Master in Photonics

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COHERENT TRANSPORT OF HOLES IN MICROTRAP ARRAYS: SCHRÖDINGER AND DE BROGLIE—BOHM ANALYSES

Albert Benseny Cases

Supervised by Dr. Jordi Mompart Penina, (UAB)

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Coherent Transport of Holes in Microtrap Arrays: Schrödinger and de Broglie–Bohm Analyses

Albert Benseny
Physics Department, Universitat Autònoma de Barcelona, Bellaterra, E-08193
E-mail: albert.benseny@uab.cat

Abstract.
We present a robust method to coherently remove defects in optical microtrap arrays. By using the tunneling interaction between trapped states, we create a multisite dark state that, by applying a matterwave analog of the well known stimulated Raman adiabatic passage, allows us to adiabatically remove empty sites from the system. This can be done in fermionic systems as well as in strongly interacting bosonic systems. This technique offers the possibility of cleaning quantum registers before starting the information processing. With the aid of the de Broglie–Bohm pilot-wave theory we interpret the results obtained in the numeric simulations to explain how does the transport process take place.

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1. Introduction

Quantum gases trapped in optical periodic potentials have attracted considerable attention for quantum computation since they fulfill all the basic requirements for quantum information processing [1, 2]. In fact, neutral atoms in optical periodic potentials with short-range interactions, e.g., s-wave scattering of bosons, or state selective long-range interactions, e.g., dipole-dipole interaction, do not present intrinsic limitations in their scalability. Thus, quantum registers with single-site addressing of \( \sim 100 \) qubits [3] and cluster entangled states of thousands of atoms [4] have been reported, respectively, in 2D optical microtrap arrays and 3D optical lattices. Recent report of a Fermi gas loaded into a three-dimensional optical lattice has given birth to a good candidate for a quantum device, since, by means of the Pauli principle, the number of atoms per site can be controlled [5].

On the other hand, fault-tolerant quantum computation requires close to ideal quantum systems where the preparation of the initial state and the implementation of the subsequent logic quantum gates is performed with a very high fidelity. Therefore, one needs to start with a defect-free quantum system where all sites of the lattice are occupied by one atom.

In this paper we will show how to control and manipulate the external degrees of freedom of atoms to remove holes, i.e., empty sites, from the system by adiabatically following a multi-site dark state whose shape is determined by the tunneling interaction between sites. We will numerically show the control method by applying it.
to a triple well potential and interpret the results in the de Broglie–Bohm pilot-wave theory [6, 7].

This article is structured as follows. In Section 2 we give an introduction of the de Broglie–Bohm pilot-wave theory. In Section 3 we review the previous work done in three level optics and three level atom optics, on which we will base our work. In Section 4 we present our scheme to move empty sites in arrays of optical traps. In Section 5 we show our method’s simulation results in a system of two atoms in a triple well potential and interpret them in terms of quantum trajectories. Finally, we summarize the paper’s conclusions in Section 6.

2. Basics of de Broglie–Bohm pilot-wave theory

Since we will use the de Broglie–Bohm pilot-wave theory to interpret the results of our simulations we give here an introduction to its ideas and formalism.

De Broglie–Bohm pilot-wave theory [6, 7] is a quantum theory formulation in terms of hidden variables: the positions of the particles. In this theory, particles describe definite trajectories, i.e. they have precisely defined positions at all times. These trajectories give a clear physical interpretation of the system dynamics.

The mathematical formalism of the de Broglie–Bohm theory is derived from the Schrödinger equation. The Schrödinger equation for a \( N \) particles wavefunction \( \phi(x_1, x_2, \ldots, x_N, t) \), reads (with the conventional meaning of the symbols):

\[
\frac{i\hbar}{\partial t} \phi = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \nabla_i^2 \phi + V(x_1, x_2, \ldots, x_N, t) \phi \tag{1}
\]

Since the wavefunction is complex valued, we can write it in polar form, \( \phi = R e^{iS/\hbar} \), where the modulus, \( R \), and the phase, \( S/\hbar \), are real valued functions.

