Upgrading of a wavemeter

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Abstract. The aim of this project is to upgrade a *wavemeter*, a device able to measure wavelengths with high accuracy, to use it in Quantum Optics experiments. By proper alignment of the optical elements and optimization of reference wavelength, it has been built a device able to measure wavelengths with a precision of 25 MHz, although it shows a constant offset of +0.027 nm.

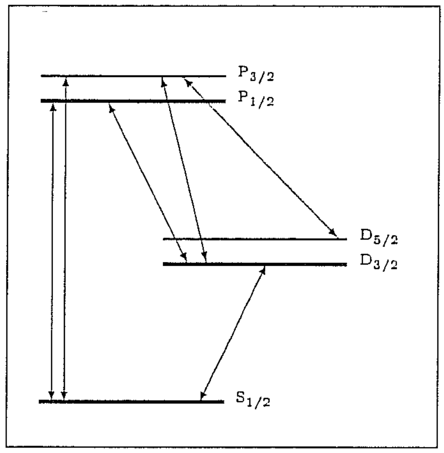
Keywords: Wavemeter, Quantum Optics, Quantum Computation

1. State of the art

The experiments in which the wavemeter will be used are focussed on the manipulation of isolated Calcium ions to work in the quantum computation field, using the ion's excited states as a source of obtaining quantum bits. Laser light with high monochromaticity is needed for different purposes:

(i) It is needed high control of the energy level in which the Ca^+ is in every moment, and this is obtained by using π and $\pi/2$ laser pulses at a very narrow and well defined wavelength, to be sure that any light pulse applied into the ion links only two of the states of our system, without interfering with the other levels. Then, when a photon is absorbed by the ion, it can be said, with very high confidence, which is the final energy state of the system.

(ii) Laser light is used to cool the ions and make them stay still at a certain position. First, ions are confined in a relatively big region by electromagnetical fields. On this stage, their movement is described roughly by the Maxwell distribution, presenting different velocities and so, not staying in a definite position. To slow the ions down –cooling them– it should be used laser light 50 MHz red de-tuned from 396.847nm, the wavelength corresponding to the transition between the $S_{1/2} \rightarrow P_{1/2}$ levels of the ion, and also a beam at 866.214 nm, resonant on the transition $D_{3/2} \rightarrow P_{1/2}$. This last beam is used to be sure that if any ion decays from $P_{1/2}$ to the $D_{3/2}$ level,



Working	R. Int.
$S_{1/2} \rightarrow P_{1/2}$ at 396.8469 nm	220
$D_{3/2} \rightarrow P_{1/2}$ at 866.214 nm	160
Project	
$S_{1/2} \rightarrow D_{3/2}$ at 729.147 nm	160
Other transitions	
$S_{1/2} \rightarrow P_{3/2}$ at 393.3663 nm	230
$D_{3/2} \rightarrow P_{3/2}$ at 849.802 nm	130
$D_{3/2} \rightarrow P_{5/2}$ at 854.209 nm	170

Figure 1. Energy levels of Ca^+. The thicker lines correspond to more relevant levels. Wavelengths of the different transitions are classified in different categories according to the disposability of lasers working in that transition: if they are currently at work, and whether they will in near future or not. Data has been obtained from [1]. It can be seen that line $S_{1/2} \rightarrow P_{3/2}$ is more intense than $S_{1/2} \rightarrow P_{1/2}$; even though, this state can decay to both $D_{3/2}$ and $D_{5/2}$, so it is not a good candidate to be used as a qbit.

it will be re-pumped again to the upper level and so, in fact, the $D_{3/2}$ level does not play a important role in the trapping process. Acting like that, ions that are moving against the propagation direction of the 396 nm laser beam will see this light at a higher frequency due to Doppler shift effect –the laser will be in the precise resonant frequency for them–, so they are likely to absorb this photons and thus get the $\hbar \boldsymbol{k}$ momentum of the photon and slow down; with laser projections along x, y and z-axis, Ca^+ ions are slowed down for all directions. After this first step, ions are "cooled" to minimize its movement. That is done in the same way as described before, using red de-tunings of around half of the FWHM of the $S_{1/2} \rightarrow P_{1/2}$ line, that is, around 10 MHz. A detailed scheme of the energy levels of Ca^+ and the wavelength corresponding to each transition is shown in figure 1

A wavemeter is a device that uses interferometry to calculate the wavelength of a given source by comparing it with a known one. Basically –and before entering in details as it will be done in section 2– it works like a Michelson's interferometer where two beams follow the same path.

