5. Dynamic analyses

Continuously moving landslides require a “dynamic” analysis instead of a classical “static” approach. Creep phenomena are not the usual stability problems of geotechnical analysis approach. The considered slopes are neither still nor ruptured: they move.

In the classical approach, soil strength is regarded as a unique resisting force. Movement may be initiated when equilibrium between driving and resisting forces is modified by a pore water pressure increase. Limit equilibrium stability analysis should only be applied to determine the safety factor of a slope in the phase immediately preceding the movement while a different kind of analysis should be used in the successive phase (Angeli, Gasparetto, Menotti et al., 1995).

5.1. General model including viscosity

In order to solve the general equations to equilibrium of forces for section in infinite slope it is necessary to consider the earth pressure forces $E$ and $E'$ equal but opposite in direction: in this way, they couldn’t appear in equilibrium of vertical section considered.

First of all you must find normal stress $\sigma$, destabilizing shear stress $\tau$ at the base of section considered in function of weight $W$ and slope angle $\alpha$:

$$\tau = W \cdot \text{sen} \alpha = \gamma \cdot l \cdot \text{sen} \alpha \quad (1)$$

$$\sigma = W \cdot \cos \alpha = \gamma \cdot l \cdot \cos^2 \alpha \quad (2)$$

in which $\gamma$ is the specific weight of the sliding mass and $l$ the depth of sliding mass from the slip surface to the ground. The stability or instability of the slope will depend on shear stress $\tau$ along slip surface. If this shear stress $\tau$ is under a “fixed value”, slope is in stability.
conditions; reverse if $\tau$ is above “fixed value”, slope is in instability conditions and in this case it would occur movement.

In order to obtain this confront it is necessary found the “fixed value” $\tau_0$ given from Mohr-Coulomb expression in effective stress:

$$\tau_0 = c' + \sigma' \tan \phi'$$

(3)

in which $c'$ is cohesion value, $\phi'$ the friction angle and $\sigma'$ effective stress obtained from:

$$\sigma' = \sigma - p_w$$

(4)

with normal stress $\sigma$ and pore water pressure $p_w$ at the base of the section.

It seems clear that pore water pressure has a big influence on stability of slope when $\tau$ is fixed without changing with course of time instead $\tau_0$ is depending to variations of pore water pressure in time. An increment of water table level in the slope and consequent growing of pore water pressure causes a decrease in $\tau_0$ and a more possibility for slope instability.

In order to establish if slope is stable or instable it is used a safety factor $F$ given from:

$$F = \frac{\tau_0}{\tau}$$

(5)

in which $\tau_0$ is destabilizing force in fact if $\tau_0$ decreases the slope is unstable; $\tau$ is resisting force depending from weight $W$ and then constant.

$$p_w \uparrow \iff \sigma' \downarrow \iff \tau_0 \downarrow \iff F \downarrow$$

Security factor $F$ defines three different situations for the slope:

$F > 1$, slope is stable and there aren’t movements;

$F = 1$, slope is in limit condition to stability;

$F < 1$, slope is unstable and it is moving.
5.1.1. General equations to visco-plastic model

In the previous paragraph have been defined equations to limit equilibrium method but it is necessary define equations controlling movements when \( F < 1 \).

When safety factor is lower than 1, \( \tau < \tau_0 \) i.e. that resisting forces are lower than destabilizing ones; it means that \( \tau - \tau_0 \neq 0 \), resulting force isn’t equal to 0 and it is the cause of movements. You can write that \( \tau - \tau_0 = ma \) wit \( m \) mass for unitary surface of slope section and \( a \) the acceleration of movement for section considered.

The dynamics of the landslide are governed by the difference between destabilising forces \( (T) \), that depend basically on weight and slope, which are constant, and resisting forces \( (T_r) \), that are sensitive to water pressure at the slip surface.

The momentum equation can be written as:

\[
T - T_r = ma \tag{6}
\]

where \( m \) is the mass and \( a \) the acceleration. In practice, a net force becomes available (difference between driving force and shearing resistance).