For the de Broglie–Bohm theory to yield the same statistical results as quantum mechanics two assumptions are needed, that make the wavefunction act as a pilot wave for the particles’ trajectories. First, that the initial distribution of possible particle positions is given by \( R^2 \), and second, that the \( S \) defines the velocity of each particle trajectory: at a time \( t \), the \( i \)-th particle velocity will be given by:

\[
\vec{v}_i = \frac{1}{m_i} \nabla_i S \bigg|_{(x_1[t], x_2[t], \ldots, x_N[t], t)} \tag{2}
\]

where \( x_i[t] \) is the position of particle \( i \) (of its trajectory) at time \( t \). Notice the dependence of the velocity of a particle on the positions of all particles.

If we cast the polar form of the wavefunction into the Schrödinger equation (1) and separate the real and imaginary parts of the resulting equation we obtain:

\[
-\frac{\partial R^2}{\partial t} = \sum_{i=1}^{N} \frac{1}{m_i} \nabla_i (R^2 \nabla_i S) \tag{3}
\]

\[
-\frac{\partial S}{\partial t} = V + \sum_{i=1}^{N} \frac{1}{2m_i} (\nabla_i S)^2 - \sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \frac{1}{R} \nabla_i^2 R \tag{4}
\]

Equation (3) is a continuity equation for the modulus of the wavefunction. This continuity equation ensures that the particle positions’ distribution is given by the modulus square of the wavefunction at any time. Equation (4) is the so-called quantum Hamilton–Jacobi equation because of its similarity with the (classical) Hamilton–Jacobi equation but with one additional term, called the quantum potential, responsible for the system characteristic quantum behaviour.
As we stated previously, in the de Broglie–Bohm theory the state of the system is given by the wavefunction and the position of all the particles. The trajectory is the time evolution of these positions. Since these positions are hidden variables we cannot know their value in a given experiment, but we know their statistics: the probability density of finding particle 1 at $x_1$, particle 2 at $x_2$, ... at a certain time $t$ is given by $R^2(x_1, x_2, ..., x_N, t)$, and their velocity is given by equation (2).

By knowing the initial wavefunction and the initial position of all the particles, one can compute the wavefunction evolution (either by the Schrödinger equation (1) or equations (3) and (4)) and then find the evolution of the positions (the trajectory) by using equation (2).

3. Three Level Optics and Three Level Atom Optics

In this section we give a review of the work done in the fields of Three Level Optics (TLO) and Three Level Atom Optics (TLAO) on which our work is based upon.

3.1. Three Level Optics

In reference [8], Bergmann et al introduced the STImulated Raman Adiabatic Passage (STIRAP) technique, a robust method to transfer population between the internal levels of a TLO system (figure 1.a). They showed that by using a counterintuitive pulse scheme, first a Stokes (S) pulse and then a Pump (P) pulse, it is possible to transfer the population from level $|A\rangle$ to level $|C\rangle$ without populating level $|B\rangle$.

![Figure 1.](image)

Figure 1. a) Three Level Optics. b) Pulse scheme to achieve STIRAP. $\Omega_P$ ($\Omega_S$) is the Rabi frequency coupling $|A\rangle$ and $|B\rangle$ ($|B\rangle$ and $|C\rangle$).

In the rotating wave approximation the Hamiltonian of the system reads:

$$
\mathcal{H} = \begin{pmatrix}
0 & \Omega_P(t) & 0 \\
\Omega_P(t) & 2\Delta_P & \Omega_S(t) \\
0 & \Omega_S(t) & 2(\Delta_P - \Delta_S)
\end{pmatrix}
$$

(5)

where $\Omega_P$ ($\Omega_S$) is the Rabi frequency coupling levels $A$ and $B$ ($B$ and $C$) and $\Delta_P$ ($\Delta_S$) is the corresponding detuning of the field with the atomic transition.