If we are able to claim that two beams follow the same path within the interferometer, and we know the wavelength of one of them, the relative number of interference fringes observed for both wavelengths will give us a very accurate measure of the unknown beam.

With accurate alignment of the components of the wavemeter, theoretically precisions below 100 MHz can be achieved –around 25 pm for the wavelengths used– which is 10 times worse than the precision needed to cool the ions. It is clear then that only with such a device it will be never achieved the accuracy needed to trap or cool the

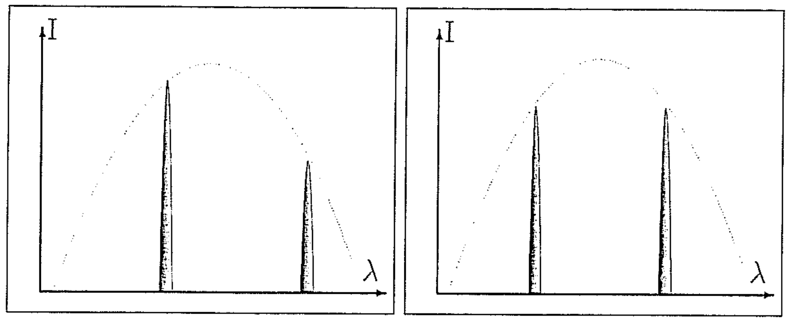


Figure 2. When the two modes have different intensities (left) their position with respect the central maximum of Ne emission (dotted line) cannot be determined. When their intensity is the same (right) their wavelength is easily determined

ions, but still it is a fundamental tool for this purpose, if we proceed in the following way:

1st- We select with the wavemeter the desired laser wavelength. Even though it will not be in the precise frequency that is needed, with an accuracy around 100 MHz the ions inside the trap will absorb part of this light, and re-emit it by spontaneous emission.

2nd- At this point, with a CCD camera focussed into the ion chamber it is visible an "ion cloud", that is, a big region where ions are emitting light by spontaneous decay. The number of photons emitted are counted with a photocounter. Since this amount of photons is expected to increase as we approach the correct frequency of the transition, we tune the laser wavelength with an *Acusto-Optical Modulator* (AOM) until the CCD shows us a bright luminous spot: that is a single ion trapped that is continuously emitting light.

Nowadays it is used in the laboratory a wavemeter with a He-Ne reference laser, that emits in two modes of the cavity. By temperature-tuning of the cavity the relative intensity of those modes can be selected, and so can be selected their frequency with respect to the gain profile of the He-Ne laser. A visual schema can be seen in figure 2. Detailed explanation of this reference laser can be found in [3] and [4].

Even though, this method of stabilizing the reference laser has some disadvantages. First, two different "reference" signals must be obtained at two different silicon photodiodes –type BPX65– with a time response around 12 ns [5]; after that this two signals must be compared electronically and then the temperature of the He-Ne cavity must be tuned to make both intensities equal. However fast you are able to do this process, it is not convenient when one wants to achieve in principle resolutions of hundreds of kHz or even GHz.

Summing up everything said before, the aim of this project will focuss on:

(i) Realignment of the whole device to obtain the best accuracy achievable.