For a local point of the landslide where infinite slope conditions apply, resisting forces can be estimated using Mohr-Coulomb criterion, depending on cohesion and friction. Forces are computed over a unit surface, and therefore shear stresses are considered in what follows:

\[
\tau - \left[ c' + (\sigma - p_w) \tan \phi \right] = ma \tag{7}
\]

The groundwater pressure is the only temporal variable in the left hand side of the equation. Therefore this equation predicts a unique value of the acceleration for each value of groundwater pressure. A one to one relationship should also exist between acceleration and position of the groundwater level, if parallel flow is assumed.

This equation is applicable for movement of a block sliding above an tilted slope with constant acceleration (infinite slope hypothesis).

In real cases is quite difficult to have available this condition because in field nothing is constant.

For example, in Vallcebre active landslide movements have been compared whit pore water pressure dates and displacements, velocities and accelerations ones. It can be noted that:

- Displacements and velocities are related at pore water pressure levels: displacements are constant if water table level is constant but if it is not constant, velocities are growing or decreasing;
- Accelerations are so small that can be considered equal to 0.

From these two notes it has been possible to realize the existence of another resisting force that contributed to stop movement in opposition to gravitational forces that instead causes movements. This fact suggests that other terms should be included in the momentum equation. When you have a variation of pore water pressure you have a variation of velocities that produces acceleration; this new force come into play in order to leave equal to 0 this produced acceleration and to obtain velocity values constant. This new force, called viscous force \( F_v \), is the base starting from different viscosity models. All models developed are depending from velocity and from some parameters called viscous parameters.
Introducing this new force, you can obtain general equation of visco-plasticity:

\[ \tau - \tau_0 = ma + F_v \]  

(8)

The corresponding momentum equation, for infinite slope conditions, substituting different terms of \( \tau \) and \( \tau_0 \) becomes:

\[ \gamma \cdot l \cdot \text{sen} \alpha \cdot \cos \alpha - \left[ c' + \left( \gamma \cdot l \cdot \cos^2 \alpha - p_w \right) \cdot \text{tg} \varphi' \right] = m \cdot a + F_v \]  

(9)

In this equation is it possible group some terms to obtain a simplified one:

\[ P = \gamma \cdot l \cdot \text{sen} \alpha \cdot \cos \alpha - \left[ c' + \left( \gamma \cdot l \cdot \cos^2 \alpha \right) \cdot \text{tg} \varphi' \right] \]

\[ Q = \text{tg} \varphi' \]

Substituting in the (9):

\[ P + Q \cdot p_w = m \cdot a + F_v \]  

(10)
5.1.2. Input parameters models

Within the hypothesis of infinite slope geometrical parameters of the slope and mechanical characteristic of soil have been proposed. In geometrical parameters are included \( l \) the depth of sliding mass, \( \alpha \) surface slope angle and mathematical elaborations of the same angle (\( \text{sen} \alpha \) and \( \cos \alpha \)). In mechanical characteristics are proposed cohesion \( c' \), friction angle \( \phi' \) in residual conditions and unit weight \( \gamma \) of the point in which model has been applied.

Finally grouped terms have been calculated as shown before; the last term is the mass \( m \) above shear stress zone as: \( m = \gamma \cdot l \cdot \cos \alpha \).

These parameters are proposed for Orvieto, Fosso San Martino and Vallcebre respectively:

**ORVIETO**

\[
\begin{align*}
&c' = 0 \quad \text{kPa} \\
&\phi' = 11.3 \quad ^\circ \\
&\tan \phi' = 0.19982 \\
&\gamma = 19.6 \quad \text{KN/mc} \\
&l = 20.8 \quad \text{m} \\
&\alpha = 8.5 \quad ^\circ \\
&\text{sen} \alpha = 0.1478 \\
&\cos \alpha = 0.9890
\end{align*}
\]

\[
\begin{align*}
P &= -20,08569 \quad \text{kPa} \\
Q &= 0.19982 - \\
m &= 41.1 \quad \text{t}
\end{align*}
\]

**FOSSO SAN MARTINO**

\[
\begin{align*}
&c' = 0 \quad \text{kPa} \\
&\phi' = 17 \quad ^\circ \\
&\tan \phi' = 0.30573 \\
&\gamma = 20.5 \quad \text{KN/mc} \\
&l = 22 \quad \text{m} \\
&\alpha = 10.2 \quad ^\circ \\
&\text{sen} \alpha = 0.1771 \\
&\cos \alpha = 0.9842
\end{align*}
\]

\[
\begin{align*}
P &= -54,95762 \quad \text{kPa} \\
Q &= 0.30573 - \\
m &= 45.2 \quad \text{t}
\end{align*}
\]