It is straightforward to check that, for $\Delta_P = \Delta_S$, there exists a dark state $|D(\Theta)\rangle \equiv \sin \Theta |A\rangle - \cos \Theta |C\rangle$, with $\tan \Theta \equiv \frac{\Omega_P}{\Omega_S}$, that is an eigenvector of the Hamiltonian (5) with eigenvalue 0. Note that this state only populates levels $|A\rangle$ and $|C\rangle$. Therefore, if the electron is in state $|A\rangle$, it is possible to transfer it to $|C\rangle$ by adiabatically changing the parameter $\Theta$ from 0 to $\frac{\pi}{2}$. This corresponds to the pulse scheme shown in figure 1.b.
3.2. Three Level Atom Optics

An extension of the STIRAP was done by Eckert et al [9] to a system of one atom in a triple well potential (see figure 1.a), considering only the ground states of the three traps: |1⟩, |2⟩ and |3⟩, for the left, middle and right traps respectively. In this simplified picture the Hamiltonian that describes the evolution of the system is:

\[
\mathcal{H} = \begin{pmatrix}
0 & J_1(t) & 0 \\
J_1(t) & 0 & J_2(t) \\
0 & J_2(t) & 0
\end{pmatrix}
\]  

(6)

where \(J_1\) (\(J_2\)) is the tunneling rate between the left and middle traps (middle and right traps). These tunneling rates depend on the distances between the traps, \(d_1\) and \(d_2\) and their shape. Therefore, by moving the traps it is possible to transfer population between them.

\[
J_i(d_i) = \frac{-1 + e^{\beta^2} \left[ 1 + d_i \sqrt{\pi} \left( 1 - \text{erf}(d_i) \right) \right]}{\sqrt{\pi} \left( e^{2d_i^2} - 1 \right) / 2d_i}
\]  

(9)

Since equation (6) has the same form as equation (5) (with \(\Delta_P = \Delta_S = 0\)) we can extend techniques from TLO to TLAO. Therefore, by approaching the traps in similar fashion as one would apply the pulses in STIRAP (see figure 2.b), one can adiabatically transport the atom from the left trap to the right trap. That corresponds to first approaching the middle and right traps and then approaching the left and middle traps, with a delay \(t_{\text{delay}}\) between the approaches. This process has been adapted to similar contexts [10, 11] where it has been named Coherent Tunnelling by Adiabatic Passage (CTAP).

In [9] Eckert et al showed that, indeed, this can be used to transfer the atom from the left trap to the right trap. They not only showed that it works in the simplified picture, just considering the groundstates of three traps (which is obvious), but also integrating the Schrödinger equation in the position space:

\[
i\hbar \frac{\partial \varphi(x, t)}{\partial t} = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) \right] \varphi(x, t)
\]  

(7)

with \(V(x)\) being the piecewise defined harmonic wells potential centered at \(x_{L}(t), x_{M} = 0, x_{R}(t)\):

\[
V(x) = \begin{cases} 
\frac{1}{2}m\omega_x^2(x - x_{L})^2 & \text{if } x < \frac{x_{M} - x_{R}}{2} \\
\frac{1}{2}m\omega_x^2(x - x_{M})^2 & \frac{x_{M} - x_{L}}{2} \leq x < \frac{x_{M} - x_{R}}{2} \\
\frac{1}{2}m\omega_x^2(x - x_{R})^2 & \text{if } x > \frac{x_{M} - x_{R}}{2}
\end{cases}
\]  

(8)

With this definition of the traps, the tunneling elements \(J_i\) in Hamiltonian (5) are:

\[
J_i(d_i) = \frac{-1 + e^{\beta^2} \left[ 1 + d_i \sqrt{\pi} \left( 1 - \text{erf}(d_i) \right) \right]}{\sqrt{\pi} \left( e^{2d_i^2} - 1 \right) / 2d_i}
\]  

(9)
where \( d_i \) is the distance between traps \( i \) and \( i + 1 \) (\( d_1 = -x_L \) and \( d_2 = x_R \)) and \( \text{erf} \) is the error function.