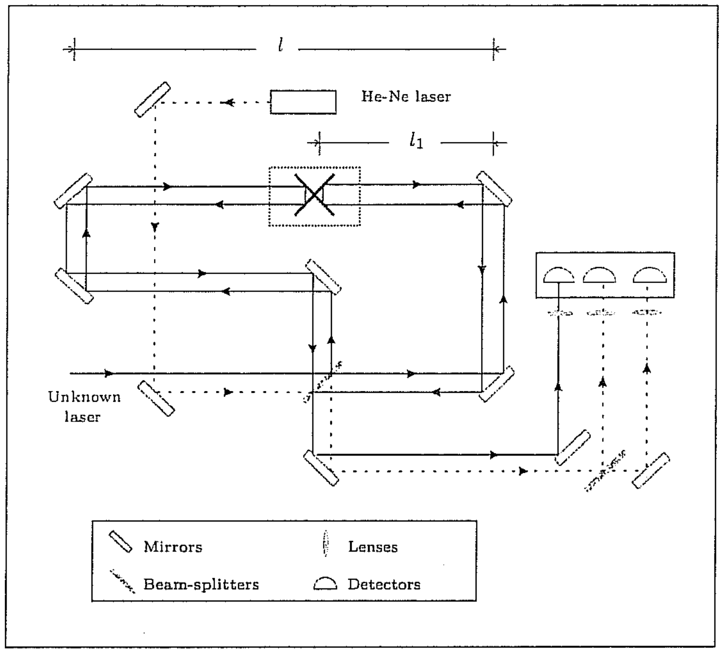


Figure 3. Beams from the Reference and Unknown laser are divided in the beam splitter into the two paths of the wavemeter, and detected afterwards. A polarizing beam splitter is used to divide the He-Ne laser depending on the polarization of the two modes to compare their relative intensities. Solid lines represent the unknown laser beam, while dotted lines show the reference one. When both are superposed, only the unknown beam is showed; reference beam follows the same path but in the opposite direction.

(ii) Substitution of the He-Ne laser for a new reference one with more accuracy and stability. See section 4

(iii) Connect the wavemeter to a computer to optimize data saving and obtaining a user-friendly display.

2. Working fundamentals

The working principle of the wavemeter is, as has been pointed in section 1, the comparative measure of two different wavelengths, one has a well known frequency and acts as a reference to obtain the wavelength of the other. An schema of the wavemeter in its actual state is shown in figure 3, and we want to substitude the reference He-Ne laser for a new. This schema is analogous to the one described in [2]. The reflectors of the laser are two mirrors at 90° designed like that because it is the easiest way for us to be sure that both lasers follow the same optical path, a crucial point for the wavemeter to work properly, as it will be discussed in section 3.2.

Let's now calculate the interference intensity that will be collected at the detector –ignoring, for the moment, the effect of the lenses. We know that the electric field for a beam with ω and $\boldsymbol{k}$, after traveling a distance x will be:

$$E(\boldsymbol{x},t) = \boldsymbol{E}_0 e^{i\left(\omega t - \boldsymbol{k}\cdot\boldsymbol{x}\right)} \tag{1}$$

Since $\boldsymbol{k}$ is always parallel to $\boldsymbol{x}$, we can substitute $\boldsymbol{kx}$ by kx in (1). The problem is now reduced to find this traveling distance x for each branch of the device; to do so, it is advisable to look for one branch in detail, following the schema draw in figure 4 for one of the branches, the total distance is:

$$\begin{aligned} x = \overline{A_1B_1} + \overline{B_1C_1} + \overline{C_1C_1'} + \overline{C_1'D_1} + \overline{D_1D_2} + \overline{D_2C_2} + \\ + \overline{C_2B_2} + \overline{B_2A_2} + \overline{A_2X_1} + \overline{X_1X_2} + \overline{X_2X} \end{aligned} \tag{2}$$

but, all the terms in (2) are constant except $\overline{C_1'D_1}$ and $\overline{D_1D_2}$, that are precisely the distance l_1 of figure 3. Those terms being constant mean that, when we incorporate them in (1) they will add a constant phase to the exponential, so the only important therm is $2 \cdot l_1$; the same process can be done for the other branch, with the change $l_1 \longrightarrow (l - l_1)$. Assuming now that the beam splitter is ideal – the amplitude of both waves is $A_0/\sqrt{2}$, being A_0 the amplitude of the original intensity– the interference between light of both paths in the detector will be (ignoring constant phases):

$$\begin{aligned} E_1 + E_2 &= \frac{A_0}{\sqrt{2}}\, e^{i(\omega t - k2l_1 + \varphi_1)} + \frac{A_0}{\sqrt{2}} \cdot e^{i(\omega t - k2(l-l_1) + \varphi_2)} \\ &= \frac{A_0}{\sqrt{2}}\, e^{i\omega t + \varphi}\left(e^{-i2kl_1} + e^{i2kl_1} + e^{-i2kl}\right) \\ &= 2\frac{A_0}{\sqrt{2}} e^{i\omega t + \varphi'} \cos(2kl_1); \\ I_{Tot.} \propto \left|E_1 + E_2\right|^2 &\Rightarrow I_{Tot.} \propto \cos^2(2kl_1) = cos^2(\frac{4\pi}{\lambda} l_1) \end{aligned} \tag{3}$$