**VALLCEBRE**

\[
\begin{align*}
&c' = 0 \quad \text{kPa} \\
&\phi' = 7.27 \quad ^\circ \\
&\tan \phi' = 0.1275708 \\
&\gamma = 20 \quad \text{KN/mc} \\
&l = 15.5 \quad \text{m} \\
&\alpha = 5.2 \quad ^\circ \\
&\text{sen} \alpha = 0.0903 \\
&\cos \alpha = 0.9959
\end{align*}
\]

\[
\begin{align*}
P &= -11,36145 \quad \text{kPa} \\
Q &= 0.127571 - \\
m &= 31.47 \quad \text{t}
\end{align*}
\]
5.1.3. Viscous models

Subsequently is proposed a viscous model: Bingham’s physic model. This model considers variations of velocity perpendicular at shear zone is:

\[
\frac{dv}{dz} = \frac{\tau - \tau_0}{\eta} \quad (\tau > \tau_0)
\]

in which \(v\) is velocity, \(z\) the thickness of shear zone measured perpendicular to the base, \(\tau\) the resistant shear stress, \(\tau_0\) limit shear stress calculated with “Mohr Coulomb criterion” above which start movements and \(\eta\) dynamic viscosity.

Hence, it is possible express the viscous force like:

\[
F_v = \eta \cdot \frac{dv}{dz}
\]

in which if you can consider a linear variation of velocity respect \(z\), you can have viscous force only in shear zone; so, for this reason viscous force is expressed in finite terms:

\[
F_v = \eta \cdot \frac{v}{z}.
\]

5.2. Data from the field

This study propose three cases of active landslides; for this reason all required data have been obtained in a different way.

**Pore water pressure.** In Orvieto landslides monitoring started in 1982 when a piezometer was installed in borehole OR in the upper half of the slope. This has been borehole considered in this study but at all monitoring locations, Casagrande-type cells were placed in the stiff clay and in the clayey debris. Even though Casagrande-type piezometers have an appreciable hydrodynamic time lag, they were regarded as being effective in measuring very slow pore water pressure changes over several years provided that tubes were accurately sealed and the borehole thoroughly filled with clay material throughout its entire length. Variations in time of piezometric levels were recorded over at least five years. In previous pages some graphs have been presented (Calvello, Cascini, Grimaldi; 2007) in which is expressed the possibility to consider modeled data in SEEP for pore water pressure because of a good correlation with recorded data from piezometers. In this way there are available data of pore water pressure from SEEP of GeoStudio for some boreholes each 7 days. This model will focus on borehole OR in the upper extremely point of the shear zone in next figure marked:

![Figure 83: Blue cross indicate point in Orvieto landslide in which visco plastic model have been applied.](image)
A sector of the left side of San Martino valley has been put under instrumental control with 9 measurement stations. Twelve electropneumatic piezometric cells have been positioned at various depths on the same vertical and in different sites of the slope profile. Piezometric levels varied with rainfall and decreased progressively from 1980 to the end of 1985. In this study has been considered borehole B with cell at 18 m in depth.

In order to correctly interpret the piezometer data, it must be considered that the initial pore pressure readings during the first few months of measurements may have been influenced by the installation procedure of the cells. That could explain the recorded highest pore pressure values of some piezometers at the beginning of the second year of measurements (January, February 1981), rather than when the rate of movements are at a maximum (May 1980).

Calvello, Cascini, Sorbino (2007) proposed a model considering that the real transient water flow in the slope is satisfactorily modeled by SEEP of GeoStudio. It means that data of pore pressures required are available from program in steps of 15 days.

In Vallcebre, pore water pressure data are furnished from depth of freatic level recorded by “data logger” for borehole S2.