### 3.3. Three Level Optics for a Hole

An interesting question is what happens in the \( \Lambda \) three-level system described in section 3.1 if we have two electrons instead of one (see figure 3.a) as studied in reference [12].

Since electrons are fermions the entire wavefunction must be antisymmetric. This means that if the spin wavefunction is symmetric, e.g., the spins of the electrons are parallel, the atomic wavefunction must be antisymmetric and therefore, by the Pauli principle, they cannot be in the same atomic state. Then, the state of the two electrons (1 and 2) can only be in the subspace generated by:

\[
|\tilde{A}\rangle = \frac{1}{\sqrt{2}} (|B\rangle_1 |C\rangle_2 - |C\rangle_1 |B\rangle_2) \\
|\tilde{B}\rangle = \frac{1}{\sqrt{2}} (|C\rangle_1 |A\rangle_2 - |A\rangle_1 |C\rangle_2) \\
|\tilde{C}\rangle = \frac{1}{\sqrt{2}} (|A\rangle_1 |B\rangle_2 - |B\rangle_1 |A\rangle_2)
\]

(10) (11) (12)

Since there are two electrons to fill three states, there will be two occupied states and an empty one, a hole. We could see this subspace of the system as a single particle three level system (\(|\tilde{A}\rangle, |\tilde{B}\rangle, |\tilde{C}\rangle\)), these states representing a hole being on level \(|A\rangle\), \(|B\rangle\) or \(|C\rangle\), respectively (see figure 3.b). In this new system, \( \Omega_P \) (\( \Omega_S \)) couples levels \(|\tilde{A}\rangle\) and \(|\tilde{B}\rangle\) (\(|\tilde{B}\rangle\) and \(|\tilde{C}\rangle\)). Then, the form that the Hamiltonian of this system takes is also (5). This means that there will also exist a dark state in the system that only populates states \(|\tilde{A}\rangle\) and \(|\tilde{C}\rangle\), and that by applying the same pulse scheme as in the STIRAP case, one can transfer the state of the system from \(|\tilde{A}\rangle\) (one electron in \(|B\rangle\) and one in \(|C\rangle\)) to \(|\tilde{C}\rangle\) (one electron in \(|A\rangle\) and one in \(|B\rangle\)).

### 4. Coherent control of defects

We now present our scheme to transfer holes in microtrap arrays. The system under investigation is sketched in figure 4.c and it consists of a 2D array of optical traps that has been faultily loaded with a quantum degenerate Fermi gas, i.e., presenting some defects such as empty sites. This paper’s main goal is to develop an efficient and robust method to coherently manipulate and transport these empty sites, by applying a CTAP technique for each defect. For this purpose, we will assume the ability to approach columns or, alternatively, rows of traps [3], such that tunneling between neighboring traps can take place.
4.1. Hole CTAP

A basic idea of our proposal can be seen as follows. Imagine the same three level potential as in TLAO, but now with two atoms instead of one: one in the middle trap and one in the right trap (figure 4.a). This means there is a atom hole in the left trap (figure 4.b). Could it be possible to adiabatically transfer this hole from the left trap to the right one, in a similar manner as Eckert et al (section 3.2) did for an atom? Analogously to the hole STIRAP case (section 3.3), we need to strongly suppress the possibility of the two atoms being in the same trap.

![Figure 4](image.png)

**Figure 4.** Three Level Atom Optics for a hole in the a) atom picture and the b) hole picture. c) Sketch of our system.

One way to do so is by taking advantage of the Pauli exclusion principle as in the previous example. If the spatial wavefunction of the atoms is antisymmetric (fermions with a symmetric spin wavefunction – bosons with an antisymmetric spin wavefunction) then it will be forbidden for them to be in the same trap.

Another way to achieve the hole transfer is by adding a strong interaction between the two atoms that would make the energy of the atoms being in the same trap higher. Since we are adiabatically following a level with energy zero, we will not populate states with higher energy.