The interference intensity, vanishes when $\frac{4\pi}{\lambda} l_1 = \left(n + \frac{1}{2}\right)\pi$, $n \in \mathbb{N}$. Imagine now that, for a given length l_1 the intensity is 0, and we know with high accuracy the wavelength λ_{ref} of our beam; then, $l_1 = \left(n + \frac{1}{2}\right)\frac{\lambda_{ref}}{4}$. Now we move the reflector into a new position $l_2 = \left(m + \frac{1}{2}\right)\frac{\lambda_{ref}}{4}$ where the interference vanishes too. The difference between those positions is:

$$l_1 - l_2 = \frac{\lambda_{ref}}{4}(n' - m') = \frac{\lambda_{ref}}{4} \cdot N_{ref} \tag{4}$$

being $N_{ref} \equiv n - m$ the difference of orders between both minimums and thus, the number of interference fringes that appear between both positions of the reflector.

Let's imagine that we have another beam whose wavelength is unknown (λ_{sig}), but that shows two interference minimums *precisely at the same positions* $l_1' = l_1$ *and* $l_2' = l_2$. If this beam follows *exactly the same path* of our reference beam, following the same steps as done before, we can write:

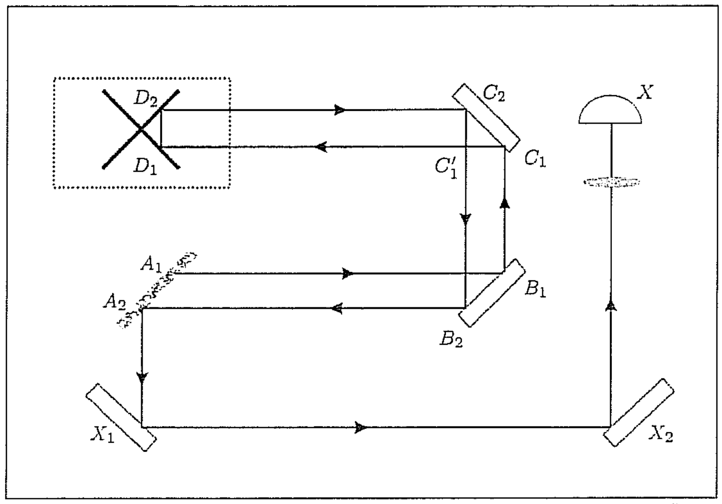


Figure 4. Detailed schema of the path followed by light in one of the branches, where the main points have been labeled.

$$l_1 - l_2 = \frac{\lambda_{sig}}{4}(n - m) \equiv \frac{\lambda_{sig}}{4} \cdot N_{sig} \tag{5}$$

Combining (4) and (5), we found: convination

$$\lambda_{sig} = \lambda_{ref} \cdot \frac{N_{ref}}{N_{sig}} \tag{6}$$

Two main conditions must be fulfilled, as it has been remarked: first, the minimums of the two beams must coincide in two positions and the optical path of both laser beams must be the same. Let's discuss this two points in detail in sections 3.1 and 3.2

3. Accuracy limitations

3.1. Number of fringes

We are working in a region of the spectrum between the near infrared to the UV, with a reference wavelength of $\lambda_{ref} = 852.106418$ nm, and a unknown wavelength close to the ones showed in figure 1. To design the wavemeter we must be sure that, when the reflector moves from 0 to l, at least, there must be two positions l_1 and l_2 where the interference for both wavelengths vanishes at the same time, as discussed before. The electronics of our device are fast enough to provide us a very accurate lecture of the number of fringes –with precision of $\pm 10^{-3}$ fringes. To minimize the relative error further, we will work with N_{sig} and N_{ref} bigger than 5×10^6. It can be seen that this minimum number of fringes will lead to one coincidence every 10 cm, wich is enough for our device whose reflectors move from 0 to 50cm, providing around 4 lectures per travel.