Three boreholes were located in the lower unit of the landslide (boreholes S2, S9 and S11), which are considered to be representative of different parts of the whole unit and where infinite slope assumptions could apply. A close relationship between the groundwater level changes and landslide activity was measured using a wire displacement meter at borehole S2. There exists a strong level of synchronism between the two records. In addition, the rate of displacement is strongly correlated with the water table data. The exception to this is the event of January 1997 (increment of velocity without increment of water table), which may be caused by other factors (for example toe erosion by the Vallcebre torrent). In this way, a point along borehole S2 has been considered the better choice. Database available for Vallcebre landslide is the most large proposed because both data of pore water pressure and displacements are transferred each 20 minutes from November 1996 until today.

In detail, pore water pressure \( p_w \) is obtained from (11):

\[
p_w = \gamma_w \cdot (l - h_w)
\]

(11)

in which \( \gamma_w \) (1000 kg/m\(^3\)) is unit weight of water, \( l \) the depth of sliding mass and \( h_w \) is groundwater level measured from piezometer in field.
**Displacements and velocities.** In Orvieto area, displacements monitoring started in 1982 with an inclinometer in borehole OR. Subsequently all area was involved by means of probe inclinometers inserted in aluminium casing. Inclinometer casings are all installed at the same locations as the piezometric boreholes and are embedded in the stiff clay for a length ensuring at least five reading steps (distance between consecutive steps is 0.61m). Measurements are performed with a Digitilt probe: results are available from 01/01/1995 to 29/12/2000 but not constant in time. Next table shows the number of date available each year:

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of displacements dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>5</td>
</tr>
<tr>
<td>1996</td>
<td>7</td>
</tr>
<tr>
<td>1997</td>
<td>10</td>
</tr>
<tr>
<td>1998</td>
<td>7</td>
</tr>
<tr>
<td>1999</td>
<td>6</td>
</tr>
<tr>
<td>2000</td>
<td>7</td>
</tr>
</tbody>
</table>

In this way, day in which any measurement has been taken is identified and it is possible to obtain velocities with finite differences method:

$$v(t_i) = \frac{d_i - d_{i-1}}{t_i - t_{i-1}}$$  \hspace{1cm} (12)

in which $d_i$ indicates a displacement at time $t_i$ and $d_{i-1}$ a previous displacement taken at $t_{i-1}$. It is necessary to obtain velocity at time $v(t_i)$. This procedure allows to derive velocities from measured displacements in field.

In Fosso San Martino, in the same borehole of piezometric cells they have been installed six inclinometers (measurements made by SINCO Digitilt Inclinometer System). In the study, results proposed belonged to borehole B and, as in the previous case, measurements are not constant in time because, as you can see in next table, in years 1982 and 1983, displacements have been monitored only one day:

<table>
<thead>
<tr>
<th>Years</th>
<th>Number of displacements dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>13</td>
</tr>
<tr>
<td>1981</td>
<td>5</td>
</tr>
<tr>
<td>1982</td>
<td>1</td>
</tr>
<tr>
<td>1983</td>
<td>1</td>
</tr>
<tr>
<td>1984</td>
<td>5</td>
</tr>
<tr>
<td>1985</td>
<td>11</td>
</tr>
</tbody>
</table>

As the preceding case, velocities have been obtained with equation (12) hence deriving from data measured in field.

Displacements in Vallcebre area are caught from wire extenziometer and GPS instruments each 20 minutes. In order to obtain horizontal displacements of cable, it is necessary treat
recorded data in field: further information about this physical procedure is found in Asensio 2001. Velocities are obtained from finite differences method (12) starting from displacements.

5.3. Regression

General equation of visco-plastic model with Bingham’s law becomes:

\[ P + Q \cdot p_w = m \cdot a + \eta \cdot \frac{v}{z} \]  \hspace{1cm} (13)

In terms of finite difference previous equation becomes:

\[ P + Q \cdot p_w(t_i) = m \cdot \frac{v_i - v_{i-1}}{t_i - t_{i-1}} + \eta \cdot \frac{v_i}{z} \]  \hspace{1cm} (14)

This equation carries out acceleration in terms of velocity otherwise it could remain a closed form. Previous equation is elaborated in terms of velocity:

\[ v_j(t_i) = \left(\frac{t_i - t_{i-1}}{m + (t_i - t_{i-1}) \cdot \frac{\eta}{z}}\right) \left[ P + p_w(t_i) \cdot Q \right] + m \cdot v_{i-1} \]  \hspace{1cm} (15)

Subsequently these velocities obtained from visco-plastic model will be indicated as \( \hat{v}_i \). These values have to be compared with measured velocities (obtained from field).