Then again, to perform CTAP for the hole (move it from the left trap to the right one) we must perform the same trap movements we used for a single particle: first approaching the middle and right traps and then approaching the left and middle ones, with a delay between the two approaches, again as in figure 2.b.

4.2. Fermionic Hubbard model

We will now generalize the previous example for transporting a hole in a three wells/two atoms system to a n wells/n−1 fermionic atoms system (with n odd).

We focus on a single empty site in a microtrap array and consider a truncated row (or column) that contains this empty site in one of its extremes (see figure 4.c), therefore dealing with a 1D array of n−1 fermions in n sites. The goal is to efficiently move this empty site to the other extreme of the array, without heating the system. The dynamics of this 1D system will be governed by the following Hamiltonian:

$$\hat{H} = \int \hat{\Psi}^\dagger (x) \left( -\frac{\hbar^2}{2m} \nabla^2 + V(x, t) \right) \hat{\Psi}(x) \, dx$$

(13)

where $\hat{\Psi}^\dagger (x)$ ($\hat{\Psi}(x)$) is the creation (annihilation) fermionic field operator and $V(x, t)$ the truncated 1D periodic optical potential whose shape can be controlled over the whole process by changing the distance between traps.

When the potential is deep and the temperature is low enough we can assume a situation in which only the lowest band is occupied. We can then expand our fermionic
Coherent Transport of Holes in Microtrap Arrays

field operators in terms of Wannier functions in a tight binding approximation [13] \( \tilde{\Psi}(x) = \sum c_i \phi_i(x) \). This expansion allows us to speak in terms of the operators \( \hat{c}_i \) and \( \hat{c}_i^\dagger \) that are the fermionic creation and annihilation operators at the different sites denoted by the index \( i \). These operators satisfy the usual fermionic anticommutation relations \( \{ \hat{c}_i, \hat{c}_j^\dagger \} = \delta_{ij} \), and \( \{ \hat{c}_i, \hat{c}_j \} = \{ \hat{c}_i^\dagger, \hat{c}_j^\dagger \} = 0 \). Thus Hamiltonian (13) reads, in the Wannier expansion:

\[
\hat{H} = -\sum_i J_i \left( \hat{c}_i^\dagger \hat{c}_{i+1} + \hat{c}_{i+1}^\dagger \hat{c}_i \right) + \sum_i \mu_i \hat{c}_i^\dagger \hat{c}_i
\]

with coefficients \( J_i \) and \( \mu_i \) defined in terms of overlap integrals of the Wannier functions:

\[
J_i = \int \phi_i^* \left( -\frac{\hbar^2}{2m} \nabla^2 + V(x,t) \right) \phi_{i+1} dx
\]

\[
\mu_i = \int \phi_i^* \left( -\frac{\hbar^2}{2m} \nabla^2 + V(x,t) \right) \phi_i dx
\]

The first term is the hopping term of a Hubbard type of Hamiltonian, that describes tunneling of atoms from one site to another. Note that since the hopping decreases with distance, we restricted it to neighboring sites. The second part describes the on-site energy, and since we are considering just a single atom per site, we can shift the ground state energy and then drop it from the Hamiltonian.

From now on, as in solid state physics, we will consider a hole as a virtual particle, and describe the system in its terms. In this context, the vacuum state for the hole (each trap has an atom in it), will be \( |\tilde{\Omega}\rangle \equiv \hat{c}_1^\dagger \hat{c}_2^\dagger \ldots \hat{c}_n^\dagger |\Omega\rangle \) with \( |\Omega\rangle \) the fermionic vacuum state. Since transitions to excited states are not allowed and we are considering \( n \) traps with \( n - 1 \) atoms, the dynamics of our system will be constrained to remain in the \( n \)-dim Hilbert space spanned by \{\( \hat{C}_i^\dagger |\tilde{\Omega}\rangle \), with \( \hat{C}_i^\dagger = \hat{c}_i \) being the hole creation operator at site \( i \). Then, in terms of the on-site hole operators, Hamiltonian (14) (dropping the last term) reads:

\[
\hat{H} = -\sum_i J_i \left( \hat{C}_{i+1}^\dagger \hat{C}_i + \hat{C}_i^\dagger \hat{C}_{i+1} \right)
\]

