Chapter 7

Analytical results and evaluation

7.1 Results presentation and analysis

7.1.1 Solution evaluation

The most straightforward way to understand and evaluate eq.(6.96) is to plot it. It can be seen that for \( z = z_p \) the excess pore pressure is always zero because the \( \cos(n\pi/2) = 0 \) which satisfies the boundary condition (6.72). The boundary condition (6.73) is also satisfied. It can be checked differentiating eq.(6.96) with respect to \( z \), a factor \( \sin(...)z \) appears and it is always zero for \( z = 0 \). To check the initial condition it is more difficult. However, looking into eq.(6.96) with more detail, the first constant factor \( \frac{1}{n^2}(\frac{u_0}{n} - \frac{u_1}{n}) \) should strike the reader’s attention. It is certainly derived from the approximation of the initial condition as a difference of a linear and a sinusoidal functions. Verruijt [28] solved a consolidation problem with the same boundary conditions but the initial condition stated constant excess pore pressure all over the domain at \( t = 0 \). For large time values he
simplified it:

\[ \frac{c_v t}{h^2} \gg 0.1 \quad \bar{u}(\bar{u}_0) \approx \frac{4}{\pi} \cos \left( \frac{\pi z}{2h} \right) \exp \left( - \frac{\pi^2 c_v t}{4 h^2} \right) \]  

which is exactly as the derived eq.(6.96) with the exception of the constant factor \(\frac{4}{\pi}\). Hence, the initial condition is the key element determining this constant value. For Verruijt’s [28] solution one can see that instantly when \(t\) and \(z\) equal 0, the cosine and exponential terms equal 1 and \(\bar{u}(0,0) = \frac{4}{\pi} \bar{u}_0\). It is characteristic of the coupled consolidation solutions to present an instantly increase in the excess pore pressure at the start of the process. This is the Mandel-Cryer effect. However, in our case it is the opposite:

\[ \bar{u}(0,0) = \frac{4}{\pi} \left( \frac{\bar{u}_0}{\pi} - \frac{u_{1c}}{3} \right) \]  

which is notably smaller than \(u_0\). It seems instead of an instantly increase the derived solution presents and instantly decrease in the value of the pore pressure. It may be explained looking back at the initial condition. The approximation itself, linear minus sinusoidal, is not bad, it could even be considered good enough. The problem is the combination with boundary condition (6.73). Eq.(6.73) states there is no flow in \(z = 0\) which is a correct way of indicating the presence of an impermeable boundary. Nevertheless, the initial condition seems to contradict it as the linear distribution leaves an area in the shape of a triangle which means that large gradients exist around \(z = 0\). This somehow inconsistency is the reason of the awkward initial value of the problem, as if eq.(6.96) could not account for the initial large gradients. The boundary and initial conditions are true (obviously the steel pile is impermeable and the maximum initial excess water pressure will be found at its contact with the soil), the complication is that of combining the non-linear pressure distribution with the boundary conditions. This is due to the fact that this model is only static and, neglecting the dynamics of the problem, this handicap is introduced. Eq.(6.96) is an exact analytical solution to an analytical problem and this process is correct; but it can not be considered an exact analytical solution to the real problem.

### 7.1.2 Derived results

The analytical solution for the consolidation equation can be plotted without taking into consideration the first constant factor, that was discussed on the previously, and the graphical solution is in accordance to available ones in the literature. It is plotted against \(T_v\). If the coefficient of consolidation is estimated, the same plot could be done in function of time in absolute terms. The coefficient was estimated in the previous chapter to corroborate the hypothesis of partial drainage, so it could be easily done. Still it was a very rough estimation and besides the initial consolidation time is unknown, for these reasons the plots are left in function of \(T_v\). It would show that, due to the high permeability of the sand, combined with the relatively long duration of the pseudostatic test, the dissipation is completed, or almost completed (degree of consolidation almost 1) when the application of the load finishes. This supports the idea derived from the experimental tests that the loading process is not fully undrained. Besides, the excess maximum pore pressure calculated for fully undrained case was \(5.2 MPa\), far more larger than the one recorded in the tests. Therefore no there was no effect of the generated excess pore pressures in the bearing capacity.