A mathematical method that allows to adjust a function to data is regression. So that you can estimate parameters value adjusting modeled results to data measured in field in the best possible way. In this case regression allows to adjust results of visco-plastic equations to data measured in field.

In this way a calibration of equations has been made in function of furnished dates like pore water pressure \( p_w \), measured velocity \( v \), geometrical and mechanical parameters of slope.

As results of this regression you can obtain viscosity parameter. This method proposes the best values of parameters to adjust visco-plastic equations.

In order to obtain a good correlation between data, two methods for regression are proposed: regression of minimum square and correlation parameter.

**Minimum square method** admits to have minimum value of sum of square differences \( E^2 \) between dates and function that represented behavior of model results. From a mathematical point of view, it is necessary to have a series of dates \( \{x_i, y_i, ..., z_i\}, \ i = 0, ..., n \) related by a function \( f \) depending some parameters \( a_j, \ j = 0, ..., m \):

\[ x_i = f(a_0, ..., a_m, y_i, ..., z_i) \]

In order to obtain the best correlation between data it is necessary to define:

\[ E^2 = \sum (x_i - \hat{x}_i)^2 = \sum \left\{ x_i - f(a_0, ..., a_m, y_i, ..., z_i) \right\}^2 \hspace{1cm} i = 0, ..., n \]
in which $x_i$ are given data from field and $\hat{x}_i$ results estimated from the function corresponding to this dates.

The minimum value has been calculated as:

$$\frac{\partial E^2}{\partial a_j} = 0 \quad j = 0, \ldots, m$$

Deriving this error $E^2$ you find minimum possible error especially since $E^2$ is trending to 0. It means that distances between visco-plastic model and data in field are so small.

It is necessary to solve a system with $m$ equations and $m$ unknown to find parameters $a_j$.

**Correlation coefficient, $r^2$** furnish a value representing goodness of arrangement between dates, $x_i$ and results of function $f, x_i$. The expression used to calculate this factor is the most simple and common:

$$r^2 = 1 - \frac{\sigma_{x,x_i}^2}{\sigma^2}$$

in which $\sigma^2$ is variance of dates $x_i$,

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}, \quad \bar{x} = \frac{\sum x_i}{n} \quad i = 0, \ldots, n$$

and $\sigma_{x,x_i}^2$ is typical fault of square estimate method that is related to $E^2$ factor:

$$\sigma_{x,x_i}^2 = \frac{\sum (x_i - \hat{x}_i)^2}{n} \quad i = 0, \ldots, n$$

in which $x_i$ are data, $\bar{x}_i$ is average and $\hat{x}_i$ values estimates from function $f$. It is possible to found a link between $r^2$ and $E^2$ in fact to have a good correlation it is necessary minimizing $r^2$ like you do with $E^2$. Using option Solver of Microsoft Excel 2007, parameters necessary to correlation have been found.
5.4. Results of visco-plastic parameters

Theoretical bases about visco-plastic behavior have been proposed and an analysis of required data have been conducted. In this paragraph you suggest results of this model applied at the three area of analysis. In particular graphs to compare modeled and measured velocities and/or displacements and the final value of viscosity (unknown value $\eta/z$) are proposed. In order to obtain them, the methodology pursuit is summarized herein:

1. Velocity measurements and pore water pressure are available data. The first step deals with time because velocity values have been recorded in some days but pore pressure are accessible in different time (pore water pressure is modeled from SEEP of GeoStudio every 15 days). For this reason, it is necessary to operate a linearization: within the hypothesis that pore water pressure are linear in time, if you have a value in $t_1$ and in $t_2$ of pressure ($P_1$ and $P_2$) you could calculate pressure $P_m$ in the same time $t_m$ in which is available velocity as shown:

\[
(t_m - t_1) : (t_2 - t_1) = (P_m - P_1) : (P_2 - P_1)
\]

\[
P_m - P_1 = \frac{(P_2 - P_1) \cdot (t_m - t_1)}{(t_2 - t_1)}
\]

\[
P_m = P_1 + \frac{(P_2 - P_1) \cdot (t_m - t_1)}{(t_2 - t_1)}
\]

In this way, monitored velocity and pressure are in the same date: the first one will be used to make comparison and pressure will be inserted in visco-plastic model.