For \( n \) odd, it is simple to check that the zero-energy eigenstate of the system is

\[
|\tilde{D}\rangle = \sum_{m=1}^{n-1} (-1)^{m+1} \prod_{j=1}^{m-1} J_{2m-2j-1} \left( \prod_{j=0}^{n-1-m} J_{2m+2j} \right) \hat{C}_{2m-1}^\dagger |\tilde{\Omega}\rangle
\]

Note that \( |\tilde{D}\rangle \) not only involves the first and last trap, but also \( \hat{C}_i^\dagger |\tilde{\Omega}\rangle \) with \( i \) odd.

From these results and given the ability to control the trap distances, we can expect to coherently “move” all the fermions so we transfer the hole from the first to the last trap of our row. To do so we assume that we are varying the tunneling elements, \( J_i \), and that transitions to first excited vibrational levels are not allowed.

More specifically, by adiabatically varying the shape of the tunneling interaction we can move the dark state from \( \hat{C}_1^\dagger |\tilde{\Omega}\rangle \) to \( \hat{C}_1^\dagger |\tilde{\Omega}\rangle \). This corresponds to the scheme shown in figures 2.b and 2.c., but now the tunneling elements and the distances are classified as either odd or even (the interaction/distance between the first and second trap is odd, for the second and third trap even, and so forth): the odd approaching sequences are superimposed, and delayed in time with respect to the even ones.
4.3. Bose–Hubbard model

In the previous treatment, we have investigated the hole dynamics in a fermionic system. However, the previous idea can also be applied to strongly interacting (“hard-core”) bosons. In the limit where bosons have an infinite on-site repulsion energy, only the states with zero or one particle per site are permitted. By mapping the Bose-Hubbard model to a XX-model and then applying the Jordan-Wigner transformation, it is possible to show that the Hamiltonian for the boson system is equivalent to Hamiltonian (14). Therefore the same physics are bound to happen in both systems. This is to be explained precisely elsewhere.

5. Numeric simulations

In what follows we will take into account the full Hamiltonian through a numerical integration of the Schrödinger equation to simulate the dynamics of two neutral atoms in the three-trap potential. The Schrödinger equation of the system reads:

\[
\frac{i \hbar}{\partial t} \psi(x_1, x_2, t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(x_1) + V(x_2) \right] \psi(x_1, x_2, t) \tag{19}
\]

with the piecewise harmonic wells potential defined in equation (8).

We present simulations for a fermionic system, although as we mentioned above the method also works for hardcore bosons (adding a contact potential term in equation (19)). The results in such a system are similar to the ones presented here.

For fermions, the states with the hole in the left, right and middle traps are, respectively:

\[
\begin{align*}
|\tilde{1}\rangle & = \frac{1}{\sqrt{2}} (|2\rangle_1|3\rangle_2 - |3\rangle_1|2\rangle_2) \\
|\tilde{2}\rangle & = \frac{1}{\sqrt{2}} (|3\rangle_1|1\rangle_2 - |1\rangle_1|3\rangle_2) \\
|\tilde{3}\rangle & = \frac{1}{\sqrt{2}} (|1\rangle_1|2\rangle_2 - |2\rangle_1|1\rangle_2)
\end{align*}
\]  

being \(|k\rangle_j\) the state of atom \(j\) in the ground state of trap \(k\). When the traps are far apart, these states are gaussians at the center of each trap.