As an example, if the unknown beam has $\lambda_{sig} = 729.147\text{nm}$ ($S_{1/2} \rightarrow D_{3/2}$ transition) will give us the following numbers: $N_{ref} = 500\ 165$, $N_{sig} = 538\ 024$ and $\delta N_{ref} = \delta N_{sig} = 10^{-3}$. The error in the determination of λ will be then:

$$\delta\lambda_{sig} = \sqrt{\left(\frac{\partial\lambda_{sig}}{\partial N_{ref}}\delta N_{ref}\right)^2 + \left(\frac{\partial\lambda_{sig}}{\partial N_{sig}}\delta N_{sig}\right)^2} = \lambda_{ref}\sqrt{\frac{(\delta N_{ref})^2}{N_{sig}^2} + \frac{N_{ref}^2(\delta N)^2}{N_{sig}^4}}$$
$$= 2\times 10^{-6}\text{nm} \quad \Longrightarrow \quad \boxed{\frac{\delta\lambda_{sig}}{\lambda_{sig}} = 3\times 10^{-9} < 10^{-8}} \tag{7}$$

3.2. Alignment

Since now, we have assumed that the interferometer is completely aligned and we are looking at the central spot of the beam, that is, we are in the ideal situation showed in figure 3. Now, we should ask ourselves what happens if, for some reason, the adjustment of the mirrors is not perfect and somehow, beams from the two branches arrive into the detector with some angle θ_i. It is clear then that the traveling distance calculated in 2 will be different and so the interference intensity of (3) will not be correct.

If the beam is tilted an angle θ small, the total length traveled by the beam will be $L' = L/cos(\theta)$, being L the total distance traveled by our beam, playing the role of x in (1) –from the output without tilting. We can define a relative displacement from the optical axis δx and for small angles $\theta \approx \delta x/L$. We can rewrite:

$$\delta L = L\left(\frac{1}{\cos\theta} - 1\right) \Rightarrow$$
$$\frac{\delta L}{L} = \frac{1}{1-\cos\theta} - 1 \approx \frac{1}{1+\frac{1}{2}\left(\frac{\delta x}{L}\right)^2} - 1 \approx \frac{1}{2}\left(\frac{\delta x}{L}\right)^2 \tag{8}$$

but equations (4) and (5) we can deduce that $\delta\lambda \propto \delta l_1 = \delta L$ so, for our $L \approx 5m$ and $\delta x \approx 0.5\text{mm}$ we have:

$$\frac{\delta\lambda}{\lambda} = \frac{\delta L}{L} \approx= \frac{1}{2}\left(\frac{\delta x}{L}\right)^2 = 5\times 10^{-9} \tag{9}$$

Another important effect of the misalignment of the device is that when the beam is tilted the interference pattern changes with the position of the reflector. This makes that the amplitude of the interference intensity also changes. To have a good align of the beam, this effect will be of great help, since the best alignment of the device will be achieved when this intensity does not change when the reflector is moving.

3.3. Refractive index on air

Both unknown and reference beam propagate though air in our device, and air may have different refractive index $-n$ and $n_{ref}-$ for each of the wavelengths. Equation (6) should be corrected with a factor n/n_{ref} becoming:

$$\lambda_{sig} = \lambda_{ref} \frac{N_{ref}}{N_{sig}} \cdot \frac{n_{sig}}{n_{ref}} \tag{10}$$

Even though we can calculate n_{ref} for the known wavelength, n will differ from one desired frequency to another and the error introduced will not be the same when measuring IR beams compared with when measuring UV. Anyhow, with pressure temperature an humidity constant, this error is always below 5×10^{-8}

3.4. Other sources of error

There are also other effects that can produce errors in the determination of our wavelength, and those are, mainly, the curvature of the wavefront and dispersion. Both of them are far bellow 10^{-8} relative error, so they are not affecting our lectures.

4. Stabilization of reference wavelength

As it has been said before, one of the fundamental aspects for the wavemeter to work properly is to have a well defined wavelength as a reference. To obtain this we will work with saturation spectroscopy of line D2 of gas of Cesium, tuning our reference laser to be resonant to one of the cross-over lines –see 4.1– of the hyperfine structure of this element.