2. Pore water pressure, geometrical parameters and mechanical characteristics are used in the model in order to obtain a new value of velocity from model; in equation (15) of visco-plastic model is involved a first value (first attempt value) of the unknown parameter of viscosity $\eta/z$. 

3. Regression of minimum square is carried out with velocities obtained from model and velocities monitored: in this way calibration of model is performed because the exact value of viscosity $\eta/z$ is extracted. This value of viscosity obtained is constant in time. The best adjustment forecast lower correlation coefficient.

4. This accurate value is inducted in model and for all dates of pore water pressure modeling you obtain the corresponding velocity. Displacements from visco-plastic model could be calculated from velocity: $d_i(t_i) = d_{i-1} + v_i(t_i - t_{i-1})$. In this way it is possible to make comparison graphs between modeled (indicate with ^) and monitored data.

Results proposed about Orvieto modeling:

$\eta/z = 2.79E+08$ [kPa*sec/m]

$r^2 = 0.983$

![Displacements](image_url)

Figure 86: Comparison between modeled and monitored displacements for Orvieto area.
In order to perform the model, only dates from 27/06/1997 have been taken into account because of the first period data are affected from influence of adapting instruments. Change of displacements are similar in performance but are translated as values. About velocities graph, the course has not the same good agreement because peaks occurs similar in magnitude but translated in time.

Results proposed about Fosso San Martino modeling:

\[ \frac{\eta}{z} = 3.90 \times 10^7 \text{ [kPa*sec/m]} \]

\[ r^2 = 0.79 \]
Data considered for Fosso San Martino landslide stopped in May 1984 because data of pore water pressure modeled in SEEP ended in this date.

Graphs resulted from comparing are not very pleased in terms of displacements and velocities. The last one is in agreement with pore pressure course in period between 1981 and 1983. This worst agreement for this case is probably due at a limit model: Orvieto and next Vallcebre landslides have both safety factor lower than unit instead Fosso San Martino area has safety factor bigger than unitary value.

Regarding Fosso San Martino results, it could be noted that probably the assumption of infinite slope model is not representative of the real conditions of the slope and/or visco-plastic model is too simple for a complex phenomena like Fosso San Martino landslides.

The reflection in which movements are due to some others factors depending of time could also be considered as attempt to explicate no satisfactory results.
Results proposed about Vallcebre modeling:

\[ \eta/z = 4.79 \times 10^7 \text{ [kPa*sec/m]} \]

\[ r^2 = 0.993 \]

Figure 90: Comparison between modeled and monitored displacements for Vallcebre area.

Figure 91: Comparison between modeled and monitored velocities for Vallcebre area.
The best results proposed regarding Vallcebre landslide: displacements has exactly the same forecasted course; velocities from the first days of 1997 have the same course but peaks from model occur with bigger magnitude. The model obtains satisfactory results for cases with safety factor lower than unit.

This simulation is successful because the parameters back-analyzed are consistent with field and laboratory data obtained in an independent manner. Thus, the landslide behavior at selected locations seems to be well reproduced taking into account of viscous component. Next steps in this analysis will include the examination of the behavior of the landslide from a global perspective, instead of considering only motion in individual boreholes. That approach, however, will likely require a considerable computational effort, because a 3D analysis and a coupled hydrological and mechanical model may be required. The existence of well-instrumented landslides to implement and validate complex numerical models is thus seen to be highly beneficial.
5.5. Viscosity

Many active landslides can have a long life if the safety factor at residual strength conditions in the slip plane remains around the value of 1. These landslides show periods of rest, periods with slow movements and accelerations, leading sometimes to catastrophic failures. Behavior of these slow moving landslides may occur due to changes in material behavior, the loading by external slope masses which have become unstable, extreme meteorological events or significant increase in external loading by man.