We start with the atoms in state \(|\tilde{1}\rangle\) (see figure 4.a), obtained by imaginary time evolution, and then perform the system evolution by integrating the time dependent Schrödinger equation (19), changing the trap distances over time as shown in figure 2.b. From figure 5.a, where we have plotted the population of \(|\tilde{1}\rangle, |\tilde{2}\rangle\) and \(|\tilde{3}\rangle\), during the evolution is obvious that the CTAP process for the hole is taking place transferring the system state from \(|\tilde{1}\rangle\) to \(|\tilde{3}\rangle\), barely populating \(|\tilde{2}\rangle\). Figure 5.b shows the fidelity of the process for different evolutions varying the minimum distance between the traps and the delay time between the two approaches. One can see that the process is very robust, since there is a large range of parameters for which the process takes place with high fidelity.

Figure 6 shows snapshots of the joint probability distribution (modulus square of the wavefunction) in the configuration space at different times during the simulation of the hole CTAP process. The graphs are shown in the \(x_1-x_2\) space, so each point of the plane means a position for both particles. Therefore the two bright spots at the initial time are due to the antisymmetrization of the wavefunction: one spot for particle 1 in the middle trap \((x_1 = 0)\), particle 2 in the right trap \((x_2 = 9\alpha^{-1})\) and vice versa for the other spot.

There are a few things we would like to emphasize about this evolution. First of all, the diagonal \(x_1 = x_2\) is forbidden because of the antisymmetrization of the
Figure 5. a) Population of the states $|\tilde{1}\rangle$ (red–solid), $|\tilde{2}\rangle$ (blue–dashed) and $|\tilde{3}\rangle$ (light blue–dotted) during the hole CTAP process for the trap approach sequence shown in figure 2.b. b) Fidelity of the CTAP process for different values of the minimum distance between the traps ($a_{\text{min}}$) and delay times between the two approaches ($t_{\text{delay}}$), showing the robustness of the process.

wavefunction. Secondly, the counter-diagonal $x_1 = -x_2$ is almost not populated. That is because the process is done in a STIRAP-like fashion, forbidding the hole to be in the middle trap. If the hole could be in the middle the atoms would then be in the outermost traps, populating the counter-diagonal. But since the atoms start in the upper and left regions of the configuration space and end in the lower and right regions, how do they manage to cross the forbidden counterdiagonal? We will look at the Bohmian trajectories of the system to answer that question.

Figure 6. Snapshots of the modulus square of the wavefunction in configuration space during the hole CTAP process at times $t = 0, 180, 240, 300, 360, 420, 480, 720 \omega^{-1}$

From the time evolution shown in figures 5 and 6 we have computed some Bohmian trajectories of the system, with their initial positions distributed on the
region where initially $|\varphi(x_1,x_2)|^2$ is larger than zero. Figure 7.a shows those trajectories in the configuration space, i.e. in the plane $x_1-x_2$. The trajectories, as one would expect, follow the wavefunction, starting in the top and right regions of the plane (where the initial wavefunction is, figure 6.a) and ending in the bottom and left regions (figure 6.h).

In the center of the time evolution, when the hole really transfers from the left trap to the right trap, the trajectories get really fast because of the quasinode in the wavefunction over the counterdiagonal. Since the population between the zones corresponding to $|1\rangle$ and $|3\rangle$ is really small, the trajectories must cross it at high speed, similarly to a classical fluid passing through a narrow conduct.

6. Conclusions and Outlook

We have presented a method to coherently transport holes in microtrap arrays. This method is robust since it comes from an extension of STIRAP.

Apart from CTAP (a STIRAP-like process) we can also simulate other TLO processes like Coherent Population Trapping, where we would place the hole in a superposition between the first and last traps, or Electromagnetically Induced Transparency, where we would inhibit the transfer of the hole.

Furthermore, our method can also be used to create a single atom diode or a single atom transistor. Since the behaviour of the system will be different depending on the spin wavefunction of our atoms, we can use this in our advantage to control whether some transport can take be allowed or forbidden.

We have used the de Broglie–Bohm theory to clarify some aspects of how does this transport take place. Finally, we would also like to emphasize the value of alternative interpretations of quantum mechanics, such as de Broglie–Bohm formalism in bringing out certain aspects of the theory that may not be apparent enough in the standard interpretation.

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References