The solution can also be presented graphically by plotting the variation of excess pore pressure with depth at certain times; the resulting family of curves are called **isochrones**. They start almost perpendicular to the x-axis for \(z = 0\), at the surface of the cone, indicating there is an impermeable boundary there, and converge at \(z = 12 cm\), the end of the plastified area, where the excess pore pressure is always zero as indicated by the fully drained boundary condition. The gradient of an isochrone is related to the hydraulic
gradient by:
\[ \frac{\partial \bar{u}}{\partial z} = -\gamma w \]  
(7.3)

and from Darcy’s law, the seepage velocity is:
\[ V = -\frac{k}{\gamma w} \frac{\partial \bar{u}}{\partial z} \]  
(7.4)

At the surface of the cone there is no seepage flow, because the boundary was defined to be impermeable. But just a little deeper into the soil, for depths as 2cm, the gradient for small values of time increases abruptly and remains kind of constant for the rest of the stratum, once more explaining the large flow that occur. Isochrones normally show increasing gradients, thus increasing seepage velocities, towards the drain, in this case, because of the way the initial condition was defined, large gradients also occur close to the impermeable boundary.

The shape of the isochrones, even for \( Tv = 0 \) displays an interesting feature: immediately when consolidation starts even the top of the cone starts to drain. In 1D consolidation, it is common for isochrones properties in general to find that for small times (i.e. \( t = Tv = 0 \)) consolidation is limited to a certain depth of the layer while the for the rest pore pressures have not yet start to fall. In this cases, a critical time \( t_c \) can be defined when excess pore pressures start to decrease also at the other boundary, the non-draining one. However it seems in this case the solution starts directly at this critical time and the whole stratum drains from the beginning. This can be corroborated looking at the dissipation plot. Even for \( z = 0 \), at the top, pore pressures start to decrease as soon as consolidation begins. This large gradients can explain why the excess pore pressures dissipate so fast and do not affect pile’s bearing capacity. The figure of the dissipation also shows that the initial value at the top of the cone according to the derived consolidation solution is not the 5.2 MPa that had been estimated, \( \bar{u}(0, 0) \neq u_0 \). This feature of the solution was previously discussed.
7.2 Model evaluation

Despite all the limitations and assumptions that were made in the definition of the analytical model, it has been useful to corroborate the conclusions of the experimental tests. The amount of simplifications made much limits the application of the model as a predicting tool or quantitative evaluation. However, it was designed to check the experimental results, not to predict them, and, qualitatively, this has succeeded. It proves useful to achieve better scientific insight into the problem and further understand the conclusions that had been derived in the first part and why they have been so. Some questions, though, remain unanswered, among them:

- Lack of quality data from the soil properties ⇒ the coefficient of consolidation \( c_v \) can be only roughly estimated. It is decisive to determine the degree of drainage/undrainage of the problem. Right now one can only state that it is a case of ‘partial drainage’, nothing else.

- It remains unknown when drainage starts. From the experimental plots it was argued that there was a time lapse between the maximum pore pressure recorded and the maximum bearing capacity. It is indeed a problem in time, more analytical detail and understanding should be obtained.

- Dynamics, even if it is only low-frequency, occur. It would be interesting to incorporate them to the model. Information about inertia and damping should be
Figure 7.4: Isochrones contrasted with the static approach.

### 7.3 Conclusions

The analytical results support the experimental ones in the extent that:

- The loading process is not fully undrained.

- Consolidation during loading can explain the fact that excess pore pressures experimentally occur but they do not affect ultimate pile bearing capacity.

- No dynamics were taken into consideration in the analytical model. This could be the explanation for the awkward first term in the consolidation equation. Therefore, although the obtained conclusions do not differ from those extracted from the experimental tests, dynamics should be considered into the model.

- The analytical model can not be used to predict or estimate the behavior of a pile test. It is only static and it has been seen the strong limitation this assumption introduced in the consolidation results. The model considers only the soil under the pile tip, if the bearing capacity wants to be estimated one need to account for the shaft resistance of the pile and the surrounding soil too. Yet, it is a good approach to represent the experiments carried out and gain understanding.
Figure 7.5: Dissipation of excess pore pressures
Part III

Numerical modeling