The fact that landslides show velocities not indefinitely increasing can be explained by a viscous drag component of the material. It means that the velocity of the landslide is related by different viscous laws to excessive shear stresses. Different viscous laws have been developed to describe this relationship. The most well known is the Bingham law showing constant linear relation and a yielding point (in this equation thickness of the shear zone has been considered unitary):

\[ v = \frac{1}{\eta} \cdot (\tau - \tau_0) \]

where \( v \) is the velocity of movement, \( \tau \) is the existing shear stress, \( \tau_0 \) is the shear strength at a speed of zero and \( \eta \) is the dynamic viscosity.

The analysis of the groundwater records in combination with displacements shows, at first sight, an irregular response with respect to the onset of movement and the pore pressure fluctuations. The first step of this analysis is to obtain a threshold in fluctuation of pore water pressure from which movements starts or stops. However, from this threshold in terms of pore water pressure, a correspondent threshold in terms of safety factor has been found. In other words, the mobility of this slow moving landslides is analyzed over a period in which observations of the groundwater level and displacement measurements were available in order to identify critical thresholds in the mobility of this landslide. The attention has been focused in these periods in which mobility stops or begins and a first attempt relating a temporal trend and variations in the viscosity of the material has been conducted. In fact, this idea was born from a physical consideration: when pore water pressure is lower than a threshold in a period of time, safety factor in the same time is growing and movements stop, in this period viscous component has a big effect and one could expect a major value of this parameter. Whereas, when movements start, viscosity is decreasing. This idea has been summarized in a viscous index \( (a) \) that is a coefficient one can use to amplify viscous value or decrease it when necessary. This viscosity index has been inserted in previous visco – plastic models presented:

\[ \nu_i(t_i) = \frac{(t_i - t_{i-1}) \cdot [P + p_w(t_i) \cdot Q] + m \cdot v_{i-1}}{m + (t_i - t_{i-1}) \cdot \left( a \cdot \frac{\eta}{z} \right)} \]  \hspace{1cm} (16)

In which \( P \) and \( Q \) are factors defined in equations 9 depending from all slope properties, \( p_w \) is the pore water pressure changing in time, \( m \) is the mass calculated as in paragraph 5.1.2., \( \nu \) is velocity, parameter of viscosity \( \eta/z \).

In this way new results for three analyzed cases are proposed. The aim of this paragraph was to identify critical thresholds for the onset of movement in the mobility of this landslide. A second aim was to assess the temporal variation of the viscosity index and to find out whether there is a trend to strain softening of the material.
5.5.1. Orvieto

In Orvieto application, results proposed from viscosity index are the worst of three analyzed cases. What happens in this landslide could be explained considering some uncertainties in monitoring data in which, during period in which movements are recorded, negative values of velocities are proposed and it has not any physical sense. For this site, evaluation of viscosity index has been played as “a mathematical game” to adjust course of monitored velocities (or displacements) with the one of modeling data. Results are subsequently proposed:

**Figure 92:** Displacements results of viscosity index application in visco – plastic model.

**Figure 93:** Velocities results of viscosity index application in visco – plastic model.
The change of viscosity index is proposed in order to have an idea of their magnitude: they may vary during the study period. Therefore for each moving period with many consecutive days the viscosity index was calculated as a constant value:

This graph also shows that the viscosity index is not constant in time. This might be due to the fact that the landslide is not a rigid block. The safety factor is given for the total landslide while the local safety factor might change due to changes in stress distribution of the elastic body. Since the measured velocities are point observations the velocities have to be correlated with the local safety factor. The not unique relationship might also be explained by a change in the behavior of material in time (strain softening).
5.5.2. Fosso San Martino

Figure 95 shows the fluctuation of the relative pore water pressure above the sliding surface observed during the measuring periods. Blue circles underlines the periods in which movement was detected.

![Pore water pressure graph](image)

**Figure 95: Course of pore water pressure and correspondent threshold.**

It can be observed from the graph that movements take place when pore water pressure (average in many consecutive days) is bigger than a threshold value of 196 kPa. In fact, in periods of time in which average of pore water pressure is lower than this threshold, no movement has been recorded.

Fosso San Martino case represents the best application of viscosity index proposed:

![Displacements graph](image)

**Figure 96: Displacements results of viscosity index application in visco – plastic model.**
Following the same reasoning, in two periods (30 May 1980 – 13 August 1980 and 26 March 1981 – 11 December 1983) mobility in Fosso San Martino landslide is stopped and in these periods, viscosity index are so high. The adjust of graphs of monitored and modeled data is powerful to explain validity of viscous – plastic model taking into account changing in viscosity property of clayey materials.

