push-off specimen expressed in MPa.
Load step of 0.209 mm.

Figure 5.6 shows the shear stresses in the specimen for a load step of 0.254 mm. At this step the specimen is already cracked as it will be seen in next point. Note the increase shear stress in the interface from Figure 5.5.

Figure 5.6: Shear stress in push-off specimen expressed in MPa.
Load step of 0.254 mm.

Lastly, an advanced loading stage is shown in Figure 5.7, in which the load step is 3.44 mm.
In this figure cracking is clearly visible. Note that specimen rotates around z axis located in the middle of the interface instead of sliding along the discontinuity. This fact will be commented in-depth in point 5.3.2.

5.3.2. Compressive and Tensile Stresses

Before sliding, tensile strength must be reached in the interface. After the specimen cracks at the tensile strength, the faces of the crack may slide, one with respect the other. In this point we see how the tensile strength is reached in the centre of the discontinuity. This fact does not agree with test experiments [1] in which the crack starts to grow and increases in width from the top of the interface. Figure 5.8 and 5.9 show the tensile stress distribution along the interface and the $\sigma_{xx}$ in the specimen respectively for a load step of 0.209.
Figure 5.8: Tensile stress (in MPa) distribution along the interface.
Load step of 0.209 mm.

Note in Figure 5.8 that the maximum shear stress take a value of 3.71 MPa, which is smaller than the tensile strength of the material, that implies, as it was said in point 5.3.1, that the specimen has not cracked yet.

As it can be seen from Figure 5.8, there are two zones subjected to compressive stresses located at both extremes. These compressions appear because in terms of horizontal stresses, the push off tests respond as the Brazilian splitting-tension test, in which there is a small compressed zone at
each end of the interface due to the load action. In fact we can consider that in the push-off test also, the load acts over the interface.

5.3.3. Shear Stress-Displacements Plots

A final use of the Non-Linear finite element model is to plot the representative curves of push-off tests (see Chapter 3), it means, shear stress vs. vertical and horizontal displacements. Vertical vs. horizontal displacement may be plotted as well. In all cases, shear stress has been calculated as the applied load divided by area defined by the edges of the two notches. This is the way that experimental curve have been plotted in Chapter 3.

Figure 5.10 shows shear stress vs. vertical and horizontal displacement in a push-off specimen. Although the curves seem to be linear, there is a soft inflexion in both curves at shear stress level of 5.54 MPa. At this stress level, the matrix cracks, so concrete starts to behave non-linearly. This fact may be seen clearly in Figure 5.11, whose axis has been reduced.

![Graph showing shear stress vs. vertical and horizontal displacement](image)

Figure 5.10: Shear stress vs. vertical and horizontal displacement.
These curves may be compared with the experimental ones and the results are represented in Figures 5.12 and 5.13 respectively. Note that in both figures the maximum shear stress plotted is 12 MPa.
Figure 5.13: Shear stress vs. horizontal displacement. Comparison between modelling and experimental results.

Note that the model curve fits quite well to experimental curve in the elastic phase. However, in pre-peak non-linearity, the differences increase following the reasoning discussed in section 5.3.2.

The hardening curve produced can be explained through Figure 5.14 in which the vertical displacement is plotted vs. the horizontal one. As can be seen from the figure, for a certain vertical displacement, the horizontal displacement is much smaller in the model than the experiments, so the shear stress transmitted across the crack is much higher in the model. Note that, as has been said before, this only occurs when the elastic regime has been crossed (see Figure 5.13 in which horizontal axis has been reduced to 0.030 mm).

Figure 5.14: Vertical vs. horizontal displacement.
Figure 5.15: Vertical vs. horizontal displacements.