4.1. Sasturation Spectroscopy. Cross-over lines

Understanding of saturation spectroscopy needs of some study of Lamb-dip effect, Doppler broadening of the absorbtion linewidth of an element, and non linear spectroscopy. Here we present a intuitive explanation for the presence of the cross-over lines when working with hyperfine structure of Cs addressing the reader for more information in the references [6].

Basically the method consists on scanning the D2 line of *Cs* with a narrow-band laser. The laser is directed into a cell full of gas of Cesium, that has a mirror in its end, so the light is reflected back.

Atoms of Cs have different velocities that fit into a Maxwell distribution; that means that, due to Doppler shift, for each wavelength ω exists a certain group of atoms with a velocity v_z (z is the propagation direction of the laser beam) that will absorb our wavelength, following the relation $\omega = \omega_i + kv_z$, being ω_i the resonant frequency to a certain hyperfine transition of the atom. After crossing the cesium gas chamber, the beam is reflected back ($k \to -k$), and so it will be absorbed by the group of atoms that have a velocity $-v_z$. In general then, each wavelength is absorbed by two groups of atoms with the same absolute velocity, but in opposite directions. This always holds except for two cases:

(i) $v_z = 0$: For those atoms with no velocity respect to the laser beam propagation, the beam is absorbed normally when the laser enters the gas chamber, and so some atoms are excited when $\omega = \omega_i$; later the beam is reflected back, but there are some atoms that have already been promoted to an excited level *so they cannot absorb any other photon at that frequency*, and the total absortion after the forward-backward trip inside the camber is lower than for other frequencies.

(ii) $\omega = \frac{\omega_i+\omega_j}{2}$: when we use a frequency between two resonant transitions, we have two possible absorptions when the laser enters the chamber in the forward direction:

$$\frac{\omega_i + \omega_j}{2} = \omega_i + kv_1^{(f)} \Rightarrow v_1^{(f)} = \frac{\omega_j - \omega_i}{2k}$$
$$\frac{\omega_i + \omega_j}{2} = \omega_j + kv_2^{(f)} \Rightarrow v_2^{(f)} = \frac{\omega_i - \omega_j}{2k}$$

then light is reflected back, and doing again the same process, we find the group of atoms that will absorb the backward light are defined by the velocities:

$$v_1^{(b)} = \frac{\omega_i - \omega_j}{2k} = v_2^{(f)}$$
$$v_2^{(b)} = \frac{\omega_j - \omega_i}{2k} = v_1^{(f)}$$

meaning that some of the atoms that in principle are able to absorb our beam once it has been reflected in the mirror, have already been excited by the laser in its forward propagation, and so some of them have been promoted to an upper level and cannot absorb more photons of our beam. This will be seen as a peak in the absorption curve, leading to the existence of absorption lines at frequencies that do not correspond directly to energy difference between the eigenstates of the atom, the so-called *cross-over lines*. In fact, those lines have stronger intensity that the "natural" ones, and so we use the line from $F = 3 \rightarrow F' = 3 - 4$ ($F' = 3 - 4$ has the energy between $F' = 3$ and $F' = 4$), since it is the strongest for Cs.

5. Experimental results

Finally, we compare the measures of different wavelengths measured with our wavemeter and the measures obtained with a WS-7 Fizeau interferometer from High Finesse Laser and Electronic Systems [7] with an accuracy of 60 Mhz. Results can be seen in table 1

It can be seen that our wavemeter shows an offset of around +0.027 nm that remains more or less constant for all wavelengths, and an accuracy of 25 MHz on average, 4 times better than the 100 MHz for our purposes –see section 1.

WS-7 interferometer	Wavemeter	Offset	$\Delta\nu$
854.310 9(7) nm	854.337 87 ± 0.000 06 nm	0.027 nm	24.644 MHz
793.700 0(0) nm	793.725 50 ± 0.000 02 nm	0.025 nm	9.517 MHz
849.800 3(9) nm	849.829 0 ± 0.000 1 nm	0.028 nm	41.510 MHz

Table 1. Lectures of different wavelengths with the Fizeau interferometer and the wavemeter. Difference between both measures and accuracy found for the wavemeter.

6. References

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