Landslide monitoring and laboratory tests are of great importance to analyze the relation between the velocity and the safety factor.

In literature, Salt (1988) showed that most of his ring shear results can be described by the following simple relationship between the inverse of the safety factor \( \frac{1}{F} = \frac{\tau}{\tau_0} \) and the velocity of the landslide.

Van Asch (1994) plotted the results of different ring shear tests in a \( \frac{1}{F} \)-log \( v \). His graphs showed different viscous strength of the materials; in other words: different values for the viscosity index.

The same elaboration has proposed in this work relating Fosso San Martino landslide: safety factor calculated within infinite slope hypothesis and imposing residual condition has been correlated with logarithm of velocity obtained from visco – plastic model taking into account viscosity index.

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**Figure 97: Velocities results of viscosity index application in visco – plastic model.**

![Velocities Graph](image)
Figure 98 gives an example of an $1/F-log v$ plot. For this case there seems to be a slight tendency of the curve to bend in a horizontal direction for $1/F > 1.09$. The movements of this landslide are controlled by a rate dependent viscous strength of the material which shows a linear relationship between the shear stress ratio (equal to $1/F$) and the logarithmic velocity of the landslide at least within a certain range of the safety factor.
5.5.3. Vallcebre

In order to furnish a threshold in terms of pore water pressure in correlation at viscosity index, subsequently criterion is proposed:

<table>
<thead>
<tr>
<th>pwp [kPa]</th>
<th>viscosity index</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 108</td>
<td>1.5</td>
</tr>
<tr>
<td>= 106/107</td>
<td>1.6</td>
</tr>
<tr>
<td>&gt; 99</td>
<td>1.7</td>
</tr>
<tr>
<td>= 98/99</td>
<td>2</td>
</tr>
<tr>
<td>&lt; 97</td>
<td>&gt;2.5</td>
</tr>
</tbody>
</table>

When daily pore pressure measured in site falls into a range, corresponding viscosity index is found and, as one may expect, with low pressure it has been imposed a high value of index. Following this criterion, the adjust of graphs in terms of displacements and velocities reproduces an adequate way the actual behavior of these parameters. Results are shown in figure 99 and 100:

Figure 99: Course of displacements in visco – plastic model taking into account viscosity index.
The same consideration related to logarithm of velocity, inverse of safety factor and viscosity index proposed to Fosso San Martino analysis has been proposed other times for Vallcebre case:

This graph underlines a growing trend of velocity (in logarithmic scale) in relation with the inverse of safety factor: as one may expect, when $F$ decreases the slope is under instability condition and the inverse when $F$ grows; instability condition implies an increment in terms of movements and velocities. When a critical velocity is achieved, inverse of safety factor grows indefinitely and $F$ pursues very low values.
A relationship between the inverse safety factor and the viscosity index can be observed in figure 102. The figure shows a decrease in the viscosity index (decrease in rate dependent strength) with a growing of the inverse safety factor or in other terms a growing in the height of the water table:

\[ p_w \uparrow \iff F \downarrow \iff \frac{1}{F} \uparrow \iff \text{viscosity index} \downarrow \]

In other words a decrease of groundwater pressure means an increase in the viscous strength of the material.

Van Asch (1994) suggested that high viscosity indexes measured in the field indicate creep deformation in a larger zone of viscous material with a higher viscosity while low index values indicate viscous deformation concentrated in a narrow shear band which is also simulated by ring shear tests in the laboratory.

He compared the results of a number of lab tests for different materials from different authors with results obtained from field monitoring of landslides. The data from the field are obtained from direct measurements relating the log velocity of the slide to the safety factor of the slide. It appeared that the materials in the field seem to have in most cases a tenfold higher viscosity index compared with indices obtained from the laboratory. An explanation might be that creep movements in the landslides are not concentrated in a shear band of limited thickness but in a larger (stiffer) creep zone (Van Asch, 1994).

In order to understand in the better way the role of viscosity in clayey soils of Vallcebre site, some test of laboratory are pursuit and were conducted in subsequent paragraph.